MATCHING
SUPPLY WITH DEMAND



An Introduction to
Operations Management





Third Edition



CACHON | TERWIESCH

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MATCHING SUPPLY WITH DEMAND: AN INTRODUCTION TO OPERATIONS MANAGEMENT, THIRD EDITION

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To the teachers, colleagues, and professionals who shared with us their knowledge.

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About the Authors

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Professor Cachon is the Fred R. Sullivan Professor of Operations and Information Management at The Wharton School of the University of Pennsylvania, where he teaches a variety of undergraduate, MBA, executive, and Ph.D. courses in operations management. His research focuses on supply chain management, and operations strategy; in particular, how new technologies enhance and transform competitiveness via operations. He is a Distinguished Fellow and past president of the Manufacturing and Service Operations Society. His articles have appeared in *Harvard Business Review, Management Science, Marketing Science, Manufacturing & Service Operations Management,* and *Operations Research.* He has been on the editorial review board of five leading journals in operations management, is a former editor of *Manufacturing & Service Operations Management,* and is the current editor of *Management Science.* He has consulted with a wide range of companies, including 4R Systems, Ahold, Americold, Campbell Soup, Gulfstream Aerospace, IBM, Medtronic, and O'Neill.

Before joining The Wharton School in July 2000, Professor Cachon was on the faculty at the Fuqua School of Business, Duke University. He received a Ph.D. from The Wharton School in 1995.

He is an avid proponent of bicycle commuting (and other environmentally friendly modes of transportation). Along with his wife and four children he enjoys hiking, skiing, fishing, snorkeling, and scuba diving.

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Professor Terwiesch is the Andrew M. Heller Professor at the Wharton School of the University of Pennsylvania. He also is a professor in Wharton's Operations and Information Management Department as well as a Senior Fellow at the Leonard Davis Institute for Health Economics. His research on operations management, research and development, and innovation management appears in many of the leading academic journals, including *Management Science, Operations Research, Marketing Science, and Organization Science.* He has received numerous teaching awards for his courses in Wharton's MBA and executive education programs.

Professor Terwiesch has researched with and consulted for various organizations, including a project on concurrent engineering for BMW, supply chain management for Intel and Medtronic, and product customization for Dell. Most of his current work relates to health care and innovation management. In the health care arena, some of Professor Terwiesch's recent projects include the analysis of capacity allocation for cardiac surgery procedures at the University of California–San Francisco and at Penn Medicine, the impact of emergency room crowding on hospital capacity and revenues (also at Penn Medicine), and the usage of intensive care beds in the Children's Hospital of Philadelphia. In the innovation area, recent projects include the management of the clinical development portfolio at Merck, the development of open innovation systems, and the design of patient-centered care processes in the Veterans Administration hospital system.

Professor Terwiesch's latest book, *Innovation Tournaments,* outlines a novel, process-based approach to innovation management. The book was featured by *BusinessWeek, the Financial Times,* and the *Sloan Management Review.*

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Along with our students, we would like to thank our co-teachers in the core: Naren Agrawal, Krishnan Anand, Omar Besbes, Morris Cohen, Marshall Fisher, Richard Lai, Chris Lee, Pranab Majumder, Serguei Netessine, Kathy Pearson, Taylor Randall, Nicolas Reinecke, Daniel Snow, Stephan Spinler, Anita Tucker, Karl Ulrich, Senthil Veeraraghavan, and Yu-Sheng Zheng. In addition to useful pedagogical advice and quality testing, they shared many of their own practice problems and questions.

This book is not the first book in Operations Management, nor will it be the last. We hope we have incorporated the best practices of existing books while introducing our own innovations. The book by Anupindi et al. as well as the article by Harrison and Loch were very helpful to us, as they developed the process view of operations underlying Chapters 2 through 8. The book by Chase and Aquilano was especially useful for Chapter 10. We apply definitions and terminology from those sources whenever possible without sacrificing our guiding principles.

We also have received some indirect and direct assistance from faculty at other universities. Garrett van Ryzin's (Columbia) and Xavier de Groote's (INSEAD) inventory notes were influential in the writing of Chapters 2 and 14, and the revenue management note by Serguei Netessine (Wharton) and Rob Shumsky (Dartmouth) was the starting point for Chapter 16. The process analysis, queuing, and inventory notes and articles written by Martin Lariviere (Northwestern), Michael Harrison (Stanford), and Christoph Loch (INSEAD) were also influential in several of our chapters. Martin, being a particularly clever question designer, was kind enough to share many of his questions with us.

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The many cases and practical examples that illustrate the core concepts of this book reflect our extensive collaboration with several companies, including the University of Pennsylvania Hospital System in the Philadelphia region, the Circored plant in Trinidad, the Xootr factory in New Hampshire, the An-ser call center in Wisconsin, the operations group at O'Neill in California, and the supply chain group at Medtronic in Minnesota. We have benefited from countless visits and meetings with their management teams. We thank the people of these organizations, whose role it is to match supply and demand in the "real world," for sharing their knowledge, listening to our ideas, and challenging our models. Special thanks go to Jeff Salomon and his team (Interventional Radiology), Karl Ulrich (Xootr), Allan Fromm (An-ser), Cherry Chu and John Pope (O'Neill), and Frederic Marie and John Grossman (Medtronic). Allan Fromm deserves extra credit, as he was not only willing to share with us his extensive knowledge of service operations that he gathered as a CEO of a call center company but also proofread the entire manuscript and tackled most of the practice problems. Special thanks also to the McKinsey operations practice, in particular Stephen Doig, John Drew, and Nicolas Reinecke, for sharing their practical experience on Lean Operations and the Toyota Production System.

We especially thank our friend, colleague, and cycling partner Karl Ulrich, who has been involved in various aspects of the book, starting from its initial idea to the last details of the design process, including the cover design.

Through each edition of this text we have been supported by a fantastic team at McGraw Hill: Scott Isenberg, Cynthia Douglas, Colin Kelley, Karthryn Mikulic, Dick Hercher, Danielle Andries, and Erin Melloy.

Finally, we thank our family members, some of whom were surely unwilling reviewers who nevertheless performed their family obligation with a cheerful smile.

Gérard Cachon

Christian Terwiesch

Preface

This book represents our view of the essential body of knowledge for an introductory operations management course. It has been successfully used with all types of students, from freshmen taking an introductory course in operations management, to MBAs, to executive MBAs, and even PhD students.

Our guiding principle in the development of *Matching Supply with Demand* has been "real operations, real solutions." "Real operations" means that most of the chapters in this book are written from the perspective of a specific company so that the material in this text will come to life by discussing it in a real-world context. Companies and products are simply easier to remember than numbers and equations. We have chosen a wide variety of companies, small and large, representing services, manufacturing, and retailing alike. While obviously not fully representative, we believe that—taken together—these cases provide a realistic picture of operations management problems today.

"Real solutions" means that we do not want equations and models to merely provide students with mathematical gymnastics for the sake of an intellectual exercise. We feel that professional training, even in a rigorous academic setting, requires tools and strategies that students can implement in practice. We achieve this by demonstrating how to apply our models from start to finish in a realistic operational setting. For example, we do not assume the existence of inputs such as a demand forecast or a cost parameter; we actually explain how these inputs can be obtained in practice. Furthermore, we openly address the implementation challenges of each model/strategy we discuss so that students know what to expect when the "rubber hits the pavement."

To fully deliver on "real operations, real solutions," we also must adhere to the principle of "real simple." Do not worry; "real simple" does not mean plenty of "blah-blah" without any analytical rigor. Quite the contrary. To us, "real simple" means hard analysis that is made easy to learn. This is crucial for an operations text. Our objective is to teach business leaders, not tacticians. Thus, we need students to be able to quickly develop a foundation of formal models so that they have the time to explore the big picture, that is, how operations can be transformed to provide an organization with sustainable competitive advantage and/or superior customer service. Students that get bogged down in details, equations, and analysis are not fully capturing the valuable insights they will need in their future career.

So how do we strive for "real simple"? First, we recognize that not every student comes to this material with an engineering/math background. As a result, we tried to use as little mathematical notation as possible, to provide many real-world examples, and to adhere to consistent terminology and phrasing. Second, we provide various levels of detail for each analysis. For example, every little step in an analysis is described in the text via an explicit example; then a summary of the process is provided in a "how to" exhibit, a brief listing of key notation and equations is provided at the end of each chapter, and, finally, solved practice problems are offered to reinforce learning. While we do humbly recognize, given the quantitative sophistication of this text, that "much simpler" might be more accurate than "real simple," we nevertheless hope that students will be pleasantly surprised to discover that their analytical capabilities are even stronger than they imagined.

The initial version of *Matching Supply with Demand* made its debut in portions of the operations management core course at Wharton in the 2002–2003 academic year. This edition incorporates the feedback we have received over the last 10 years from many students, executives, and colleagues, both at Wharton and abroad.

Gérard Cachon

Christian Terwiesch

Changes to This Edition

The third edition has benefited from the comments and suggestions from students, faculty, and practitioners from around the world. The book is now translated into Chinese and Korean, and what once was written as an MBA textbook has been taught to undergraduate students, MBA students, doctoral students, and executives.

The changes that we implemented were substantial, touching almost every chapter of the book. The changes can be broken up into three categories: an update of data and case examples, the addition of three chapters related to content that was not previously covered in the book, and an overall streamlining of the exposition of the existing content.

Many things have happened since we wrote the second edition three years ago. Companies have gone out of business, and new business models were invented. Toyota, the only company that has a chapter dedicated to it in this book, has gone through a major crisis with quality problems in it vehicles. Sadly enough, history also repeated itself: We used the 2007 Japanese earthquake as a motivating example on the first page of the second edition. Now, as we write the third edition, we had to witness the devastating effects of the 2011 earthquake and the effects it had on people and business. To respond to the need to stay current, we have updated data and case examples throughout the book.

We decided to add three new chapters to this book. The first new chapter is about project management—a topic that is taught in many operations courses but was previously absent from the book. The second new chapter is about sustainable operation, a topic of rapidly growing interest in academia and in practice. We also added a chapter on business model innovation. Just like the chapter on lean operations and the Toyota Production System was added as a capstone chapter for the first half of the book in the second edition, for the third edition we wanted to bring together a set of ideas that enable companies to build new business models using the lessons of matching supply with demand. The chapter was fun to write, and we hope it will also be fun to read.

We have seen many textbooks grow thick over multiple editions—nothing is more painful to an author than deleting text he or she wrote before. As much as we were committed to update the content of the book and to add fresh and relevant content, we also wanted to keep the time constraints of our readers in mind. We took some content out of the book (we will make it available on our book website, www.cachon-terwiesch.net) and streamlined the exposition of several tools. It is all about *lean* after all.

Brief Contents

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Chapter 1

Introduction

A central premise in economics is that prices adjust to match supply with demand: if there is excess demand, prices rise; if there is excess supply, prices fall. But while an economist may find comfort with this theory, managers in practice often do not. To them excess demand means lost revenue and excess supply means wasted resources. They fully understand that matching supply with demand is extremely difficult and requires more tools than just price adjustments.

Consider the following examples:

- In 2006, Nintendo launched the Wii game console with much success—so much success that the company could not make enough units to keep up with demand. Some entrepreneurs would wait in long lines to purchase scarce units only to turn around and sell them online for several hundred dollars over the retail price.
- In 2007, Dell lost its worldwide market share leadership to HP. Trying to regain momentum, Dell offered laptop computers to consumers in various colors. Unfortunately, problems with dust contamination in the painting process prevented Dell from ramping up production, causing long delays, which in turn caused some customers to cancel their order.
- At 4 p.m. on weekdays, it is hard to find a taxi in Manhattan because that is when taxis tend to change between shifts. Consequently, customers wait longer for a cab.
- In March 2011, a massive earthquake hit Japan, followed by devastating tsunamis. Supplies for some key automobile and electronic components were unavailable or scarce for months, disrupting production around the globe.
- In 2008, Boeing was unable to deliver on time its new 777s to Emirates Airlines because growth in demand caught its supplier of kitchen galleys off guard and short on capacity.
- In early 2002, a victim of a car crash in Germany died in a rescue helicopter after the medical team together with their dispatcher had unsuccessfully attempted to find a slot in an operating room at eight different hospitals. In the United States, every day there are thousands of patients requiring emergency care who cannot be transported to the nearest emergency room and/or have to wait considerable time before receiving care.
- The average customer to Disney World experiences only nine rides per day, in part because of long queues. To give customers a better experience (read, "more rides"), Disney developed several mechanisms to encourage customers to find rides with short or no queues.

All of these cases have in common that they suffer from a mismatch between demand and supply, with respect either to their timing or to their quantities.

This book is about how firms can design their operations to better match supply with demand. Our motivation is simply stated: By better matching supply with demand, a firm gains a significant competitive advantage over its rivals. A firm can achieve this better

match through the implementation of the rigorous models and the operational strategies we outline in this book.

To somewhat soften our challenge to economic theory, we do acknowledge it is possible to mitigate demand–supply mismatches by adjusting prices. For example, the effective market price of the Wii game console did rise due to the strong demand. But this price adjustment was neither under Nintendo's control, nor did Nintendo (or its retailers) collect the extra surplus. In other words, we view that price adjustment as a symptom of a problem, rather than evidence of a healthy system. Moreover, in many other cases, price adjustments are impossible. The time period between the initiation of demand and the fulfillment through supply is too short or there are too few buyers and sellers in the market. There simply is no market for emergency care in operating rooms, waiting times in call centers, or piston rings immediately after an earthquake.

Why is matching supply with demand difficult? The short answer is that demand can vary, in either predictable or unpredictable ways, and supply is inflexible. On average, an organization might have the correct amount of resources (people, product, and/or equipment), but most organizations find themselves frequently in situations with resources in the wrong place, at the wrong time, and/or in the wrong quantity. Furthermore, shifting resources across locations or time is costly, hence the inflexibility in supply. For example, physicians are not willing to rush back and forth to the hospital as they are needed and retailers cannot afford to immediately move product from one location to another. While it is essentially impossible to always achieve a perfect match between supply and demand, successful firms continually strive for that goal.

Table 1.1 provides a sample of industries that we will discuss in this book and describes their challenge to match supply with demand. Take the airline industry (last column in Table 1.1.). For fiscal year 2007, British Airways achieved a 76.1 percent utilization; that is, a 160-seat aircraft (the average size in their fleet) had, on average, 122 seats occupied with a paying passenger and 38 seats flying empty. If British Airways could have had four more (paying) passengers on each flight, that is, increase its utilization by about 2.5 percent, its corporate profits would have increased by close to £242 million, which is about 44 percent of its operating profit for 2007. This illustrates a critical lesson: Even a seemingly small

TABLE 1.1 Examples of Supply–Demand Mismatches

	Retailing	Iron Ore Plant	Emergency Room	Pacemakers	Air Travel
Supply	Consumer electronics	Iron ore	Medical service	Medical equipment	Seats on specific flight
Demand	Consumers buying a new video system	Steel mills	Urgent need for medical service	Heart surgeon requiring pacemaker at exact time and location	Travel for specific time and destination
Supply exceeds demand	High inventory costs; few inventory turns	Prices fall	Doctors, nurses, and infrastructure are underutilized	Pacemaker sits in inventory	Empty seat
Demand exceeds supply	Forgone profit opportunity; consumer dissatisfaction	Prices rise	Crowding and delays in the ER; potential diversion of ambulances	Forgone profit (typically not associated with medical risk)	Overbooking; customer has to take different flight (profit loss)
Actions to match supply and demand	Forecasting; quick response	If prices fall too low, production facility is shut down	Staffing to predicted demand; priorities	Distribution system holding pacemakers at various locations	Dynamic pricing; booking policies

(*continued*)

TABLE 1.1 Concluded

Managerial importance	Per-unit inventory costs for consumer electronics retailing all too often exceed net profits	Prices are so competitive that the primary emphasis is on reducing the cost of supply	Delays in treatment or transfer have been linked to death	Most products (valued $20k) spend 4–5 months waiting in a trunk of a salesperson before being used	About 30% of all seats fly empty; a 1–2% increase in seat utilization makes the difference between profits and losses
Reference	Chapter 2, The Process View of the Organization; Chapter 12, Betting on Uncertain Demand: The Newsvendor Model; Chapter 13, Assemble-to-Order, Make-to-Order, and Quick Response with Reactive Capacity	Chapter 3, Understanding the Supply Process: Evaluating Process Capacity; Chapter 4, Estimating and Reducing Labor Costs	Chapter 8, Variability and Its Impact on Process Performance: Waiting Time Problems; Chapter 9, The Impact of Variability on Process Performance: Throughput Losses	Chapter 14, Service Levels and Lead Times in Supply Chains: The Order-up-to Inventory Model	Chapter 16, Revenue Management with Capacity Controls

improvement in operations, for example, a utilization increase of 2.5 percent, can have a significant effect on a firm's profitability precisely because, for most firms, their profit (if they have a profit) is a relatively small percentage of their revenue. Hence, improving the match between supply and demand is a critically important responsibility for a firm's management.

The other examples in Table 1.1 are drawn from a wide range of settings: health care delivery and devices, retailing, and heavy industry. Each suffers significant consequences due to demand–supply mismatches, and each requires specialized tools to improve and manage its operations.

To conclude our introduction, we strongly believe that effective operations management is about effectively matching supply with demand. Organizations that take the design of their operations seriously and aggressively implement the tools of operations management will enjoy a significant performance advantage over their competitors. This lesson is especially relevant for senior management given the razor-thin profit margins firms must deal with in modern competitive industries.

1.1 Learning Objectives and Framework

In this book, we look at organizations as entities that must match the supply of what they produce with the demand for their product. In this process, we will introduce a number of quantitative models and qualitative strategies, which we collectively refer to as the "tools of operations management." By "quantitative model" we mean some mathematical procedure or equation that takes inputs (such as a demand forecast, a processing rate, etc.) and outputs a number that either instructs a manager on what to do (how much inventory to buy, how many nurses to have on call, etc.) or informs a manager about a relevant performance measure (e.g., the average time a customer waits for service, the average number of patients in the emergency room, etc.). By "qualitative strategy" we mean a guiding principle: for example, increase the flexibility of your production facilities, decrease the variety of products offered, serve customers in priority order, and so forth. The next section gives a brief description of the key models and strategies we cover. Our learning objective for

this book, put as succinctly as we can, is to teach students how and when to implement the tools of operations management.

Just as the tools of operations management come in different forms, they can be applied in different ways:

1. Operations management tools can be applied to ensure that resources are used as efficiently as possible; that is, the most is achieved with what we have.
2. Operations management tools can be used to make desirable trade-offs between competing objectives.
3. Operations management tools can be used to redesign or restructure our operations so that we can improve performance along multiple dimensions simultaneously.

We view our diverse set of tools as complementary to each other. In other words, our focus is neither exclusively on the quantitative models nor exclusively on the qualitative strategies. Without analytical models, it is difficult to move beyond the "blah-blah" of strategies and without strategies, it is easy to get lost in the minutia of tactical models. Put another way, we have designed this book to provide a rigorous operations management education for a strategic, high-level manager or consultant.

We will apply operations tools to firms that produce services and goods in a variety of environments—from apparel to health care, from call centers to pacemakers, and from kick scooters to iron ore fines. We present many diverse settings precisely because there does not exist a "standard" operational environment. Hence, there does not exist a single tool that applies to all firms. By presenting a variety of tools and explaining their pros and cons, students will gain the capability to apply this knowledge no matter what operational setting they encounter.

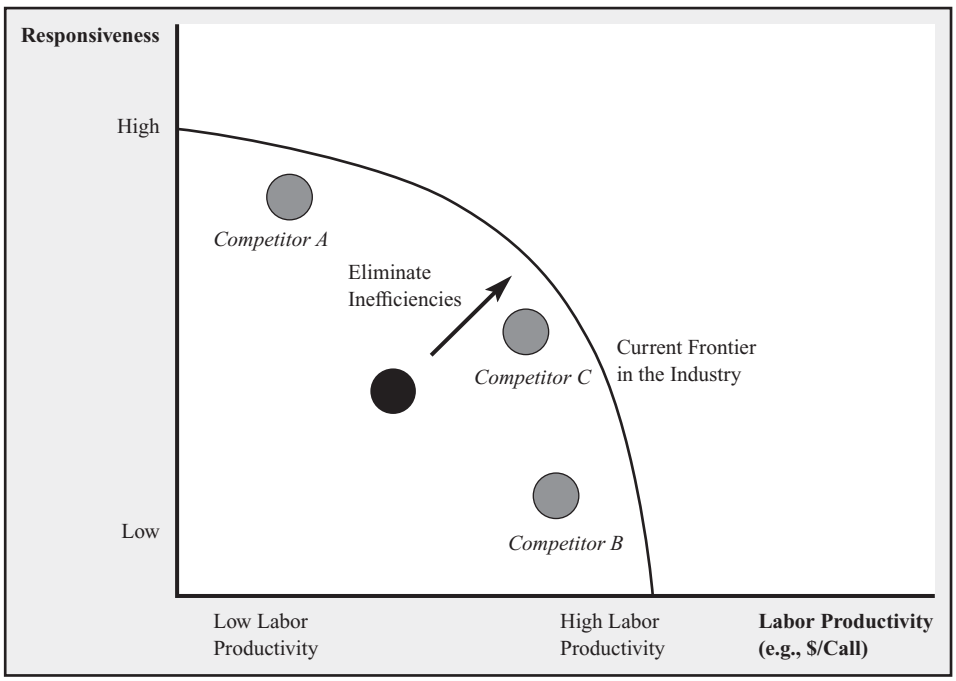
Consider how operations tools can be applied to a call center. A common problem in this industry is to find an appropriate number of customer service representatives to answer incoming calls. The more representatives we hire, the less likely incoming calls will have to wait; thus, the higher will be the level of service we provide. However, labor is the single largest driver of costs in a call center, so, obviously, having more representatives on duty also will increase the costs we incur per call.

The first use of operations management tools is to ensure that resources are used as effectively as possible. Assume we engage in a benchmarking initiative with three other call centers and find that the performance of our competitors behaves according to Figure 1.1: Competitor A is providing faster response times but also has higher costs. Competitor B has longer response times but has lower costs. Surprisingly, we find that competitor C outperforms us on both cost and service level. How can this be?

It must be that there is something that competitor C does in the operation of the call center that is smarter than what we do. Or, in other words, there is something that we do in our operations that is inefficient or wasteful. In this setting, we need to use our tools to move the firm toward the frontier illustrated in Figure 1.1. The frontier is the line that includes all benchmarks to the lower left; that is, no firm is outside the current frontier. For example, a premium service might be an important element of our business strategy, so we may choose not to compromise on service. And we could have a target that at least 90 percent of the incoming calls will be served within 10 seconds or less. But given that target, we should use our quantitative tools to ensure that our labor costs are as low as possible, that is, that we are at least on the efficiency frontier.

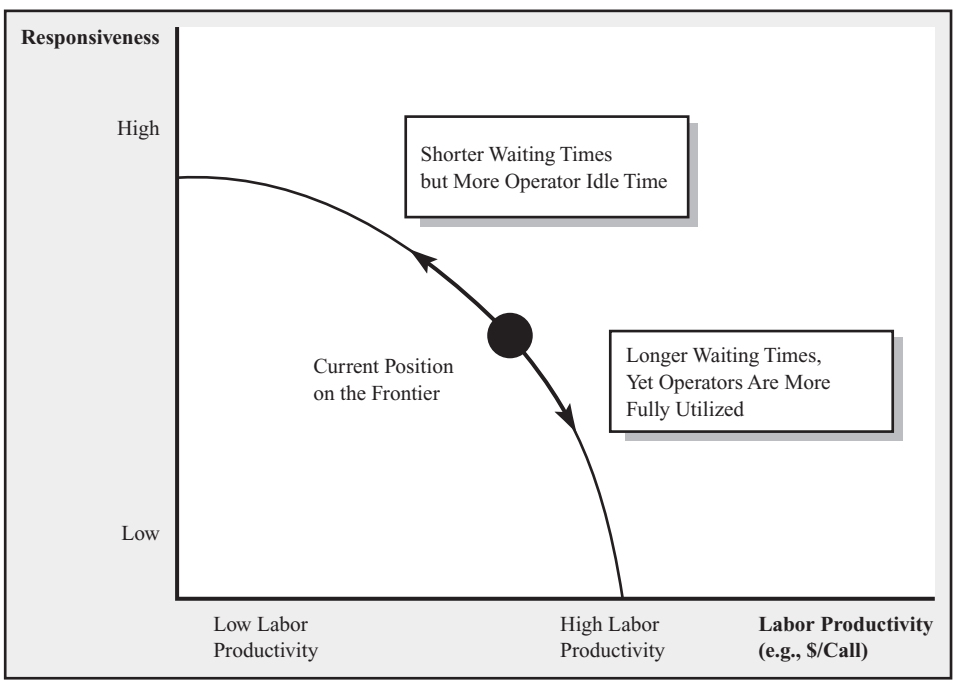
The second use of operations management tools is to find the right balance between our competing objectives, high service and low cost. This is similar to what is shown in Figure 1.2. In such a situation, we need to quantify the costs of waiting as well as the costs of labor and then recommend the most profitable compromise between these two objectives.

FIGURE 1.1
Local Improvement of Operations by Eliminating Inefficiencies



Moving to the frontier of efficiency and finding the right spot on the frontier are surely important. But outstanding companies do not stop there. The third use for our operations management tools is to fundamentally question the design of the current system itself. For example, a call center might consider merging with or acquiring another call center to gain scale economies. Alternatively, a call center might consider an investment in the development of a new technology leading to shorter call durations.

FIGURE 1.2
Trade-off between Labor Productivity and Responsiveness



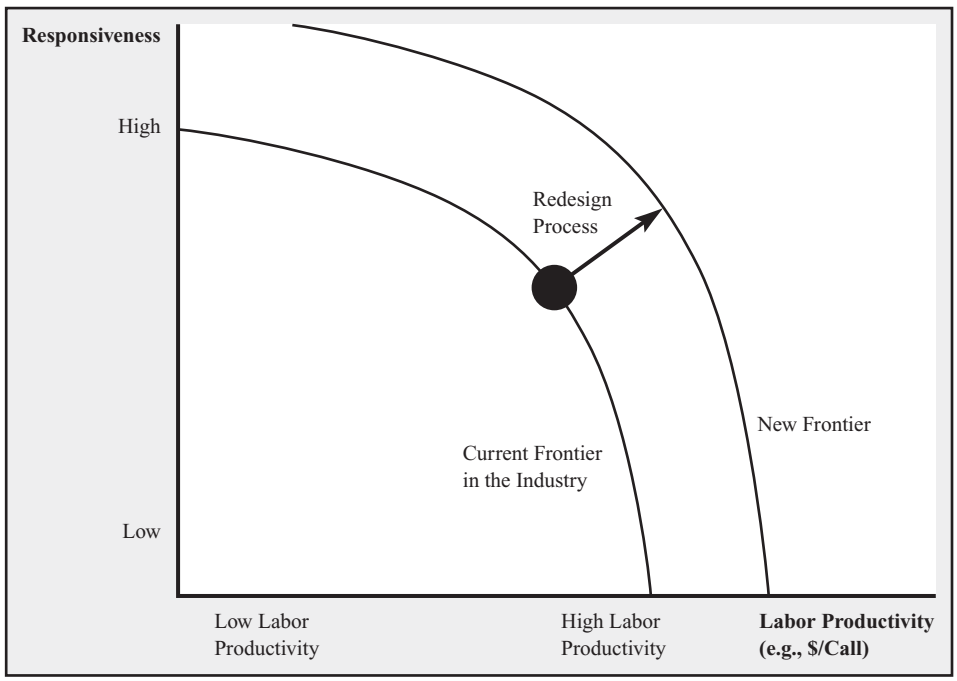


FIGURE 1.3
Redesigning the Process to Operate at an Improved Frontier

In such cases, a firm pushes the envelope, that is, moves the frontier of what previously was feasible (see Figure 1.3). Hence, a firm is able to achieve faster responsiveness and higher labor productivity. But, unfortunately, there are few free lunches: while we have improved both customer service and labor productivity, pushing out the frontier generally requires some investments in time and effort. Hence, we need to use our tools to quantify the improvements we can achieve so that we can decide whether the effort is justifiable. It is easy to tell a firm that investing in technology can lead to shorter call durations, faster service, and higher labor productivity, but is that investment worthwhile? Our objective is to educate managers so that they can provide "big ideas" and can back them up with rigorous analysis.

1.2 Road Map of the Book

This book can be roughly divided into five clusters of closely related chapters.

The first cluster, Chapters 2–7, analyzes business processes (the methods and procedures by which a service is completed or a good is produced). For the most part, the view taken in those chapters is one of process without variability in service times, production times, demand arrival, quality, and so forth. Hence, the objective is to organize the business process to maximize supply given the resources available to the firm.

Chapters 8–11 introduce variability into business process analysis. Issues include the presence of waiting times, lost demand due to poor service, and lost output due to poor quality. This cluster concludes with an overview of the Toyota Production System.

Chapters 12–15 discuss inventory control, information management, and process flexibility. Issues include demand forecasting, stocking quantities, performance measures, product design, and production flexibility.

Chapter 16 departs from a focus on the supply process and turns attention to the demand process. In particular, the chapter covers the tools of revenue management that allow a firm to better match its demand to its fixed supply.

TABLE 1.2
A High-Level Grouping of Chapters

Chapters	Theme
2–7	Process analysis without variability in service times, production rates, demand arrival, quality, etc.
8–11	Process analysis with variability in service times, production rates, demand arrival, quality, etc.
12–15	Inventory control, information management, process flexibility
16	Revenue management
17–19	Strategic operations management: supply chains, sustainability, and business models

Chapters 17–19 conclude the book with several strategic topics, including the management and control of the supply chain, sustainability, and business model innovation.

Table 1.2 summarizes these clusters.

The following provides a more detailed summary of the contents of each chapter:

• Chapter 2 defines a process, introduces the basic process performance metrics, and provides a framework for characterizing processes (the product–process matrix). Little's Law is introduced, an essential formula for understanding business processes and the link between operations management and financial accounting.

• Chapter 3 introduces process analysis tools from the perspective of a manager (as opposed to an engineer): how to determine the capacity of a process and how to compute process utilization.

• Chapter 4 looks at assembly operations with a specific focus on labor costs, an extremely important performance metric. It frequently drives location decisions (consider the current debate related to offshoring) and has—especially in service operations—a major impact on the bottom line. We define measures such as labor content, labor utilization, and idle time. We also introduce the concept of line balancing.

• Chapter 5 investigates project management, a process that is designed for a single, somewhat unique, project such as a ship, a new building, or a satellite.

• Chapter 6 connects the operational details of process analysis with key financial performance measures for a firm, such as return on invested capital. Through this chapter we discover how to make process improvement translate into enhanced financial performance for the organization.

• Chapter 7 studies production in the presence of setup times and setup costs (the EOQ model). A key issue is the impact of product variety on production performance.

• Chapter 8 explores the consequences of variability on a process. As we will discuss in the context of a call center, variability can lead to long customer waiting times and thereby is a key enemy in all service organizations. We discuss how an organization should handle the trade-off between a desire for minimizing the investment into capacity (e.g., customer service representatives) while achieving a good service experience for the customer.

• Chapter 9 continues the discussion of variability and its impact on service quality. As we will discuss in the context of emergency medicine, variability frequently can lead to situations in which demand has to be turned away because of insufficient capacity. This has substantial implications, especially in the health care environment.

• Chapter 10 details the tools of quality management, including statistical process control, six-sigma, and robust design.

• Chapter 11 describes how Toyota, via its world-famous collection of production strategies called the Toyota Production System, achieves high quality and low cost.

• Chapter 12 focuses on the management of seasonal goods with only one supply opportunity. The newsvendor model allows a manager to strike the correct balance between too much supply and too little supply.

• Chapter 13 expands upon the setting of the previous chapter by allowing additional supply to occur in the middle of the selling season. This "reactive capacity" allows a firm to better respond to early season sales information.

• Chapter 14 continues the discussion of inventory management with the introduction of lead times. The order-up-to model is used to choose replenishment quantities that achieve target availability levels (such as an in-stock probability).

• Chapter 15 highlights numerous risk-pooling strategies to improve inventory management within the supply chain: for example, location pooling, product pooling, universal design, delayed differentiation (also known as postponement), and capacity pooling.

• Chapter 16 covers revenue management. In particular, the focus is on the use of booking limits and overbooking to better match demand to supply when supply is fixed.

• Chapter 17 identifies the bullwhip effect as a key issue in the effective operation of a supply chain and offers coordination strategies for firms to improve the performance of their supply chain.

• Chapter 18 applies operations management thinking to the challenge of sustainability.

• Chapter 19 concludes the book with how operations management enables new business models. A framework is presented for understanding business model innovation that can assist in the creation of new business models.

Some of the chapters are designed to be "entry level" chapters, that is, chapters that can be read independently from the rest of the text. Other chapters are more advanced, so they at least require some working knowledge of the material in another chapter. Table 1.3 summarizes the contents of the chapters and indicates prerequisite chapters.

TABLE 1.3 Chapter Summaries and Prerequisites

Chapter	Managerial Issue	Key Qualitative Framework	Key Quantitative Tool	Prerequisite Chapters
2: The Process View of the Organization	Understanding business processes at a high level; process performance measures, inventory, flow time, and flow rate	Product–process matrix; focus on process flows	Little's Law Inventory turns and inventory costs	None
3: Understanding the Supply Process: Evaluating Process Capacity	Understanding the details of a process	Process flow diagram; finding and removing a bottleneck	Computing process capacity and utilization	Chapter 2
4: Estimating and Reducing Labor Costs	Labor costs	Line balancing; division of labor	Computing labor costs, labor utilization Minimizing idle time	Chapters 2, 3
5: Project Management	Time to project completion	Critical path	Critical path analysis	Chapters 2, 3
6: The Link between Operations and Finance	Process improvement to enhance corporate performance	Return on Invested Capital (ROIC) tree	Computing ROIC	Chapters 2, 3
7: Batching and Other Flow Interruptions: Setup Times and the Economic Order Quantity Model	Setup time and setup costs; managing product variety	Achieving a smooth process flow; deciding about setups and ordering frequency	EOQ model Determining batch sizes	Chapters 2, 3

(*continued*)

TABLE 1.3 **Concluded**

Chapter	Managerial Issue	Key Qualitative Framework	Key Quantitative Tool	Prerequisite Chapters
8: Variability and Its Impact on Process Performance: Waiting Time Problems	Waiting times in service processes	Understanding congestion; pooling service capacity	Waiting time formula	None
9: The Impact of Variability on Process Performance: Throughput Losses	Lost demand in service processes	Role of service buffers; pooling	Erlang loss formula Probability of diverting demand	Chapter 8
10: Quality Management, Statistical Process Control, and Six-Sigma Capability	Defining and improving quality	Statistical process control; six-sigma	Computing process capability; creating a control chart	None
11: Lean Operations and the Toyota Production System	Process improvement for competitive advantage	Lean operations; Toyota Production System	—	None
12: Betting on Uncertain Demand: The Newsvendor Model	Choosing stocking levels for seasonal-style goods	Improving the forecasting process	Forecasting demand The newsvendor model for choosing stocking quantities and evaluating performance measures	None
13: Assemble-to-Order, Make-to-Order, and Quick Response with Reactive Capacity	How to use reactive capacity to reduce demand–supply mismatch costs	Value of better demand information; assemble-to-order and make-to-order strategies	Reactive capacity models	Chapter 12
14: Service Levels and Lead Times in Supply Chains: The Order-up-to Inventory Model	Inventory management with numerous replenishments	Impact of lead times on performance; how to choose an appropriate objective function	The order-up-to model for inventory management and performance-measure evaluation	Chapter 12 is highly recommended
15: Risk Pooling Strategies to Reduce and Hedge Uncertainty	How to better design the supply chain or a product or a service to better match supply with demand	Quantifying, reducing, avoiding, and hedging uncertainty	Newsvendor and order-up-to models	Chapters 12 and 14
16: Revenue Management with Capacity Controls	How to manage demand when supply is fixed	Reserving capacity for high-paying customers; accepting more reservations than available capacity	Booking limit/protection level model; overbooking model	Chapter 12
17: Supply Chain Coordination	How to manage demand variability and inventory across the supply chain	Bullwhip effect; supply chain contracts	Supply chain contract model	Chapter 12
18: Sustainability	How to employ operations management techniques to a sustainability initiative	Measuring resource use and emissions	—	None
19: Business Model Innovation	How to create a business model innovation	Customer value curve and supply process transformation	—	None

Chapter 2

The Process View of the Organization

Matching supply and demand would be easy if business processes would be instantaneous and could immediately create any amount of supply to meet demand. Understanding the questions of "Why are business processes not instantaneous?" and "What constrains processes from creating more supply?" is thereby at the heart of operations management. To answer these questions, we need to take a detailed look at how business processes actually work. In this chapter, we introduce some concepts fundamental to process analysis. The key idea of the chapter is that it is not sufficient for a firm to create great products and services; the firm also must design and improve its business processes that supply its products and services.

To get more familiar with the process view of a firm, we now take a detailed look behind the scenes of a particular operation, namely the Department of Interventional Radiology at Presbyterian Hospital in Philadelphia.

2.1 Presbyterian Hospital in Philadelphia

Interventional radiology is a subspecialty field of radiology that uses advanced imaging techniques such as real-time X-rays, ultrasound, computed tomography, and magnetic resonance imaging to perform minimally invasive procedures.

Over the past decade, interventional radiology procedures have begun to replace an increasing number of standard "open surgical procedures" for a number of reasons. Instead of being performed in an operating room, interventional radiology procedures are performed in an angiography suite (see Figure 2.1). Although highly specialized, these rooms are less expensive to operate than conventional operating rooms. Interventional procedures are often safer and have dramatically shorter recovery times compared to traditional surgery. Also, an interventional radiologist is often able to treat diseases such as advanced liver cancer that cannot be helped by standard surgery.

Although we may not have been in the interventional radiology unit, many, if not most, of us have been in a radiology department of a hospital at some point in our life. From the perspective of the patient, the following steps need to take place before the patient can go home or return to his or her hospital unit. In process analysis, we refer to these steps as *activities*:

- Registration of the patient.
- Initial consultation with a doctor; signature of the consent form.

FIGURE 2.1
Example of a Procedure in an Interventional Radiology Unit

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- Preparation for the procedure.
- The actual procedure.
- Removal of all equipment.
- Recovery in an area outside the angiography suite.
- Consultation with the doctor.

Figure 2.2 includes a graphical representation of these steps, called a *Gantt diagram* (named after the 19th-century industrialist Henry Gantt). It provides several useful pieces of information.

First, the Gantt chart allows us to see the process steps and their durations, which are also called *activity times* or *processing times.* The duration simply corresponds to the length of the corresponding bars. Second, the Gantt diagram also illustrates the dependence between the various process activities. For example, the consultation with the doctor can only occur once the patient has arrived and been registered. In contrast, the preparation of the angiography suite can proceed in parallel to the initial consultation.

You might have come across Gantt charts in the context of project management. Unlike process analysis, project management is typically concerned with the completion of one single project (See Chapter 5 for more details on project management.) The most well-known concept of project management is the *critical path.* The critical path is composed of all those activities that—if delayed—would lead to a delay in the overall completion time of the project, or—in this case—the time the patient has completed his or her stay in the radiology unit.

In addition to the eight steps described in the Gantt chart of Figure 2.2, most of us associate another activity with hospital care: waiting. Strictly speaking, waiting is not really

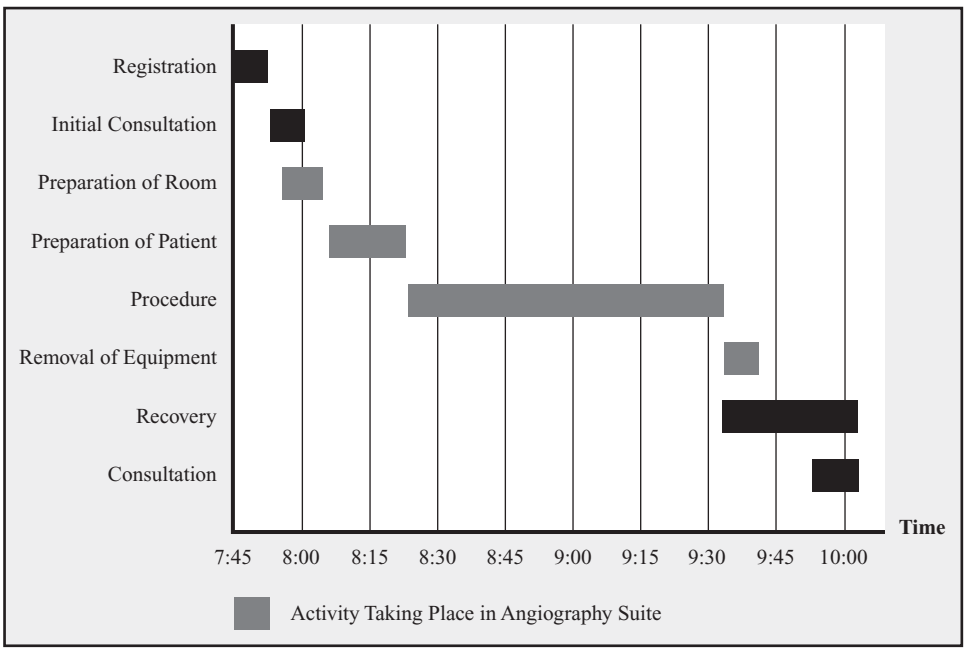


FIGURE 2.2
Gantt Chart Summarizing the Activities for Interventional Radiology

an activity, as it does not add any value to the process. However, waiting is nevertheless relevant. It is annoying for the patient and can complicate matters for the hospital unit. For this reason, waiting times take an important role in operations management. Figure 2.3 shows the actual durations of the activities for a patient arriving at 12:30, as well as the time the patient needs to wait before being moved to the angiography suite.

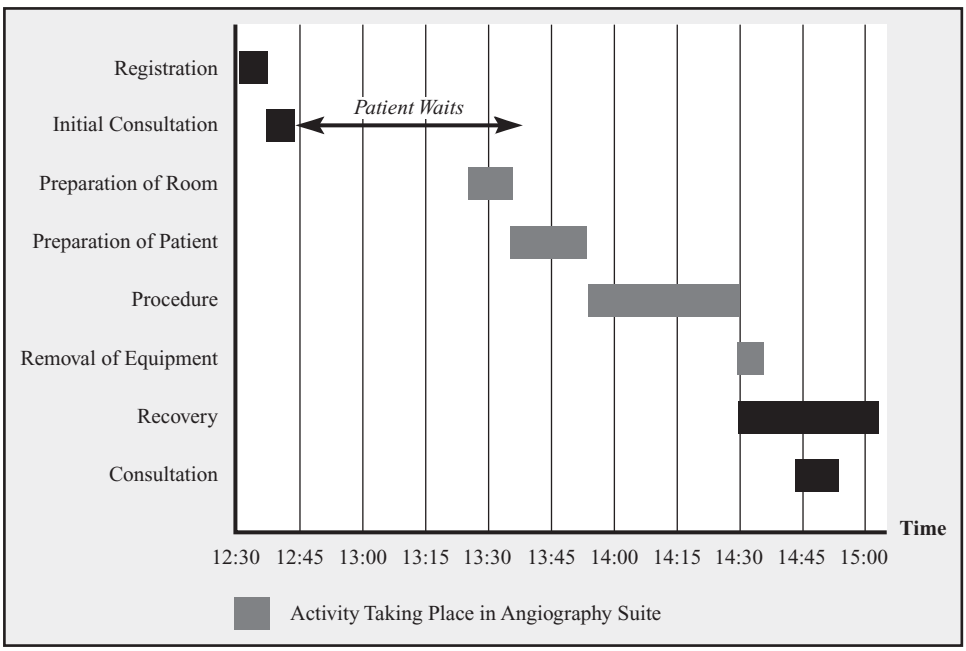


FIGURE 2.3
Gantt Chart Summarizing the Activities for a Patient Arriving at 12:30

But why is there waiting time? Waiting is—to stay in the medical language for the moment—a symptom of supply–demand mismatch. If supply would be unlimited, our visit to the hospital would be reduced to the duration of the activities outlined in Figure 2.2 (the critical path). Imagine visiting a hospital in which all the nurses, technicians, doctors, and hospital administrators would just care for you!

Given that few of us are in a position to receive the undivided attention of an entire hospital unit, it is important that we not only take the egocentric perspective of the patient, but look at the hospital operations more broadly. From the perspective of the hospital, there are many patients "flowing" through the process.

The people and the equipment necessary to support the interventional radiology process deal with many patients, not just one. We refer to these elements of the process as the *process resources.* Consider, for example, the perspective of the nurse and how she/he spends her/his time in the department of interventional radiology. Obviously, radiology from the viewpoint of the nurse is not an exceptional event, but a rather repetitive endeavor. Some of the nurse's work involves direct interaction with the patient; other work—while required for the patient—is invisible to the patient. This includes the preparation of the angiography suite and various aspects of medical record keeping.

Given this repetitive nature of work, the nurse as well as the doctors, technicians, and hospital administrators think of interventional radiology as a process, not a project. Over the course of the day, they see many patients come and go. Many hospitals, including the Presbyterian Hospital in Philadelphia, have a "patient log" that summarizes at what times patients arrive at the unit. This patient log provides a picture of demand on the corresponding day. The patient log for December 2, is summarized by Table 2.1.

Many of these arrivals were probably scheduled some time in advance. Our analysis here focuses on what happens to the patient once he/she has arrived in the interventional radiology unit. A separate analysis could be performed, looking at the process starting with a request for diagnostics up to the arrival of the patient.

Given that the resources in the interventional radiology unit have to care for 11 patients on December 2, they basically need to complete the work according to 11 Gantt charts of the type outlined in Figure 2.2. This—in turn—can lead to waiting times. Waiting times arise when several patients are "competing" for the same limited resource, which is illustrated by the following two examples.

First, observe that the critical path for a typical patient takes about 2 hours. Note further that we want to care for 11 patients over a 10-hour workday. Consequently, we will have to take care of several patients at once. This would not be a problem if we had unlimited resources, nurses, doctors, space in the angiography suites, and so forth. However,

TABLE 2.1
Patient Log on December 2

Number	Patient Name	Arrival Time	Room Assignment
1		7:35	Main room
2		7:45	
3		8:10	
4		9:30	Main room
5		10:15	Main room
6		10:30	Main room
7		11:05	
8		12:35	Main room
9		14:30	Main room
10		14:35	
11		14:40	

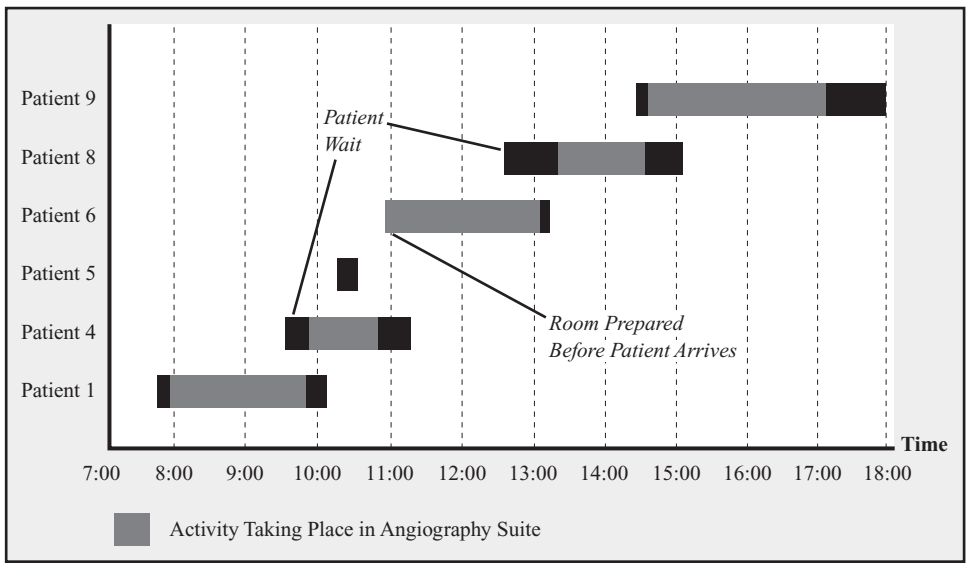


FIGURE 2.4
Time Patient Spent in the Interventional Radiology Unit (for Patients Treated in Main Room Only), Including Room Preparation Time

given the resources that we have, if the Gantt charts of two patients are requesting the same resource simultaneously, waiting times result. For example, the second patient might require the initial consultation with the doctor at a time when the doctor is in the middle of the procedure for patient 1. Note also that patients 1, 4, 5, 6, 8, and 9 are assigned to the same room (the unit has a main room and a second room used for simpler cases), and thus they are also potentially competing for the same resource.

A second source of waiting time lies in the unpredictable nature of many of the activities. Some patients will take much longer in the actual procedure than others. For example, patient 1 spent 1:50 hours in the procedure, while patient 9 was in the procedure for 2:30 hours (see Figure 2.4). As an extreme case, consider patient 5, who refused to sign the consent form and left the process after only 15 minutes.

Such uncertainty is undesirable for resources, as it leaves them "flooded" with work at some moments in the day and "starved" for work at other moments. Figure 2.5 summarizes at what moments in time the angiography suite was used on December 2.

By now, we have established two views to the interventional radiology:

- The view of the patient for whom the idealized stay is summarized by Figure 2.2. Mismatches between supply and demand from the patient's perspective mean having a unit of demand (i.e., the patient) wait for a unit of supply (a resource).
- The view of the resources (summarized by Figure 2.5), which experience demand–supply mismatches when they are sometimes "flooded" with work, followed by periods of no work.

As these two perspectives are ultimately two sides of the same coin, we are interested in bringing these two views together. This is the fundamental idea of process analysis.

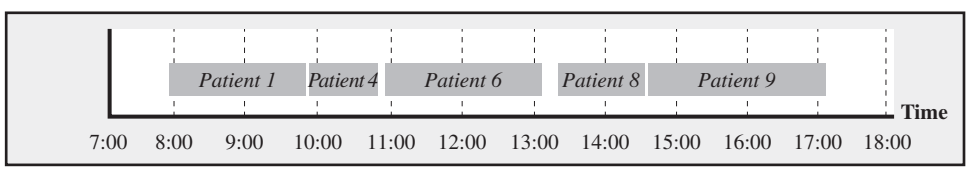


FIGURE 2.5
Usage of the Main Room

2.2 Three Measures of Process Performance

At the most aggregate level, a process can be thought of as a "black box" that uses *resources* (labor and capital) to transform *inputs* (undiagnosed patients, raw materials, unserved customers) into *outputs* (diagnosed patients, finished goods, served customers). This is shown in Figure 2.6. Chapter 3 explains the details of constructing figures like Figure 2.6, which are called *process flow diagrams.* When analyzing the processes that lead to the supply of goods and services, we first define our unit of analysis.

In the case of the interventional radiology unit, we choose patients as our *flow unit.* Choosing the flow unit is typically determined by the type of product or service the supply process is dealing with; for example, vehicles in an auto plant, travelers for an airline, or gallons of beer in a brewery.

As suggested by the term, flow units flow through the process, starting as input and later leaving the process as output. With the appropriate flow unit defined, we next can evaluate a process based on three fundamental process performance measures:

- The number of flow units contained within the process is called the *inventory* (in a production setting, it is referred to as *work-in-process, WIP*). Given that our focus is not only on production processes, inventory could take the form of the number of insurance claims or the number of tax returns at the IRS. There are various reasons why we find inventory in processes, which we discuss in greater detail below. While many of us might initially feel uncomfortable with the wording, the inventory in the case of the interventional radiology unit is a group of patients.
- The time it takes a flow unit to get through the process is called the *flow time.* The flow time takes into account that the item (flow unit) may have to wait to be processed because there are other flow units (inventory) in the process potentially competing for the same resources. Flow time is an especially important performance metric in service environments or in other business situations that are sensitive to delays, such as make-to-order production, where the production of the process only begins upon the arrival of the customer order. In a radiology unit, flow time is something that patients are likely to care about: it measures the time from their arrival at the interventional radiology unit to the time patients can go home or return to their hospital unit.
- Finally, the rate at which the process is delivering output (measured in [flow units/unit of time], e.g., units per day) is called the *flow rate* or the *throughput rate.* The maximum rate with which the process can generate supply is called the *capacity* of the process. For December 2, the throughput of the interventional radiology unit was 11 patients per day.

Table 2.2 provides several examples of processes and their corresponding flow rates, inventory levels, and flow times.

You might be somewhat irritated that we have moved away from the idea of supply and demand mismatch for a moment. Moreover, we have not talked about profits so far.

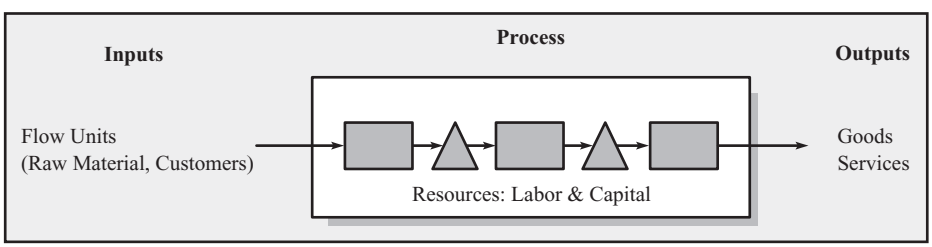


FIGURE 2.6
The Process View of an Organization

TABLE 2.2
Examples of Flow Rates, Inventories, and Flow Times

	U.S. Immigration	Champagne Industry	MBA Program	Large PC Manufacturer
Flow unit	Application for immigration benefit	Bottle of champagne	MBA student	Computer
Flow rate/ throughput	Approved or rejected visa cases: 6.3 million per year	260 million bottles per year	600 students per year	5,000 units per day
Flow time	Average processing time: 7.6 months	Average time in cellar: 3.46 years	2 years	10 days
Inventory	Pending cases: 4.0 million cases	900 million bottles	1,200 students	50,000 computers

However, note that increasing the maximum flow rate (capacity) avoids situations where we have insufficient supply to match demand. From a profit perspective, a higher flow rate translates directly into more revenues (you can produce a unit faster and thus can produce more units), assuming your process is currently *capacity constrained,* that is, there is sufficient demand that you could sell any additional output you make.

Shorter flow times reduce the time delay between the occurrence of demand and its fulfillment in the form of supply. Shorter flow times therefore also typically help to reduce demand–supply mismatches. In many industries, shorter flow times also result in additional unit sales and/or higher prices, which makes them interesting also from a broader management perspective.

Lower inventory results in lower working capital requirements as well as many quality advantages that we explore later in this book. A higher inventory also is directly related to longer flow times (explained below). Thus, a reduction in inventory also yields a reduction in flow time. As inventory is the most visible indication of a mismatch between supply and demand, we will now discuss it in greater detail.

2.3 Little's Law

Accountants view inventory as an asset, but from an operations perspective, inventory often should be viewed as a liability. This is not a snub on accountants; inventory *should* be an asset on a balance sheet, given how accountants define an asset. But in common speech, the word *asset* means "desirable thing to have" and the dictionary defines *liability* as "something that works to one's disadvantage." In this sense, inventory can clearly be a liability. This is most visible in a service process such as a hospital unit, where patients in the waiting room obviously cannot be counted toward the assets of the health care system.

Let's take another visit to the interventional radiology unit. Even without much medical expertise, we can quickly find out which of the patients are currently undergoing care from some resource and which are waiting for a resource to take care of them. Similarly, if we took a quick walk through a factory, we could identify which parts of the inventory serve as raw materials, which ones are work-in-process, and which ones have completed the production process and now take the form of finished goods inventory.

However, taking a single walk through the process—dishwasher factory or interventional radiology unit—will not leave us with a good understanding of the underlying operations. All it will give us is a snapshot of what the process looked like at one single

moment in time. Unfortunately, it is this same snapshot approach that underlies most management (accounting) reports: balance sheets itemize inventory into three categories (raw materials, WIP, finished goods); hospital administrators typically distinguish between pre- and postoperative patients. But such snapshots do not tell us *why* these inventories exist in the first place! Thus, a static, snapshot approach neither helps us to analyze business processes (why is there inventory?) nor helps us to improve them (is this the right amount of inventory?).

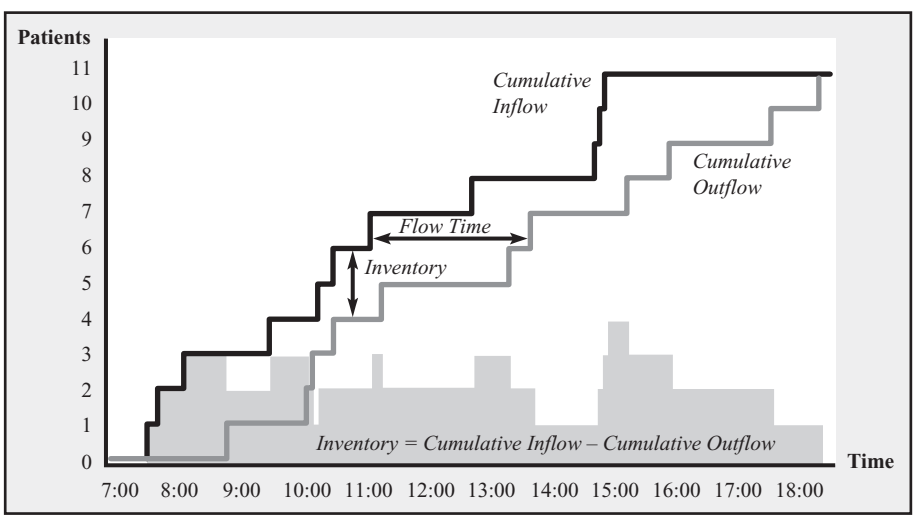
Now, imagine that instead of our single visit to the hospital unit, we would be willing to stay for some longer period of time. We arrive early in the morning and make ourselves comfortable at the entrance of the unit. Knowing that there are no patients in the interventional radiology unit overnight, we then start recording any arrival or departure of patients. In other words, we collect data concerning the patient inflow and outflow.

At the end of our stay, we can plot a graph similar to Figure 2.7. The upper of the two curves illustrates the cumulative number of patients who have entered the unit. The curve begins at time zero (7:00) and with zero patients. If we had done the same exercise in a unit with overnight patients, we would have recorded our initial patient count there. The lower of the two curves indicates the cumulative number of patients who have left the unit. Figure 2.7 shows us that by noon, seven patients have arrived, of which five have left the unit again.

At any given moment in time, the *vertical distance* between the upper curve and the lower curve corresponds to the number of patients in the interventional radiology unit, or—abstractly speaking—the inventory level. Thus, although we have not been inside the interventional radiology unit this day, we are able to keep track of the inventory level by comparing the cumulative inflow and outflow. For example, the inventory at noon consisted of two patients.

We also can look at the *horizontal distance* between the two lines. If the patients leave the unit in the same order they entered it, the horizontal gap would measure the exact amount of time each patient spent in the interventional radiology unit. More generally, given that the length of stay might vary across patients and patients do not necessarily leave the unit in the exact same sequence in which they entered it, the average gap between the two lines provides the average length of stay.

FIGURE 2.7
Cumulative Inflow and Outflow



Thus, Figure 2.7 includes all three of the basic process performance measures we discussed on the previous page: flow rate (the slope of the two graphs), inventory (the vertical distance between the two graphs), and flow time (the horizontal distance between the two graphs).

Based on either the graph or the patient log, we can now compute these performance measures for December 2. We already know that the flow rate was 11 patients/day.

Next, consider inventory. Inventory changes throughout the day, reflecting the differences between inflow and outflow of patients. A "brute force" approach to compute average inventory is to count the inventory at every moment in time throughout the day, say every five minutes, and then take the average. For December 2, this computation yields an average inventory of 2.076 patients.

Next, consider the flow time, the time a patient spends in the unit. To compute that information, we need to add to the patient log, Table 2.1, the time each patient left the interventional radiology unit. The difference between arrival time and departure time would be the flow time for a given patient, which in turn would allow us to compute the average flow time across patients. This is shown in Table 2.3 and is in many ways similar to the two graphs in Figure 2.7. We can easily compute that on December 2, the average flow time was 2 hours, 4 minutes, and 33 seconds, or 2.076 hours.

At this point, you might ask: "Does the average inventory always come out the same as the average flow time?" The answer to this question is a resounding *no.* However, the fact that the average inventory was 2.076 patients and the average flow time was 2.076 hours is no coincidence either.

To see how inventory and flow time relate to each other, let us review the three performance measures, flow rate, flow time, and inventory:

- Flow rate = 11 patients per day, which is equal to one patient per hour.
- Flow time = 2.076 hours.
- Inventory = 2.076 patients.

Thus, while inventory and flow time do not have to—and, in fact, rarely are—equal, they are linked in another form. We will now introduce this relationship as Little's Law (named after John D. C. Little).

$$\text{Average inventory} = \text{Average flow rate} \times \text{Average flow time} \qquad \text{(Little's Law)}$$

Many people think of this relationship as trivial. However, it is not. Its proof is rather complex for the general case (which includes—among other nasty things—variability) and by mathematical standards is very recent.

TABLE 2.3
Calculation of Average Flow Time

Number	Patient Name	Arrival Time	Departure Time	Flow Time
1		7:35	8:50	1:15
2		7:45	10:05	2:20
3		8:10	10:10	2:00
4		9:30	11:15	1:45
5		10:15	10:30	0:15
6		10:30	13:35	3:05
7		11:05	13:15	2:10
8		12:35	15:05	2:30
9		14:30	18:10	3:40
10		14:35	15:45	1:10
11		14:40	17:20	2:40
			Average	2:04:33

Little's Law is useful in finding the third performance measure when the other two are known. For example, if you want to find out how long patients in a radiology unit spend waiting for their chest X-ray, you could do the following:

1. Observe the inventory of patients at a couple of random points during the day, giving you an average inventory. Let's say this number is seven patients: four in the waiting room, two already changed and waiting in front of the procedure room, and one in the procedure room.
2. Count the procedure slips or any other records showing how many patients were treated that day. This is the day's output. Let's say there were 60 patients over a period of 8 hours; we could say that we have a flow rate of 60/8 = 7.5 patients/hour.
3. Use Little's Law to compute Flow time = Inventory/Flow rate = 7/7.5 = 0.933 hour = 56 minutes. This tells us that, on average, it takes 56 minutes from the time a patient enters the radiology unit to the time his or her chest X-ray is completed. Note that this information would otherwise have to be computed by collecting additional data (e.g., see Table 2.3).

When does Little's Law hold? The short answer is *always.* For example, Little's Law does not depend on the sequence in which the flow units (e.g., patients) are served (remember FIFO and LIFO from your accounting class?). (However, the sequence could influence the flow time of a particular flow unit, e.g., the patient arriving first in the morning, but not the average flow time across all flow units.) Furthermore, Little's Law does not depend on randomness: it does not matter if there is variability in the number of patients or in how long treatment takes for each patient; all that matters is the average flow rate of patients and the average flow time.

In addition to the direct application of Little's Law, for example, in the computation of flow time, Little's Law is also underlying the computation of inventory costs as well as a concept known as inventory turns. This is discussed in the following section.

2.4 Inventory Turns and Inventory Costs

Using physical units as flow units (and, hence, as the inventory measure) is probably the most intuitive way to measure inventory. This could be vehicles at an auto retailer, patients in the hospital, or tons of oil in a refinery.

However, working with physical units is not necessarily the best method for obtaining an aggregate measure of inventory across different products: there is little value to saying you have 2,000 units of inventory if 1,000 of them are paper clips and the remaining 1,000 are computers. In such applications, inventory is often measured in some monetary unit, for example, $5 million worth of inventory.

Measuring inventory in a common monetary unit facilitates the aggregation of inventory across different products. This is why total U.S. inventory is reported in dollars. To illustrate the notion of monetary flow units, consider Kohl's Corp, a large U.S. retailer. Instead of thinking of Kohl's stores as sodas, toys, clothes, and bathroom tissues (physical units), we can think of its stores as processes transforming goods valued in monetary units into sales, which also can be evaluated in the form of monetary units.

As can easily be seen from Kohl's balance sheet, on January 31, 2011, the company held an inventory valued at $3.036 billion (see Table 2.4). Given that our flow unit now is the "individual dollar bill," we want to measure the flow rate through Kohl's operation.

The direct approach would be to take "sales" as the resulting flow. Yet, this measure is inflated by Kohl's gross profit margin; that is, a dollar of sales is measured in sales dol-

TABLE 2.4 Excerpts from Financial Statements of Kohl's and Walmart (All Numbers in Millions)

Source: Taken from 10-K filings.

	2011	2010	2009	2008	2007
Kohl's					
Revenue	$ 18,391	$ 17,178	$ 16,389	$ 16,474	$ 15,544
Cost of Goods Sold	$ 11,359	$ 10,679	$ 10,332	$ 10,459	$ 9,890
Inventory	$ 3,036	$ 2,923	$ 2,799	$ 2,856	$ 2,588
Net Income	$ 1,114	$ 991	$ 885	$ 1,084	$ 1,109
Walmart					
Revenue	$418,952	$ 405,046	$ 401,244	$374,526	$344,992
Cost of Goods Sold	$307,646	$2,97,500	$2,99,419	$280,198	$258,693
Inventory	$ 36,318	$ 33,160	$ 34,511	$ 35,180	$ 33,685
Net Income	$ 16,389	$ 14,335	$ 13,188	$ 12,884	$ 12,036

lars, while a dollar of inventory is measured, given the present accounting practice, in a cost dollar. Thus, the appropriate measure for flow rate is the cost of goods sold, or COGS for short.

With these two measures—flow rate and inventory—we can apply Little's Law to compute what initially might seem a rather artificial measure: how long does the average flow unit (dollar bill) spend within the Kohl's system before being turned into sales, at which point the flow units will trigger a profit intake. This corresponds to the definition of flow time.

$$\text{Flow rate} = \text{Cost of goods sold} = \$11,359 \text{ million/year}$$
$$\text{Inventory} = \$3,036 \text{ million}$$

Hence, we can compute flow time via Little's Law as

$$\text{Flow time} = \frac{\text{Inventory}}{\text{Flow rate}}$$
$$= \$3,036 \text{ million}/\$11,359 \text{ million/year} = 0.267 \text{ year} = 97 \text{ days}$$

Thus, we find that it takes Kohl's—on average—97 days to translate a dollar investment into a dollar of—hopefully profitable—revenues.

This calculation underlies the definition of another way of measuring inventory, namely in terms of *days of supply.* We could say that Kohl's has 97 days of inventory in their process. In other words, the average item we find at Kohl's spends 97 days in Kohl's supply chain.

Alternatively, we could say that Kohl's turns over its inventory 365 days/year/97 days = 3.74 times per year. This measure is called *inventory turns.* Inventory turns is a common benchmark in the retailing environment and other supply chain operations:

$$\text{Inventory turns} = \frac{1}{\text{Flow time}}$$

TABLE 2.5
Inventory Turns and Margins for Selected Retail Segments

Source: Based on Gaur, Fisher, and Raman 2005.

Retail Segment	Examples	Annual Inventory Turns	Gross Margin
Apparel and accessory	Ann Taylor, GAP	4.57	37%
Catalog, mail-order	Spiegel, Lands End	8.60	39%
Department stores	Sears, JCPenney	3.87	34%
Drug and proprietary stores	Rite Aid, CVS	5.26	28%
Food stores	Albertson's, Safeway, Walmart	10.78	26%
Hobby, toy/game stores	Toys R Us	2.99	35%
Home furniture/equipment	Bed Bath & Beyond	5.44	40%
Jewelry	Tiffany	1.68	42%
Radio, TV, consumer electronics	Best Buy, CompUSA	4.10	31%
Variety stores	Kohl's, Walmart, Target	4.45	29%

To illustrate this application of Little's Law further, consider Walmart, one of Kohl's competitors. Repeating the same calculations as outlined on the previous page, we find the following data about Walmart:

$$\begin{aligned}
\text{Cost of goods sold} &= \$307,646 \text{ million/year} \\
\text{Inventory} &= \$36,318 \text{ million} \\
\text{Flow time} &= \$36,318 \text{ million}/\$307,646 \text{ million/year} \\
&= 0.118 \text{ year} = 43.1 \text{ days} \\
\text{Inventory turns} &= 1/43.1 \text{ turns/day} \\
&= 365 \text{ days/year} \times 1/43.1 \text{ turns/day} = 8.47 \text{ turns per year}
\end{aligned}$$

Thus, we find that Walmart is able to achieve substantially higher inventory turns than Kohl's. Table 2.5 summarizes inventory turn data for various segments of the retailing industry. Table 2.5 also provides information about gross margins in various retail settings (keep them in mind the next time you haggle for a new sofa or watch!).

Inventory requires substantial financial investments. Moreover, the inventory holding cost is substantially higher than the mere financial holding cost for a number of reasons:

- Inventory might become obsolete (think of the annual holding cost of a microprocessor).
- Inventory might physically perish (you don't want to think of the cost of holding fresh roses for a year).
- Inventory might disappear (also known as theft or shrink).
- Inventory requires storage space and other overhead cost (insurance, security, real estate, etc.).
- There are other less tangible costs of inventory that result from increased wait times (because of Little's Law, to be discussed in Chapter 8) and lower quality (to be discussed in Chapter 11).

Given an annual cost of inventory (e.g., 20 percent per year) and the inventory turn information as computed above, we can compute the per-unit inventory cost that a process (or a supply chain) incurs. To do this, we take the annual holding cost and divide it by the number of times the inventory turns in a year:

$$\text{Per-unit inventory costs} = \frac{\text{Annual inventory costs}}{\text{Annual inventory turns}}$$

Exhibit 2.1

CALCULATING INVENTORY TURNS AND PER-UNIT INVENTORY COSTS

1. Look up the value of inventory from the balance sheet.
2. Look up the cost of goods sold (COGS) from the earnings statement; do *not* use sales!
3. Compute inventory turns as

$$\text{Inventory turns} = \frac{\text{COGS}}{\text{Inventory}}$$

4. Compute per-unit inventory costs as

$$\text{Per-unit inventory costs} \quad \frac{\text{Annual inventory costs}}{\text{Inventory turns}}$$

Note: The annual inventory cost needs to account for the cost of financing the inventory, the cost of depreciation, and other inventory-related costs the firm considers relevant (e.g., storage, theft).

For example, a company that works based on a 20 percent annual inventory cost and that turns its inventory six times per year incurs per-unit inventory costs of

$$\frac{20\% \text{ per year}}{6 \text{ turns per year}} = 3.33\%$$

In the case of Kohl's (we earlier computed that the inventory turns 3.74 times per year), and assuming annual holding costs of 20 percent per year, this translates to inventory costs of about 5.35 percent of the cost of goods sold (20%/3.74 = 5.35). The calculations to obtain per unit inventory costs are summarized in Exhibit 2.1.

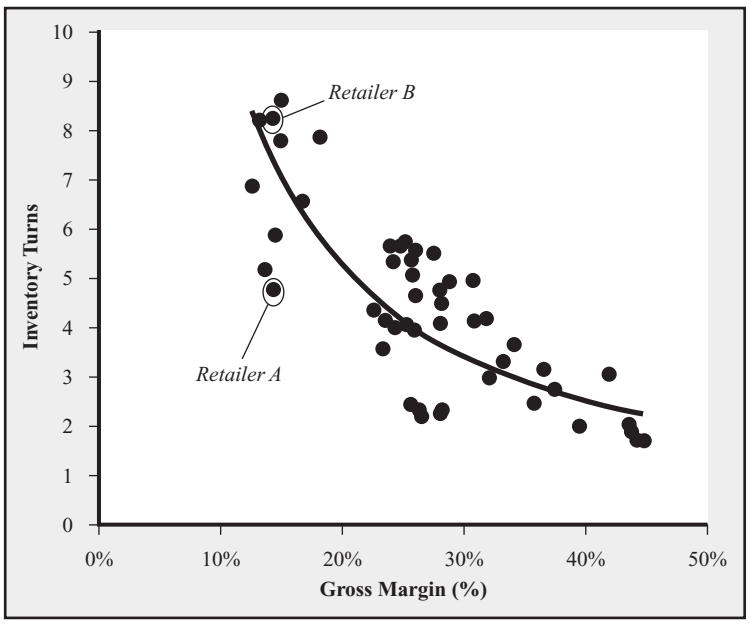
To stay in the retailing context a little longer, consider a retailer of consumer electronics who has annual inventory costs of 30 percent (driven by financial costs and obsolescence). Assuming the retailer turns its inventory about four times per year (see Table 2.5.), we obtain a per-unit inventory cost of 30%/4 = 7.5%. Consider a TV in the retailer's assortment that is on the shelf with a price tag of $300 and is procured by the retailer for $200. Based on our calculation, we know that the retailer incurs a $200 × 7.5% = $15 inventory cost for each such TV that is sold. To put this number into perspective, consider Figure 2.8.

Figure 2.8 plots the relationship between gross margin and inventory turns for consumer electronics retailers (based on Gaur, Fisher, and Raman 2005). Note that this graph does not imply causality in this relationship. That is, the model does not imply that if a firm increases its gross margin, its inventory turns will decline automatically. Instead, the way to look at Figure 2.8 is to think of gross margin for a given set of products as being fixed by the competitive environment. We can then make two interesting observations:

- A retailer can decide to specialize in products that turn very slowly to increase its margins. For example, Radio Shack is known for its high margins, as they carry many products in their assortment that turn only once or twice a year. In contrast, Best Buy is carrying largely very popular items, which exposes the company to stiffer competition and lower gross margins.
- For a given gross margin, we observe dramatic differences concerning inventory turns. For example, inventory turns vary between four and nine times for a 15 percent gross margin. Consider retailer A and assume that all retailers work with a 30 percent annual holding cost. Based on the annual inventory turns of 4.5, retailer A faces a 6.66 percent per-unit inventory cost. Now, compare this to competing retailer B, who turns its inventory eight times per year. Thus, retailer B operates with 3.75 percent per-unit inventory

FIGURE 2.8
Relationship between Inventory Turns and Gross Margin

Source: Based on Gaur, Fisher, and Raman 2005.



costs, almost a 3 percent cost advantage over retailer A. Given that net profits in this industry segment are around 2 percent of sales, such a cost advantage can make the difference between profits and bankruptcy.

2.5 Five Reasons to Hold Inventory

While Little's Law allows us to compute the average inventory in the process (as long as we know flow time and flow rate), it offers no help in answering the question we raised previously: Why is there inventory in the process in the first place? To understand the need for inventory, we can no longer afford to take the black-box perspective and look at processes from the outside. Instead, we have to look at the process in much more detail.

As we saw from Figure 2.7, inventory reflected a deviation between the inflow into a process and its outflow. Ideally, from an operations perspective, we would like Figure 2.7 to take the shape of two identical, straight lines, representing process inflow and outflow. Unfortunately, such straight lines with zero distance between them rarely exist in the real world. De Groote (1994) discusses five reasons for holding inventory, that is, for having the inflow line differ from the outflow line: (1) the time a flow unit spends in the process, (2) seasonal demand, (3) economies of scale, (4) separation of steps in a process, and (5) stochastic demand. Depending on the reason for holding inventory, inventories are given different names: pipeline inventory, seasonal inventory, cycle inventory, decoupling inventory/ buffers, and safety inventory. It should be noted that these five reasons are not necessarily mutually exclusive and that, in practice, there typically exist more than one reason for holding inventory.

Pipeline Inventory

This first reason for inventory reflects the time a flow unit has to spend in the process in order to be transformed from input to output. Even with unlimited resources, patients still need to spend time in the interventional radiology unit; their flow time would be the length of the critical path. We refer to this basic inventory on which the process operates as *pipeline inventory.*

For the sake of simplicity, let's assume that every patient would have to spend exactly 1.5 hours in the interventional radiology unit, as opposed to waiting for a resource to become available, and that we have one patient arrive every hour. How do we find the pipeline inventory in this case?

The answer is obtained through an application of Little's Law. Because we know two of the three performance measures, flow time and flow rate, we can figure out the third, in this case inventory: with a flow rate of one patient per hour and a flow time of 1.5 hours, the average inventory is

$$\text{Inventory} = 1[\text{patient/hour}] \times 1.5[\text{hours}] = 1.5 \text{ patients}$$

which is the number of patients undergoing some value-adding activity. This is illustrated by Figure 2.9.

In certain environments, you might hear managers make statements of the type "we need to achieve zero inventory in our process." If we substitute Inventory = 0 into Little's Law, the immediate result is that a process with zero inventory is also a process with zero flow rate (unless we have zero flow time, which means that the process does not do anything to the flow unit). Thus, as long as it takes an operation even a minimum amount of time to work on a flow unit, the process will always exhibit pipeline inventory. There can be no hospital without patients and no factory can operate without some work in process!

Little's Law also points us toward the best way to reduce pipeline inventory. As reducing flow rate (and with it demand and profit) is typically not a desirable option, the *only* other way to reduce pipeline inventory is by reducing flow time.

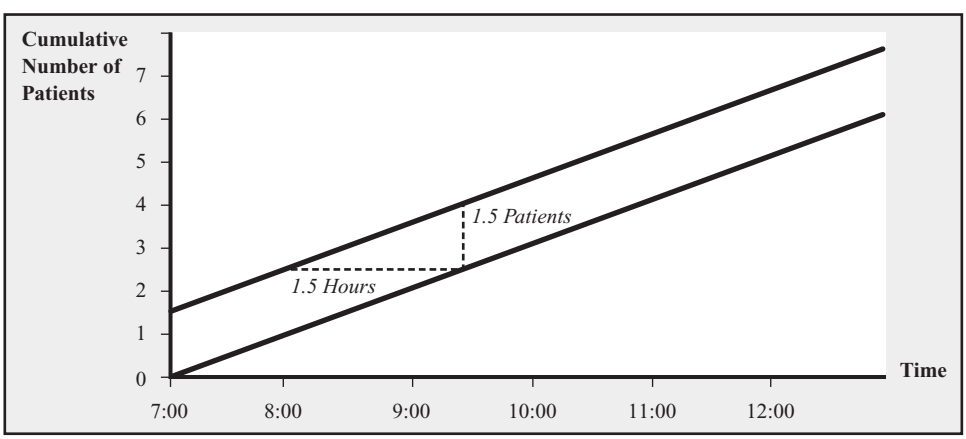
Seasonal Inventory

Seasonal inventory occurs when capacity is rigid and demand is variable. Two examples illustrate this second reason for inventory. Campbell's Soup sells more chicken noodle soup in January than in any other month of the year (see Chapter 17)—not primarily because of cold weather, but because Campbell's discounts chicken noodle soup in January. June is the next biggest sales month, because Campbell's increases its price in July.

So much soup is sold in January that Campbell's starts production several months in advance and builds inventory in anticipation of January sales. Campbell's could wait longer to start production and thereby not build as much inventory, but it would be too costly to assemble the needed capacity (equipment and labor) in the winter only to dismantle that capacity at the end of January when it is no longer needed.

In other words, as long as it is costly to add and subtract capacity, firms will desire to smooth production relative to sales, thereby creating the need for seasonal inventory.

FIGURE 2.9
Pipeline Inventory



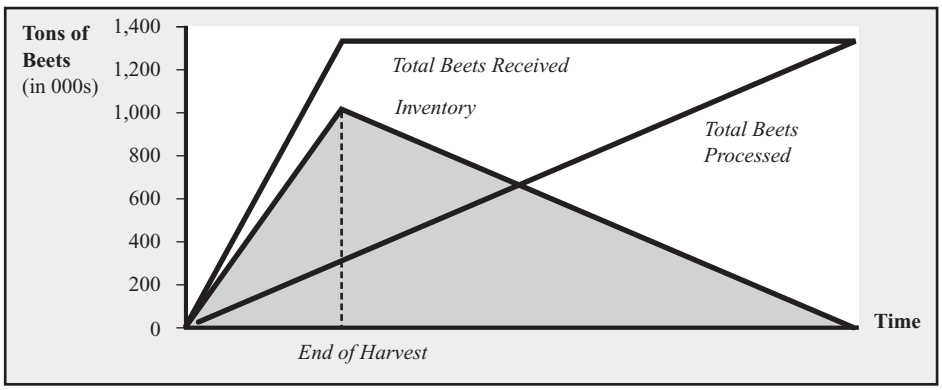


FIGURE 2.10
Seasonal Inventory—Sugar

An extreme case of seasonal inventory can be found in the agricultural and food processing sector. Due to the nature of the harvesting season, Monitor Sugar, a large sugar cooperative in the U.S. Midwest, collects all raw material for their sugar production over a period of six weeks. At the end of the harvesting season, they have accumulated—in the very meaning of the word—a pile of sugar beets, about 1 million tons, taking the form of a 67-acre sugar beets pile.

Given that food processing is a very capital-intense operation, the process is sized such that the 1.325 million tons of beets received and the almost 1 million tons of inventory that is built allow for a nonstop operation of the production plant until the beginning of the next harvesting season. Thus, as illustrated by Figure 2.10, the production, and hence the product outflow, is close to constant, while the product inflow is zero except for the harvesting season.

Cycle Inventory

Throughout this book, we will encounter many situations in which it is economical to process several flow units collectively at a given moment in time to take advantage of scale economies in operations.

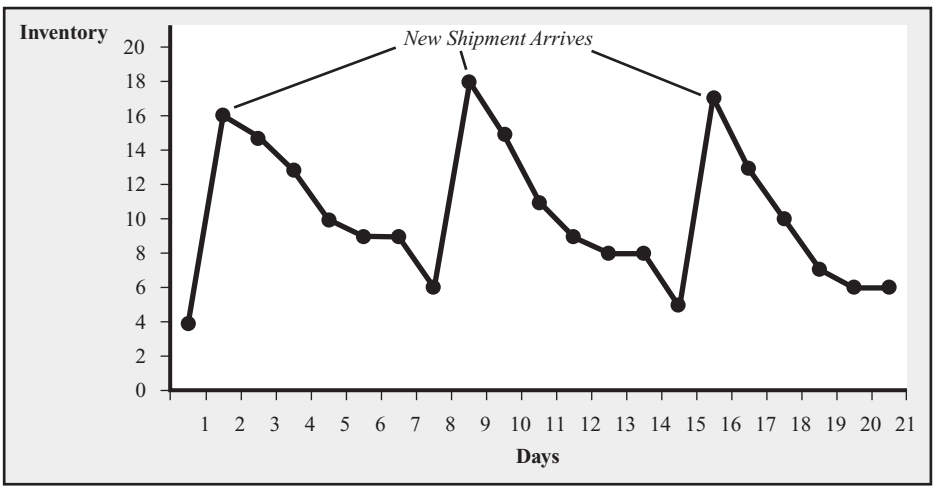
The scale economics in transportation processes provide a good example for the third reason for inventory. Whether a truck is dispatched empty or full, the driver is paid a fixed amount and a sizeable portion of the wear and tear on the truck depends on the mileage driven, not on the load carried. In other words, each truck shipment incurs a fixed cost that is independent of the amount shipped. To mitigate the sting of that fixed cost, it is tempting to load the truck completely, thereby dividing the fixed cost across the largest number of units.

In many cases, this indeed may be a wise decision. But a truck often carries more product than can be immediately sold. Hence, it takes some time to sell off the entire truck delivery. During that interval of time, there will be inventory. This inventory is labeled *cycle inventory* as it reflects that the transportation process follows a certain shipment cycle (e.g., a shipment every week).

Figure 2.11 plots the inventory level of a simple tray that is required during the operation in the interventional radiology unit. As we can see, there exists a "lumpy" inflow of units, while the outflow is relatively smooth. The reason for this is that—due to the administrative efforts related to placing orders for the trays—the hospital only places one order per week.

The major difference between cycle inventory and seasonal inventory is that seasonal inventory is due to temporary imbalances in supply and demand due to variable demand (soup) or variable supply (beets) while cycle inventory is created due to a cost motivation.

FIGURE 2.11
Cycle Inventory



Decoupling Inventory/Buffers

Inventory between process steps can serve as buffers. An inventory buffer allows management to operate steps independently from each other. For example, consider two workers in a garment factory. Suppose the first worker sews the collar onto a shirt and the second sews the buttons. A buffer between them is a pile of shirts with collars but no buttons. Because of that buffer, the first worker can stop working (e.g., to take a break, repair the sewing machine, or change thread color) while the second worker keeps working. In other words, buffers can absorb variations in flow rates by acting as a source of supply for a downstream process step, even if the previous operation itself might not be able to create this supply at the given moment in time.

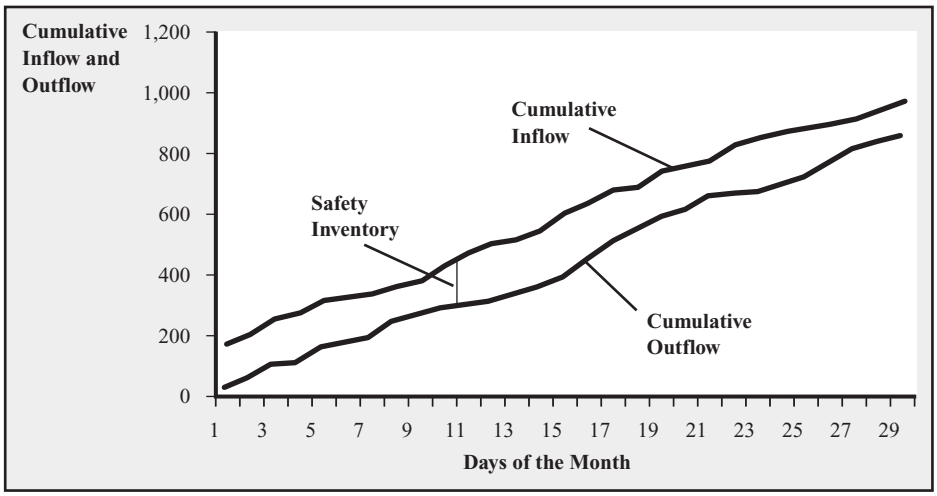
An automotive assembly line is another example of a production process that uses buffers to decouple the various stations involved with producing the vehicle. In the absence of such buffers, a disruption at any one station would lead to a disruption of all the other stations, upstream and downstream. Think of a bucket brigade to fight a fire: There are no buffers between firefighters in a bucket brigade, so nobody can take a break without stopping the entire process.

Safety Inventory

The final reason for inventory is probably the most obvious, but also the most challenging: stochastic demand. Stochastic demand refers to the fact that we need to distinguish between the predicted demand and the actually realized demand. In other words, we typically face variation in demand relative to our demand prediction. Note that this is different from variations in predictable demand, which is called *seasonality,* like a sales spike of Campbell's chicken noodle soup in January. Furthermore, stochastic demand can be present along with seasonal demand: January sales can be known to be higher than those for other months (seasonal demand) and there can be variation around that known forecast (stochastic demand).

Stochastic demand is an especially significant problem in retailing environments or at the finished goods level of manufacturers. Take a book retailer that must decide how many books to order of a given title. The book retailer has a forecast for demand, but forecasts are (at best) correct on average. Order too many books and the retailer is faced with leftover inventory. Order too few and valuable sales are lost. This trade-off can be managed, as we will discover in Chapter 12, but not eliminated (unless there are zero forecast errors).

FIGURE 2.12
Safety Inventory at a Blood Bank



The resulting inventory thereby can be seen as a way to hedge against the underlying demand uncertainty. It might reflect a one-shot decision, for example, in the case of a book retailer selling short-life-cycle products such as newspapers or magazines. If we consider a title with a longer product life cycle (e.g., children's books), the book retailer will be able to replenish books more or less continuously over time.

Figure 2.12 shows the example of the blood bank in the Presbyterian Hospital in Philadelphia. While the detailed inflow and consumption of blood units vary over the course of the month, the hospital always has a couple of days of blood in inventory. Given that blood perishes quickly, the hospital wants to keep only a small inventory at its facility, which it replenishes from the regional blood bank operated by the Red Cross.

2.6 The Product–Process Matrix

Processes leading to the supply of goods or services can take many different forms. Some processes are highly automated, while others are largely manual. Some processes resemble the legendary Ford assembly line, while others resemble more the workshop in your local bike store. Empirical research in operations management, which has looked at thousands of processes, has identified five "clusters" or types of processes. Within each of the five clusters, processes are very similar concerning variables such as the number of different product variants they offer or the production volume they provide. Table 2.6 describes these different types of processes.

By looking at the evolution of a number of industries, Hayes and Wheelwright (1979) observed an interesting pattern, which they referred to as the product–process matrix (see Figure 2.13). The product–process matrix stipulates that over its life cycle, a product typically is initially produced in a job shop process. As the production volume of the product increases, the production process for the product moves from the upper left of the matrix to the lower right.

For example, the first automobiles were produced using job shops, typically creating one product at a time. Most automobiles were unique; not only did they have different colors or add-ons, but they differed in size, geometry of the body, and many other aspects. Henry Ford's introduction of the assembly line corresponded to a major shift along the diagonal of the product–process matrix. Rather than producing a couple of products in a job shop, Ford produced thousands of vehicles on an assembly line.

TABLE 2.6
Process Types and Their Characteristics

	Examples	Number of Different Product Variants	Product Volume (Units/Year)
Job shop	• Design company • Commercial printer • Formula 1 race car	High (100+)	Low (1–100)
Batch process	• Apparel sewing • Bakery • Semiconductor wafers	Medium (10–100)	Medium (100–100k)
Worker-paced line flow	• Auto assembly • Computer assembly	Medium (1–50)	High (10k–1M)
Machine-paced line flow	• Large auto assembly	Low (1–10)	High (10k–1M)
Continuous process	• Paper mill • Oil refinery • Food processing	Low (1–10)	Very high

Note that the "off-diagonals" in the product–process matrix (the lower left and the upper right) are empty. This reflects that it is neither economical to produce very high volumes in a job shop (imagine if all of the millions of new vehicles sold in the United States every year were handcrafted in the same manner as Gottlieb Daimler created the first automobile) nor does it make sense to use an assembly line in order to produce only a handful of products a year.

We have to admit that few companies—if any—would be foolish enough to produce a high-volume product in a job shop. However, identifying a process type and looking at the product–process matrix is more than an academic exercise in industrial history. The usefulness of the product–process matrix lies in two different points:

1. Similar process types tend to have similar problems. For example, as we will discuss in Chapter 4, assembly lines tend to have the problem of line balancing (some workers working harder than others). Batch-flow processes tend to be slow in responding to

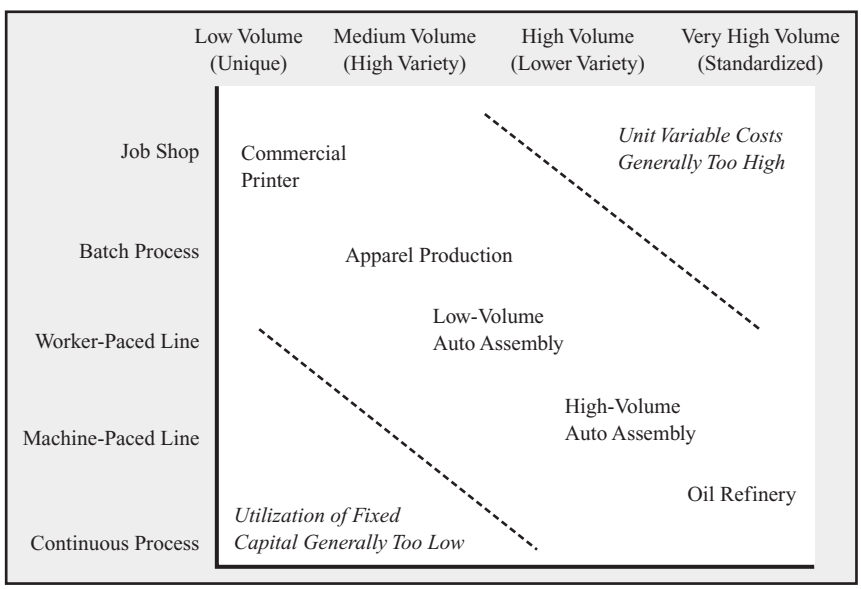


FIGURE 2.13
Product–Process Matrix

Source: Hayes and Wheelwright (1979).

customer demand (see Chapter 7). Thus, once you know a process type, you can quickly determine what type of problems the process is likely to face and what solution methods are most appropriate.

2. The "natural drift" of industries toward the lower right of Figure 2.13 enables you to predict how processes are likely to evolve in a particular industry. Consider, for example, the case of eye surgery. Up until the 1980s, corrective eye surgery was done in large hospitals. There, doctors would perform a large variety of very different eye-related cases. Fifteen years later, this situation had changed dramatically. Many highly specialized eye clinics have opened, most of them focusing on a limited set of procedures. These clinics achieve high volume and, because of the high volume and the lower variety of cases, can operate at much higher levels of efficiency. Similarly, semiconductor production equipment used to be assembled on a one-by-one basis, while now companies such as Applied Materials and Kulicke & Soffa operate worker-paced lines.

2.7 Summary

In this chapter, we emphasized the importance of looking at the operations of a firm not just in terms of the products that the firm supplies, but also at the processes that generate the supply. Looking at processes is especially important with respect to demand–supply mismatches. From the perspective of the product, such mismatches take the form of waiting times; from the perspective of the process, they take the form of inventory.

For any process, we can define three fundamental performance measures: inventory, flow time, and flow rate. The three measures are related by Little's Law, which states that the average inventory is equal to the average flow time multiplied by the average flow rate.

Little's Law can be used to find any of the three performance measures, as long as the other two measures are known. This is specifically important with respect to flow time, which is in practice frequently difficult to observe directly.

A measure related to flow time is inventory turns. Inventory turns, measured by 1/(flow time), captures how fast the flow units are transformed from input to output. It is an important benchmark in many industries, especially retailing. Inventory turns are also the basis of computing the inventory costs associated with one unit of supply.

2.8 Further Reading

De Groote (1994) is a very elegant note describing the basic roles of inventory. This note, as well as many other notes and articles by de Groote, takes a very "lean" perspective to operations management, resembling much more the tradition of economics as opposed to engineering.

Gaur, Fisher, and Raman (2005) provide an extensive study of retailing performance. They present various operational measures, including inventory turns, and show how they relate to financial performance measures.

The Hayes and Wheelwright (1979) reference is widely recognized as a pioneering article linking operations aspects to business strategy. Subsequent work by Hayes, Wheelwright, and Clark (1988) established operations as a key source for a firm's competitive advantage.

2.9 Practice Problems

Q2.1* **(Dell)** What percentage of cost of a Dell computer reflects inventory costs? Assume Dell's yearly inventory cost is 40 percent to account for the cost of capital for financing the inventory, the warehouse space, and the cost of obsolescence. In other words, Dell incurs a cost of $40 for a $100 component that is in the company's inventory for one entire year. In 2001, Dell's 10-k reports showed that the company had $400 million in inventory and COGS of $26,442 million.

Q2.2 **(Airline)** Consider the baggage check-in of a small airline. Check-in data indicate that from 9 a.m. to 10 a.m., 255 passengers checked in. Moreover, based on counting the number of

(* indicates that the solution is at the end of the book)

passengers waiting in line, airport management found that the average number of passengers waiting for check-in was 35. How long did the average passenger have to wait in line?

Q2.3 **(Inventory Cost)** A manufacturing company producing medical devices reported $60,000,000 in sales over the last year. At the end of the same year, the company had $20,000,000 worth of inventory of ready-to-ship devices.

a. Assuming that units in inventory are valued (based on COGS) at $1,000 per unit and are sold for $2,000 per unit, how fast does the company turn its inventory? The company uses a 25 percent per year cost of inventory. That is, for the hypothetical case that one unit of $1,000 would sit exactly one year in inventory, the company charges its operations division a $250 inventory cost.

b. What—in absolute terms—is the per unit inventory cost for a product that costs $1,000?

Q2.4** **(Apparel Retailing)** A large catalog retailer of fashion apparel reported $100,000,000 in revenues over the last year. On average, over the same year, the company had $5,000,000 worth of inventory in their warehouses. Assume that units in inventory are valued based on cost of goods sold (COGS) and that the retailer has a 100 percent markup on all products.

a. How many times each year does the retailer turn its inventory?

b. The company uses a 40 percent per year cost of inventory. That is, for the hypothetical case that one item of $100 COGS would sit exactly one year in inventory, the company charges itself a $40 inventory cost. What is the inventory cost for a $30 (COGS) item? You may assume that inventory turns are independent of the price.

Q2.5 **(LaVilla)** LaVilla is a village in the Italian Alps. Given its enormous popularity among Swiss, German, Austrian, and Italian skiers, all of its beds are always booked in the winter season and there are, on average, 1,200 skiers in the village. On average, skiers stay in LaVilla for 10 days.

a. How many new skiers are arriving—on average—in LaVilla every day?

b. A study done by the largest hotel in the village has shown that skiers spend on average $50 per person on the first day and $30 per person on each additional day in local restaurants. The study also forecasts that—due to increased hotel prices—the average length of stay for the 2003/2004 season will be reduced to five days. What will be the percentage change in revenues of local restaurants compared to last year (when skiers still stayed for 10 days)? Assume that hotels continue to be fully booked!

Q2.6 **(Highway)** While driving home for the holidays, you can't seem to get Little's Law out of your mind. You note that your average speed of travel is about 60 miles per hour. Moreover, the traffic report from the WXPN traffic chopper states that there is an average of 24 cars going in your direction on a one-quarter mile part of the highway. What is the flow rate of the highway (going in your direction) in cars per hour?

Q2.7 **(Industrial Baking Process)** Strohrmann, a large-scale bakery in Pennsylvania, is laying out a new production process for their packaged bread, which they sell to several grocery chains. It takes 12 minutes to bake the bread. How large an oven is required so that the company is able to produce 4,000 units of bread per hour (measured in the number of units that can be baked simultaneously)?

Q2.8** **(Mt. Kinley Consulting)** Mt. Kinley is a strategy consulting firm that divides its consultants into three classes: associates, managers, and partners. The firm has been stable in size for the last 20 years, ignoring growth opportunities in the 90s, but also not suffering from a need to downsize in the recession at the beginning of the 21st century. Specifically, there have been—and are expected to be—200 associates, 60 managers, and 20 partners.

The work environment at Mt. Kinley is rather competitive. After four years of working as an associate, a consultant goes "either up or out"; that is, becomes a manager or is dismissed from the company. Similarly, after six years, a manager either becomes a partner or is dismissed. The company recruits MBAs as associate consultants; no hires are made at the manager or partner level. A partner stays with the company for another 10 years (a total of 20 years with the company).

a. How many new MBA graduates does Mt. Kinley have to hire every year?

b. What are the odds that a new hire at Mt. Kinley will become partner (as opposed to being dismissed after 4 years or 10 years)?

Q2.9 **(Major U.S. Retailers)** The following table shows financial data (year 2004) for Costco Wholesale and Walmart, two major U.S. retailers.

	Costco	Walmart
	($ Millions)	($ Millions)
Inventories	$ 3,643	$ 29,447
Sales (net)	$48,106	$286,103
COGS	$41,651	$215,493

Source: Compustat, WRDS.

Assume that both companies have an average annual holding cost rate of 30 percent (i.e., it costs both retailers $3 to hold an item that they procured for $10 for one entire year).

a. How many days, on average, does a product stay in Costco's inventory before it is sold? Assume that stores are operated 365 days a year.

b. How much lower is, on average, the inventory cost for Costco compared to Walmart of a household cleaner valued at $5 COGS? Assume that the unit cost of the household cleaner is the same for both companies and that the price and the inventory turns of an item are independent.

Q2.10 **(McDonald's)** The following figures are taken from the 2003 financial statements of McDonald's and Wendy's.[1] Figures are in million dollars.

	McDonald's	Wendy's
Inventory	$ 129.4	$ 54.4
Revenue	17,140.5	3,148.9
Cost of goods sold	11,943.7	1,634.6
Gross profit	5,196.8	1,514.4

a. In 2003, what were McDonald's inventory turns? What were Wendy's inventory turns?

b. Suppose it costs both McDonald's and Wendy's $3 (COGS) per their value meal offerings, each sold at the same price of $4. Assume that the cost of inventory for both companies is 30 percent a year. Approximately how much does McDonald's save in inventory cost *per value meal* compared to that of Wendy's? You may assume the inventory turns are independent of the price.

[1] Example adopted from an About.com article (http://beginnersinvest.about.com/cs/investinglessons/1/blles3mcwen.htm). Financial figures taken from Morningstar.com.

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 3

Understanding the Supply Process: Evaluating Process Capacity

In the attempt to match supply with demand, an important measure is the maximum amount that a process can produce in a given unit of time, a measure referred to as the *process capacity*. To determine the process capacity of an operation, we need to analyze the operation in much greater detail compared to the previous chapter. Specifically, we need to understand the various activities involved in the operation and how these activities contribute toward fulfilling the overall demand.

In this chapter, you will learn how to perform a process analysis. Unlike Chapter 2, where we felt it was sufficient to treat the details of the operation as a black box and merely focus on the performance measures inventory, flow time, and flow rate, we now will focus on the underlying process in great detail.

Despite this increase in detail, this chapter (and this book) is not taking the perspective of an engineer. In fact, in this chapter, you will learn how to take a fairly technical and complex operation and simplify it to a level suitable for managerial analysis. This includes preparing a process flow diagram, finding the capacity and the bottleneck of the process, computing the utilization of various process steps, and computing a couple of other performance measures.

We will illustrate this new material with the Circored plant, a joint venture between the German engineering company Lurgi AG and the U.S. iron ore producer Cleveland Cliffs. The Circored plant converts iron ore (in the form of iron ore fines) into direct reduced iron (DRI) briquettes. Iron ore fines are shipped to the plant from mines in South America; the briquettes the process produces are shipped to various steel mills in the United States.

The example of the Circored process is particularly useful for our purposes in this chapter. The underlying process is complex and in many ways a masterpiece of process engineering (see Terwiesch and Loch [2002] for further details). At first sight, the process is so complex that it seems impossible to understand the underlying process behavior without a

detailed background in engineering and metallurgy. This challenging setting allows us to demonstrate how process analysis can be used to "tame the beast" and create a managerially useful view of the process, avoiding any unnecessary technical details.

3.1 How to Draw a Process Flow Diagram

The best way to begin any analysis of an operation is by drawing a *process flow diagram.* A process flow diagram is a graphical way to describe the process and it will help us to structure the information that we collect during the case analysis or process improvement project. Before we turn to the question of how to draw a process flow diagram, first consider alternative approaches to how we could capture the relevant information about a process.

Looking at the plant from above (literally), we get a picture as is depicted in Figure 3.1. At the aggregate level, the plant consists of a large inventory of iron ore (input), the plant itself (the resource), and a large inventory of finished briquettes (output). In many ways, this corresponds to the black box approach to operations taken by economists and many other managerial disciplines.

In an attempt to understand the details of the underlying process, we could turn to the engineering specifications of the plant. Engineers are interested in a detailed description of the various steps involved in the overall process and how these steps are functioning. Such descriptions, typically referred to as specifications, were used in the actual construction of the plant. Figure 3.2 provides one of the numerous specification drawings for the Circored process.

Unfortunately, this attempt to increase our understanding of the Circored process is also only marginally successful. Like the photograph, this view of the process is also a rather static one: It emphasizes the equipment, yet provides us with little understanding of how the iron ore moves through the process. In many ways, this view of a process is similar to taking the architectural drawings of a hospital and hoping that this would lead to insights about what happens to the patients in this hospital.

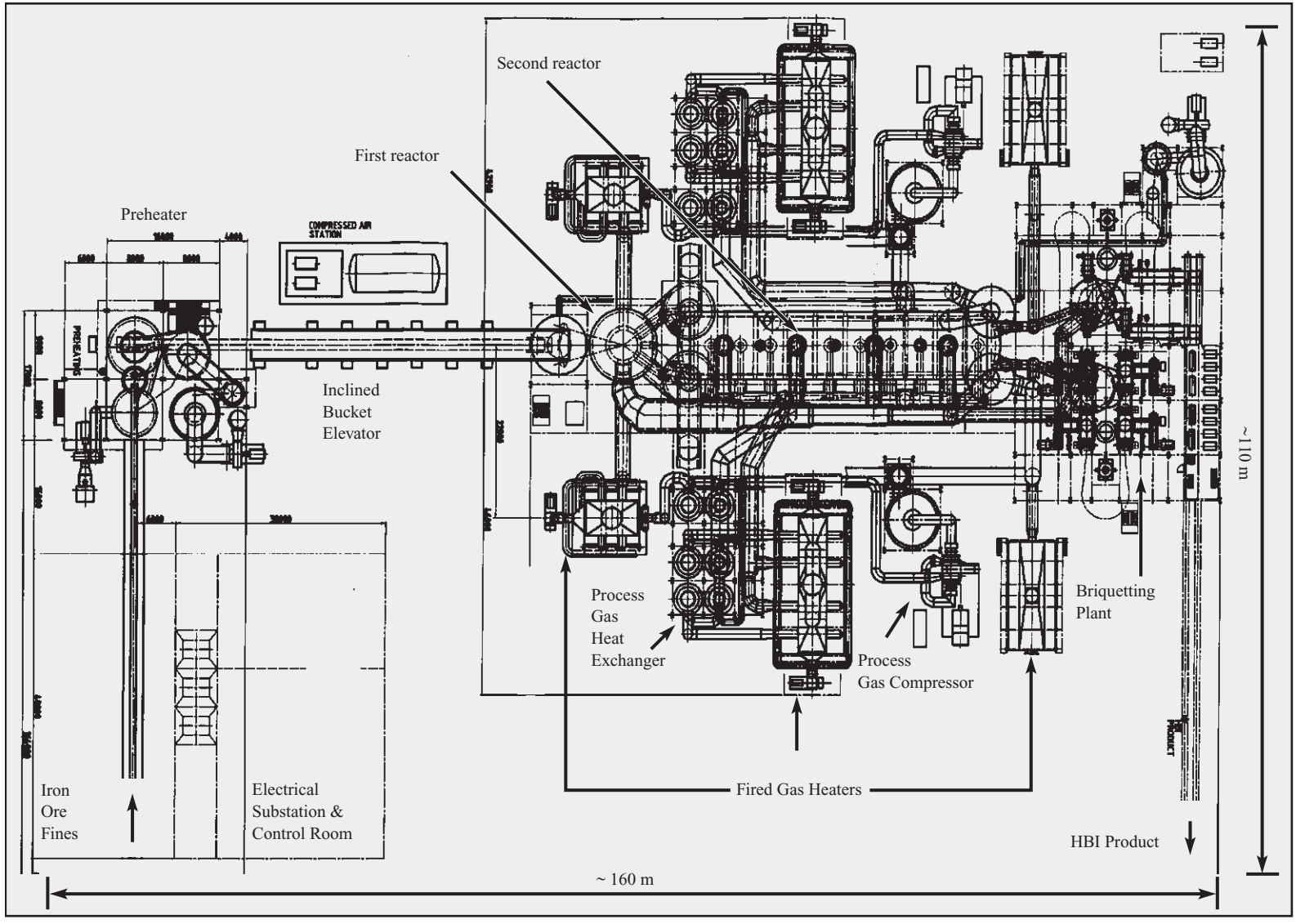
In a third—and final—attempt to get our hands around this complex process, we change our perspective from the one of the visitor to the plant (photo in Figure 3.1) or the engineers who built the plant (drawing in Figure 3.2) to the perspective of the iron ore itself

FIGURE 3.1
Photo of the Circored Plant

Source: Terwiesch and Loch 2002.

FIGURE 3.2 **Engineering Drawing**

Source: Terwiesch and Loch 2002.



and how it flows through the process. Thus, we define a unit of iron ore—a ton, a pound, or a molecule—as our flow unit and "attach" ourselves to this flow unit as it makes its journey through the process. This is similar to taking the perspective of the patient in a hospital, as opposed to taking the perspective of the hospital resources. For concreteness, we will define our flow unit to be a ton of iron ore.

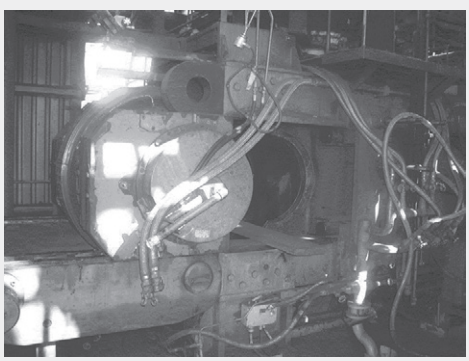
To draw a process flow diagram, we first need to focus on a part of the process that we want to analyze in greater detail; that is, we need to define the *process boundaries* and an appropriate level of detail. The placement of the process boundaries will depend on the project we are working on. For example, in the operation of a hospital, one project concerned with patient waiting time might look at what happens to the patient waiting for a lab test (e.g., check-in, waiting time, encounter with the nurse). In this project, the encounter with the doctor who requested the lab test would be outside the boundaries of the analysis. Another project related to the quality of surgery, however, might look at the encounter with the doctor in great detail, while either ignoring the lab or treating it with less detail.

A process operates on flow units, which are the entities flowing through the process (e.g., patients in a hospital, cars in an auto plant, insurance claims at an insurance company). A process flow diagram is a collection of boxes, triangles, and arrows (see Figure 3.3). Boxes stand for process activities, where the operation adds value to the flow unit. Depending on the level of detail we choose, a process step (a box) can itself be a process.

Triangles represent waiting areas or *buffers* holding inventory. In contrast to a process step, inventories do not add value; thus, a flow unit does not have to spend time in them. However, as discussed in the previous chapter, there are numerous reasons why the flow unit might spend time in inventory even if it will not be augmented to a higher value there.

FIGURE 3.3
Elements of a Process

Source: Terwiesch and Loch 2002.



Activities

- Carried out by resources
- Add value and are required for completion of the flow unit
- May or may not carry inventory
- Have a capacity (maximum number of flow units that can flow through the activity within a unit of time)

Arrows

- Indicate the flow of the flow unit
- Multiple flow unit types possible (see Section 3.6)

Inventory/Buffers

- Do *not* have a capacity; however, there might be a limited number of flow units that can be put in this inventory space at any moment of time
- Multiple flow unit types possible

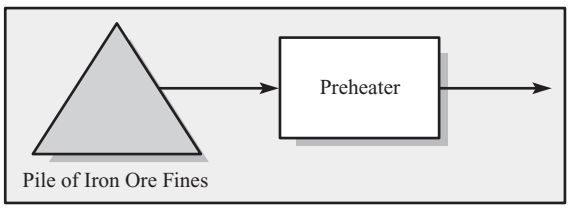


FIGURE 3.4
Process Flow Diagram, First Step

The arrows between boxes and triangles represent the route the flow unit takes through the process. If there are different flow units that take different routes through the process, it can be helpful to use different colors for the different routes. An example of this is given at the end of this chapter.

In the Circored plant, the first step the flow unit encounters in the process is the *preheater*, where the iron ore fines (which have a texture like large-grained sand) are dried and heated. The heating is achieved through an inflow of high-pressured air, which is blown into the preheater from the bottom. The high-speed air flow "fluidizes" the ore, meaning that the mixed air–ore mass (a "sandstorm") circulates through the system as if it was a fluid, while being heated to a temperature of approximately 850–900°C.

However, from a managerial perspective, we are not really concerned with the temperature in the preheater or the chemical reactions happening therein. For us, the preheater is a resource that receives iron ore from the initial inventory and processes it. In an attempt to take record of what the flow unit has experienced up to this point, we create a diagram similar to Figure 3.4.

From the preheater, a large bucket elevator transports the ore to the second process step, the *lock hoppers.* The lock hoppers consist of three large containers, separated by sets of double isolation valves. Their role is to allow the ore to transition from an oxygen-rich environment to a hydrogen atmosphere.

Following the lock hoppers, the ore enters the *circulating fluid bed reactor*, or *first reactor,* where the actual reduction process begins. The reduction process requires the ore to be in the reactor for 15 minutes, and the reactor can hold up to 28 tons of ore.

After this first reduction, the material flows into the *stationary fluid bed reactor*, or *second reactor.* This second reaction takes about four hours. The reactor is the size of a medium two-family home and contains 400 tons of the hot iron ore at any given moment in time. In the meantime, our diagram from Figure 3.4. has extended to something similar to Figure 3.5.

A couple of things are worth noting at this point:

- When creating Figure 3.5, we decided to omit the bucket elevator. There is no clear rule on when it is appropriate to omit a small step and when a step would have to be included in the process flow diagram. A reasonably good rule of thumb is to only include those process steps that are likely to affect the process flow or the economics of the process. The bucket

FIGURE 3.5 **Process Flow Diagram (to Be Continued)**

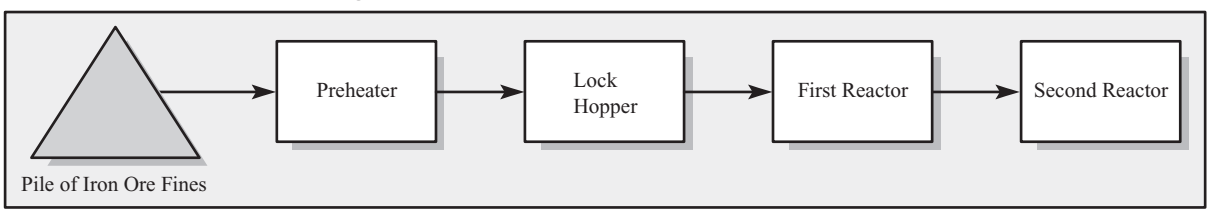
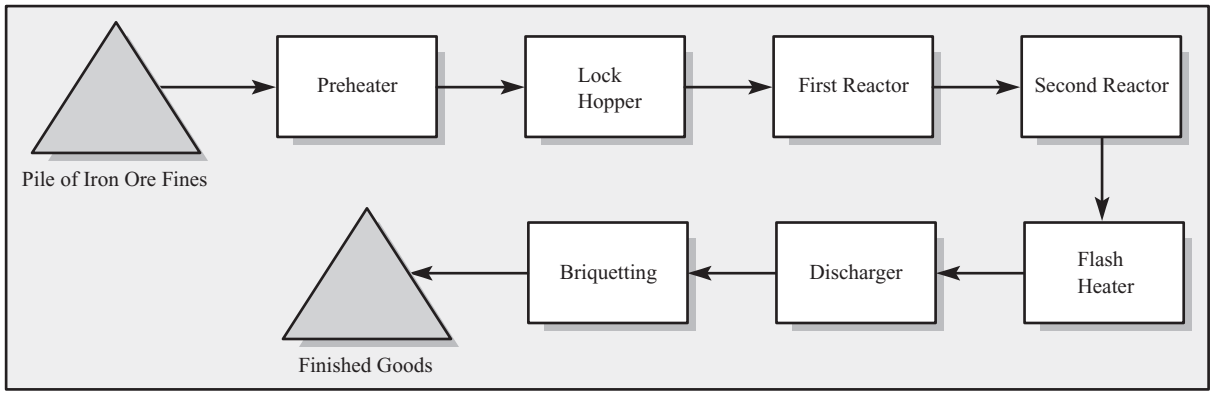


FIGURE 3.6 **Completed Process Flow Diagram for the Circored Process**



elevator is cheap, the flow units spend little time on it, and this transportation step never becomes a constraint for the process. So it is not included in our process flow diagram.

• The reaction steps are boxes, not triangles, although there is a substantial amount of ore in them, that is, they do hold inventory. The reduction steps are necessary, value-adding steps. No flow unit could ever leave the system without spending time in the reactors. This is why we have chosen boxes over triangles here.

Following the second reactor, the reduced iron enters the *flash heater*, in which a stream of high-velocity hydrogen carries the DRI to the top of the plant while simultaneously reheating it to a temperature of 685°C.

After the flash heater, the DRI enters the *pressure let-down system (discharger).* As the material passes through the discharger, the hydrogen atmosphere is gradually replaced by inert nitrogen gas. Pressure and hydrogen are removed in a reversal of the lock hoppers at the beginning. Hydrogen gas sensors assure that material leaving this step is free of hydrogen gas and, hence, safe for briquetting.

Each of the three *briquetting* machines contains two wheels that turn against each other, each wheel having the negative of one-half of a briquette on its face. The DRI is poured onto the wheels from the top and is pressed into briquettes, or iron bars, which are then moved to a large pile of finished goods inventory.

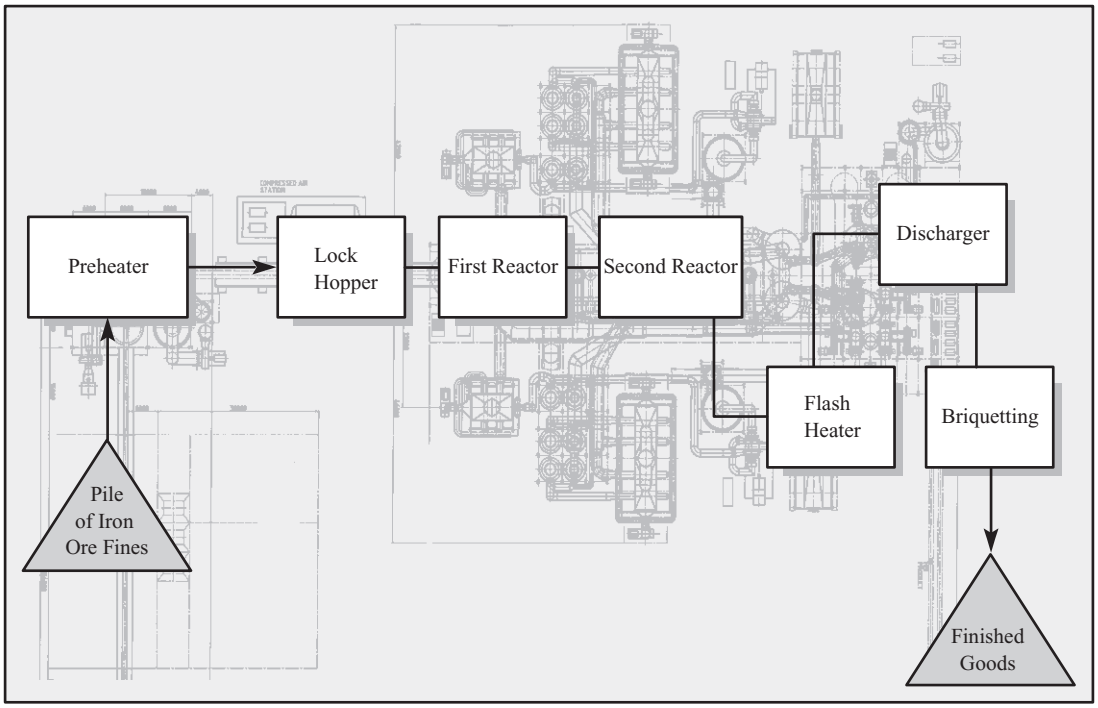
This completes our journey of the flow unit through the plant. The resulting process flow diagram that captures what the flow unit has experienced in the process is summarized in Figure 3.6.

When drawing a process flow diagram, the sizes and the exact locations of the arrows, boxes, and triangles do not carry any special meaning. For example, in the context of Figure 3.6, we chose a "U-shaped" layout of the process flow diagram, as otherwise we would have had to publish this book in a larger format.

In the absence of any space constraints, the simplest way to draw a process flow diagram for a process such as Circored's is just as one long line. However, we should keep in mind that there are more complex processes; for example, a process with multiple flow units or a flow unit that visits one and the same resource multiple times. This will be discussed further at the end of the chapter.

Another alternative in drawing the process flow diagram is to stay much closer to the physical layout of the process. This way, the process flow diagram will look familiar for engineers and operators who typically work off the specification drawings (Figure 3.2) and it might help you to find your way around when you are visiting the "real" process. Such an approach is illustrated by Figure 3.7.

FIGURE 3.7 **Completed Process Flow Diagram for the Circored Process**



3.2 Bottleneck, Process Capacity, and Flow Rate (Throughput)

From a supply perspective, the most important question that arises is how much direct reduced iron the Circored process can supply in a given unit of time, say one day. This measure is the *capacity* of the process, which we also call the *process capacity.* Not only can capacity be measured at the level of the overall process, it also can be measured at the level of the individual resources that constitute the process. Just as we defined the process capacity, we define the capacity of a resource as the maximum amount the resource can produce in a given time unit.

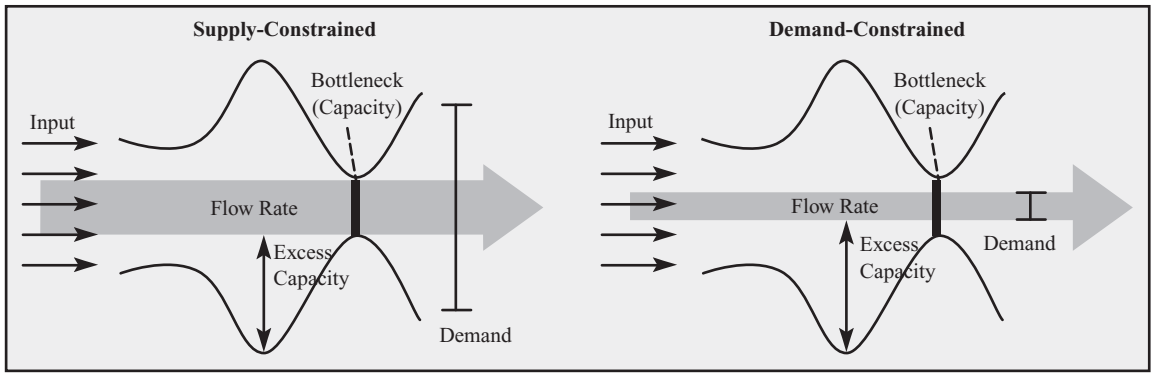
Note that the process capacity measures how much the process *can* produce, opposed to how much the process actually *does* produce. For example, consider a day where—due to a breakdown or another external event—the process does not operate at all. Its capacity would be unaffected by this, yet the flow rate would reduce to zero. This is similar to your car, which might be able to drive at 130 miles per hour (capacity), but typically—or better, hopefully—only drives at 65 miles per hour (flow rate).

As the completion of a flow unit requires the flow unit to visit every one of the resources in the process, the overall process capacity is determined by the resource with the smallest capacity. We refer to that resource as the *bottleneck.* It provides the weakest link in the overall process chain, and, as we know, a chain is only as strong as its weakest link. More formally, we can write the process capacity as

$$\text{Process capacity} = \text{Minimum}\{\text{Capacity of resource } 1, \ldots, \text{Capacity of resource } n\}$$

where there are a total of n resources. How much the process actually does produce will depend not only on its capability to create supply (process capacity), but also on the demand for its output as well as the availability of its input. As with capacity, demand and the available input should be measured as rates, that is, as flow units per unit of time. For this process, our flow unit is one ton of ore, so we could define the available input and the demand in terms of tons of ore per hour.

FIGURE 3.8 **Supply-Constrained (left) and Demand-Constrained (right) Processes**



The combination of available input, demand, and process capacity yields the rate at which our flow unit actually flows through the process, called the *flow rate:*

$$\text{Flow rate} = \text{Minimum}\{\text{Available input, Demand, Process capacity}\}$$

If demand is lower than supply (i.e., there is sufficient input available and the process has enough capacity), the process would produce at the rate of demand, independent of the process capacity. We refer to this case as *demand-constrained.* Note that in this definition demand also includes any potential requests for the accumulation of inventory. For example, while the demand for Campbell's chicken noodle soup might be lower than process capacity for the month of November, the process would not be demand-constrained if management decided to accumulate finished goods inventory in preparation for the high sales in the month of January. Thus, demand in our analysis refers to everything that is demanded from the process at a given time.

If demand exceeds supply, the process is *supply-constrained.* Depending on what limits product supply, the process is either input-constrained or capacity-constrained.

Figure 3.8 summarizes the concepts of process capacity and flow rate, together with the notion of demand- versus supply-constrained processes. In the case of the supply-constrained operation, there is sufficient input; thus, the supply constraint reflects a capacity constraint.

To understand how to find the bottleneck in a process and thereby determine the process capacity, consider each of the Circored resources. Note that all numbers are referring to tons of process output. The actual, physical weight of the flow unit might change over the course of the process.

Finding the bottleneck in many ways resembles the job of a detective in a crime story; each activity is a "suspect," in the sense that it could potentially constrain the overall supply of the process:

- The preheater can process 120 tons per hour.
- The lock hoppers can process 110 tons per hour.
- The analysis of the reaction steps is somewhat more complicated. We first observe that at any given moment of time, there can be, at maximum, 28 tons in the first reactor. Given that the iron ore needs to spend 15 minutes in the reactor, we can use Little's Law (see Chapter 2) to see that the maximum amount of ore that can flow through the reactor—and spend 15 minutes in the reactor—is

$$28 \text{ tons} = \text{Flow rate} \times 0.25 \text{ hour} \Rightarrow \text{Flow rate} = 112 \text{ tons/hour}$$

Thus, the capacity of the first reactor is 112 tons per hour. Note that a shorter reaction time in this case would translate to a higher capacity.

TABLE 3.1
Capacity Calculation

Process Step	Calculations	Capacity
Preheater		120 tons per hour
Lock hoppers		110 tons per hour
first reactor	Little's Law: Flow rate = 28 tons/0.25 hour	112 tons per hour
Second reactor	Little's Law: Flow rate = 400 tons/4 hours	100 tons per hour
Flash heater		135 tons per hour
Discharger		118 tons per hour
Briquetting machine	Consists of three machines: 3 × 55 tons per hour	165 tons per hour
Total process	Based on bottleneck, which is the stationary reactor	**100 tons per hour**

- We can apply a similar logic for the second reactor, which can hold up to 400 tons:

$$400 \text{ tons} = \text{Flow rate} \times 4 \text{ hours} \Rightarrow \text{Flow rate} = 100 \text{ tons/hour}$$

Thus, the capacity (the maximum possible flow rate through the resource) of the second reactor is 100 tons per hour.

- The flash heater can process 135 tons per hour.
- The discharger has a capacity of 118 tons per hour.
- Each of the three briquetting machines has a capacity of 55 tons per hour. As the briquetting machines collectively form one resource, the capacity at the briquetting machines is simply 3 × 55 tons per hour = 165 tons per hour.

The capacity of each process step is summarized in Table 3.1.

Following the logic outlined above, we can now identify the first reactor as the bottleneck of the Circored process. The overall process capacity is computed as the minimum of the capacities of each resource (all units are in tons per hour):

$$\text{Process capacity} = \text{Minimum}\{120, 110, 112, 100, 135, 118, 165\} = 100$$

3.3 How Long Does It Take to Produce a Certain Amount of Supply?

There are many situations where we need to compute the amount of time required to create a certain amount of supply. For example, in the Circored case, we might ask, "How long does it take for the plant to produce 10,000 tons?" Once we have determined the flow rate of the process, this calculation is fairly straightforward. Let X be the amount of supply we want to fulfill. Then,

$$\text{Time to fullfill } X \text{ units} = \frac{X}{\text{Flow rate}}$$

To answer our question,

$$\text{Time to produce } 10{,}000 \text{ tons} = \frac{10{,}000 \text{ tons}}{100 \text{ tons/hour}} = 100 \text{ hours}$$

Note that this calculation assumes the process is already producing output, that is, the first unit in our 10,000 tons flows out of the process immediately. If the process started empty,

it would take the first flow unit time to flow through the process. Chapter 4 provides the calculations for that case.

Note that in the previous equation we use flow rate, which in our case is capacity because the system is supply-constrained. However, if our system were demand-constrained, then the flow rate would equal the demand rate.

3.4 Process Utilization and Capacity Utilization

Given the first-of-its-kind nature of the Circored process, the first year of its operation proved to be extremely difficult. In addition to various technical difficulties, demand for the product (reduced iron) was not as high as it could be, as the plant's customers (steel mills) had to be convinced that the output created by the Circored process would be of the high quality required by the steel mills.

While abstracting from details such as scheduled maintenance and inspection times, the plant was designed to achieve a process capacity of 876,000 tons per year (100 tons per hour $\times$ 24 hours/day $\times$ 365 days/year, see above), the demand for iron ore briquettes was only 657,000 tons per year. Thus, there existed a mismatch between demand and potential supply (process capacity).

A common measure of performance that quantifies this mismatch is utilization. We define the *utilization* of a process as

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}}$$

Utilization is a measure of how much the process *actually produces* relative to how much it *could produce* if it were running at full speed (i.e., its capacity). This is in line with the example of a car driving at 65 miles per hour (flow rate), despite being able to drive at 130 miles per hour (capacity): the car utilizes $65/130 = 50$ percent of its potential.

Utilization, just like capacity, can be defined at the process level or the resource level. For example, the utilization of the process is the flow rate divided by the capacity of the process. The utilization of a particular resource is the flow rate divided by that resource's capacity.

For the Circored case, the resulting utilization is

$$\text{Utilization} = \frac{657{,}000 \text{ tons per year}}{876{,}000 \text{ tons per year}} = 0.75 = 75\%$$

In general, there are several reasons why a process might not produce at 100 percent utilization:

- If demand is less than supply, the process typically will not run at full capacity, but only produce at the rate of demand.
- If there is insufficient supply of the input of a process, the process will not be able to operate at capacity.
- If one or several process steps only have a limited availability (e.g., maintenance and breakdowns), the process might operate at full capacity while it is running, but then go into periods of not producing any output while it is not running.

Given that the bottleneck is the resource with the lowest capacity and that the flow rate through all resources is identical, the bottleneck is the resource with the highest utilization.

In the case of the Circored plant, the corresponding utilizations are provided by Table 3.2. Note that all resources in a process with only one flow unit have the same flow

TABLE 3.2
Utilization of the Circored Process Steps Including Downtime

Process Step	Calculations	Utilization
Preheater	657,000 tons/year/[120 tons/hour × 8,760 hours/year]	62.5%
Lock hoppers	657,000 tons/year/[110 tons/hour × 8,760 hours/year]	68.2%
First reactor	657,000 tons/year/[112 tons/hour × 8,760 hours/year]	66.9%
Second reactor	657,000 tons/year/[100 tons/hour × 8,760 hours/year]	75.0%
Flash heater	657,000 tons/year/[135 tons/hour × 8,760 hours/year]	55.6%
Discharger	657,000 tons/year/[118 tons/hour × 8,760 hours/year]	63.6%
Briquetting	657,000 tons/year/[165 tons/hour × 8,760 hours/year]	45.5%
Total process	657,000 tons/year/[100 tons/hour × 8,760 hours/year]	**75%**

rate, which is equal to the overall process flow rate. In this case, this is a flow rate of 657,000 tons per year.

Measuring the utilization of equipment is particularly common in capital-intensive industries. Given limited demand and availability problems, the bottleneck in the Circored process did not operate at 100 percent utilization. We can summarize our computations graphically, by drawing a utilization profile. This is illustrated by Figure 3.9.

Although utilization is commonly tracked, it is a performance measure that should be handled with some care. Specifically, it should be emphasized that the objective of most businesses is to maximize profit, not to maximize utilization. As can be seen in Figure 3.9, there are two reasons in the Circored case for why an individual resource might not achieve 100 percent utilization, thus exhibiting excess capacity.

- First, given that no resource can achieve a higher utilization than the bottleneck, every process step other than the bottleneck will have a utilization gap relative to the bottleneck.

FIGURE 3.9 **Utilization Profile**

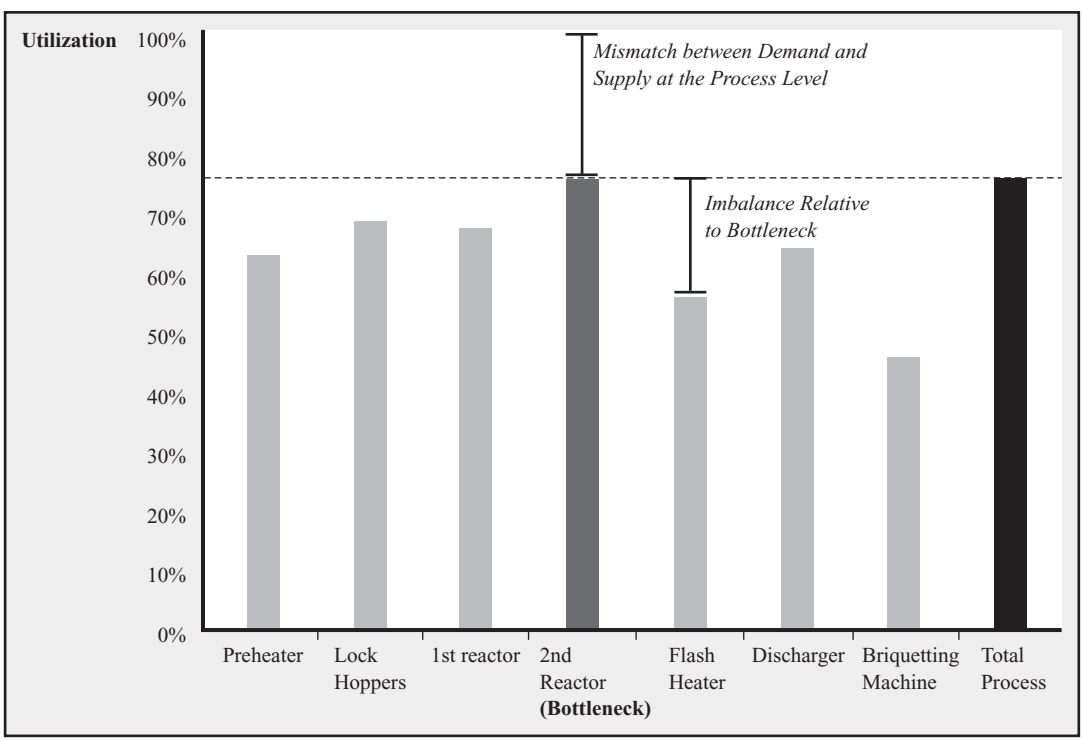


TABLE 3.3
Utilization of the Circored Process Steps Assuming Unlimited Demand and No Downtime

Process Step	Calculations	Utilization
Preheater	100/120	83.3%
Lock hoppers	100/110	90.9%
First reactor	100/112	89.3%
Second reactor	100/100	100.0%
Flash heater	100/135	74.1%
Discharger	100/118	84.7%
Briquetting machine	100/165	60.6%
Total process	**100/100**	**100%**

- Second, given that the process might not always be capacity-constrained, but rather be input- or demand-constrained, even the bottleneck might not be 100 percent utilized. In this case, every resource in the process has a "base level" of excess capacity, corresponding to the difference between the flow rate and the bottleneck capacity.

Note that the second reason disappears if there is sufficient market demand and full resource availability. In this case, only the bottleneck achieves a 100 percent utilization level. If the bottleneck in the Circored plant were utilized 100 percent, we would obtain an overall flow rate of 876,000 tons per year, or, equivalently 100 tons per hour. The resulting utilization levels in that case are summarized in Table 3.3.

3.5 Workload and Implied Utilization

Given the way we defined utilization (the ratio between flow rate and capacity), utilization can never exceed 100 percent. Thus, utilization only carries information about excess capacity, in which case utilization is strictly less than 100 percent. In contrast, we cannot infer from utilization by how much demand exceeds the capacity of the process. This is why we need to introduce an additional measure.

We define the *implied utilization* of a resource as

$$\text{Implied utilization} = \frac{\text{Demand}}{\text{Capacity}}$$

The implied utilization captures the mismatch between what could flow through the resource (demand) and what the resource can provide (capacity). Sometimes the "demand that could flow through a resource" is called the *workload.* So you can also say that the implied utilization of a resource equals its workload divided by its capacity.

Assume that demand for the Circored ore would increase to 1,095,000 tons per year (125 tons per hour). Table 3.4 calculates the resulting levels of implied utilization for the Circored resources.

TABLE 3.4
Implied Utilization of the Circored Process Steps Assuming a Demand of 125 Tons per Hour and No Downtime

Process Step	Calculations	Implied Utilization	Utilization
Preheater	125/120	104.2%	83.3%
Lock hoppers	125/110	113.6%	90.9%
First reactor	125/112	111.6%	89.3%
Second reactor	125/100	125%	100.0%
Flash heater	125/135	92.6%	74.1%
Discharger	125/118	105.9%	84.7%
Briquetting machine	125/165	75.8%	60.6%
Total process	**125/100**	**125%**	**100%**

Several points in the table deserve further discussion:

• Unlike utilization, implied utilization can exceed 100 percent. Any excess over 100 percent reflects that a resource does not have the capacity available to meet demand.

• The fact that a resource has an implied utilization above 100 percent does not make it the bottleneck. As we see in Table 3.4, it is possible to have several resources with an implied utilization above 100 percent. However, there is only one bottleneck in the process! This is the resource where the implied utilization is the highest. In the Circored case, this is—not surprisingly—the first reactor. Would it make sense to say that the process has several bottlenecks? No! Given that we can only operate the Circored process at a rate of 100 tons per hour (the capacity of the first reactor), we have ore flow through every resource of the process at a rate of 100 tons per hour. Thus, while several resources have an implied utilization above 100 percent, all resources other than the first reactor have excess capacity (their utilizations in Table 3.4 are below 100 percent). That is why we should not refer to them as bottlenecks.

• Having said this, it is important to keep in mind that in the case of a capacity expansion of the process, it might be worthwhile to add capacity to these other resources as well, not just to the bottleneck. In fact, depending on the margins we make and the cost of installing capacity, we could make a case to install additional capacity for all resources with an implied utilization above 100 percent. In other words, once we add capacity to the current bottleneck, our new process (with a new bottleneck) could still be capacity-constrained, justifying additional capacity to other resources.

3.6 Multiple Types of Flow Units

Choosing an appropriate flow unit is an essential step when preparing a process flow diagram. While, for the examples we have discussed so far, this looked relatively straightforward, there are many situations that you will encounter where this choice requires more care. The two most common complications are

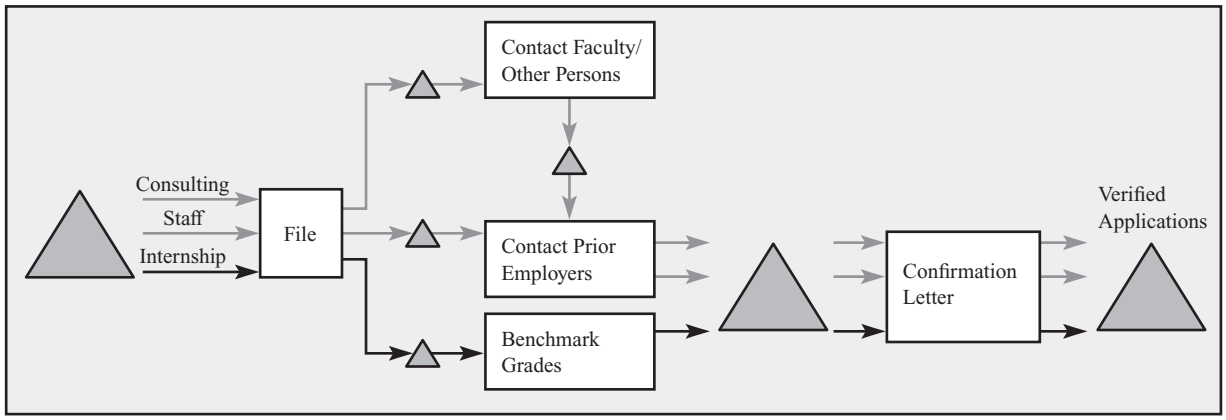
- The flow of the unit moving through the process breaks up into multiple flows. For example, in an assembly environment, following an inspection step, good units continue to the next processing step, while bad units require rework.
- There are multiple types of flow units, representing, for example, different customer types. In an emergency room, life-threatening cases follow a different flow than less complicated cases.

The critical issue in choosing the flow unit is that you must be able to express all demands and capacities in terms of the chosen flow unit. For example, in the Circored process, we chose one ton of ore to be the flow unit. Thus, we had to express each resource's capacity and the demand in terms of tons of ore. Given that the process only makes ore, the choice of the flow unit was straightforward. However, consider the following example involving multiple product or customer types. An employment verification agency receives resumés from consulting firms and law firms with the request to validate information provided by their job candidates.

Figure 3.10 shows the process flow diagram for this agency. Note that while the three customer types share the first step and the last step in the process (filing and sending confirmation letter), they differ with respect to other steps:

- For internship positions, the agency provides information about the law school/business school the candidate is currently enrolled in as well as previous institutions of higher education and, to the extent possible, provides information about the applicant's course choices and honors.

FIGURE 3.10 **Process Flow Diagram with Multiple Product Types**



- For staff positions, the agency contacts previous employers and analyzes the letters of recommendation from those employers.
- For consulting/lawyer positions, the agency attempts to call former supervisors and/or colleagues in addition to contacting the previous employers and analyzes the letters of recommendation from those employers.

As far as demand, this process receives 3 consulting, 11 staff, and 4 internship applications per hour. Table 3.5 also provides the capacities of each activity, in applications per hour. Given that the workload on each activity as well as all of the capacities can be expressed in terms of "applications per hour," we can choose "one application" as our flow unit, despite the fact that there are multiple types of applications.

The next step in our process analysis is to find the bottleneck. In this setting this is complicated by the *product mix* (different types of customers flowing through one process). For example, the process step "contact persons" might have a very long processing time, resulting in a low capacity for this activity. However, if the workload on this activity (applications per hour) is also very low, then maybe this low capacity is not an issue.

TABLE 3.5 **Finding the Bottleneck in the Multiproduct Case**

	Processing Time	Number of Workers	Capacity	Workload [Applications/Hour]				Implied Utilization
				Consulting	Staff	Interns	Total	
File	3 [min./appl.]	1	1/3 [appl./min.] = 20 [appl./hour]	3	11	4	18	18/20 = 90%
Contact persons	20 [min./appl.]	2	2/20 [appl./min.] = 6 [appl./hour]	3	0	0	3	3/6 = 50%
Contact employers	15 [min./appl.]	3	3/15 [appl./min.] = 12 [appl./hour]	3	11	0	14	14/12 = 117%
Grade/school analysis	8 [min./appl.]	2	2/8 [appl./min.] = 15 [appl./hour]	0	0	4	4	4/15 = 27%
Confirmation letter	2 [min./appl.]	1	1/2 [appl./min.] = 30 [appl./hour]	3	11	4	18	18/30 = 60%

To find the bottleneck and to determine capacity in a multiproduct situation, we need to compare each activity's capacity with its demand. The analysis is given in Table 3.5.

To compute the demand on a given activity as shown in Table 3.5, it is important to remember that some activities (e.g., filing the applications) are requested by all product types, whereas others (e.g., contacting faculty and former colleagues) are requested by one product type. This is (hopefully) clear by looking at the process flow diagram.

To complete our analysis, divide each activity's demand by its capacity to yield each activity's implied utilization. This allows us to find the busiest resource. In this case, it is "contact prior employers", so this is our bottleneck. As the implied utilization is above 100 percent, the process is capacity-constrained.

The flow unit "one application" allowed us to evaluate the implied utilization of each activity in this process, but it is not the only approach. Alternatively, we could define the flow unit as "one minute of work." This might seem like an odd flow unit, but it has an advantage over "one application." Before explaining its advantage, let's figure out how to replicate our analysis of implied utilization with this new flow unit.

As before, we need to define our demands and our capacities in terms of our flow unit. In the case of capacity, each worker has "60 minutes of work" available per hour. (By definition, we all do!) So the capacity of an activity is (Number of workers) $\times$ 60 [minutes/hour]. For example "contact persons" has two workers. So its capacity is $2 \times 60 = 120$ minutes of work per hour. Each worker has 60 "minutes of work" available per hour, so two of them can deliver 120 minutes of work.

Now turn to the demands. There are 11 staff applications to be processed each hour and each takes 3 minutes. So the demand for staff applications is $11 \times 3 = 33$ minutes per hour. Now that we know how to express the demands and the capacities in terms of the "minutes of work," the implied utilization of each activity is again the ratio of the amount demanded from the activity to the activity's capacity. Table 3.6 summarizes these calculations. As we would expect, this method yields the same implied utilizations as the "one application" flow unit approach.

So if "one application" and "one minute of work" give us the same answer, how should we choose between these approaches? In this situation, you would work with the approach that you find most intuitive (which is probably "one application," at least initially) because they both allow us to evaluate the implied utilizations. However, the "one minute of work" approach is more robust. To explain why, suppose it took 3 minutes to file a staff application, 5 minutes to file a consulting application, and 2 minutes to file an internship application. In this case, we get into trouble if we define the flow unit to be "one application"—with that flow unit, we cannot express the capacity of the file activity! If we receive only internship applications, then filing could process $60/2 = 30$ applications per hour. However, if we receive only consulting applications, then filing can only process $60/5 = 12$ applications per hour. The number of applications per hour that filing can process depends on the mix of applications! The "minute of work" flow unit completely solves that problem—no matter what mix of applications is sent to filing, with one worker, filing has 60 minutes of work available per hour. Similarly, for a given mix of applications, we can also evaluate the workload on filing in terms of minutes of work (just as is done in Table 3.6.

To summarize, choose a flow unit that allows you to express all demands and capacities in terms of that flow unit. An advantage of the "minute of work" (or "hour of work," "day of work," etc.) approach is that it is possible to do this even if there are multiple types of products or customers flowing through the process.

So what is the next step in our process analysis? We have concluded that it is capacity-constrained because the implied utilization of "contact employers" is greater

TABLE 3.6 Using "One Minute of Work" as the Flow Unit to Find the Bottleneck in the Multiproduct Case

	Processing Time	Number of Workers	Capacity	Workload [Minutes/Hour]				Implied Utilization
				Consulting	Staff	Interns	Total	
File	3 [min./appl.]	1	60 [min./hour]	3 × 3	11 × 3	4 × 3	54	54/60 = 90%
Contact persons	20 [min./appl.]	2	120 [min./hour]	3 × 20	0	0	60	60/120 = 50%
Contact employers	15 [min./appl.]	3	180 [min./hour]	3 × 15	11 × 15	0	210	210/180 = 117%
Grade/school analysis	8 [min./appl.]	2	120 [min./hour]	0	0	4 × 8	32	32/120 = 27%
Confirmation letter	2 [min./appl.]	1	60 [min./hour]	3 × 2	11 × 2	4 × 2	36	36/60 = 60%

than 100 percent—it is the bottleneck. Given that it is the only activity with an implied utilization greater than 100 percent, if we are going to add capacity to this process, "contact employers" should be the first candidate—in the current situation, they simply do not have enough capacity to handle the current mix of customers. Notice, if the mix of customers changes, this situation might change. For example, if we started to receive fewer staff applications (which have to flow through "contact employers") and more internship applications (which do not flow through "contact employers") then the workload on "contact employers" would decline, causing its implied utilization to fall as well. Naturally, shifts in the demands requested from a process can alter which resource in the process is the bottleneck.

Although we have been able to conclude something useful with our analysis, one should be cautious to not conclude too much when dealing with multiple types of products or customers. To illustrate some potential complications, consider the following example. At the international arrival area of a major U.S. airport, 15 passengers arrive per minute, 10 of whom are U.S. citizens or permanent residents and 5 are visitors.

The immigration process is organized as follows. Passengers disembark their aircraft and use escalators to arrive in the main immigration hall. The escalators can transport up to 100 passengers per minute. Following the escalators, passengers have to go through immigration. There exist separate immigration resources for U.S. citizens and permanent residents (they can handle 10 passengers per minute) and visitors (which can handle 3 visitors per minute). After immigration, all passengers pick up their luggage. Luggage handling (starting with getting the luggage off the plane and ending with moving the luggage onto the conveyor belts) has a capacity of 10 passengers per minute. Finally, all passengers go through customs, which has a capacity of 20 passengers per minute.

We calculate the implied utilization levels in Table 3.7. Notice when evaluating implied utilization we assume the demand on luggage handling is 10 U.S. citizens and 5 visitors even though we know (or discover via our calculations) that it is not possible for 15 passengers to arrive to luggage handling per minute (there is not enough capacity in immigration). We do this because we want to compare the potential demand on each resource with its capacity to assess its implied utilization. Consequently, we can evaluate each resource's implied utilization in isolation from the other resources.

Based on the values in Table 3.7, the bottleneck is immigration for visitors because it has the highest implied utilization. Furthermore, because its implied utilization is

TABLE 3.7
Calculating Implied Utilization in Airport Example

Resource	Demand for U.S. Citizens and Permanent Residents [Pass./Min.]	Demand for Visitors [Pass./Min.]	Capacity [Pass./Min.]	Implied Utilization
Escalator	10	5	100	15/100 = 15%
Immigration—U.S. residents	10	0	10	10/10 = 100%
Immigration—visitors	0	5	3	5/3 = 167%
Luggage handling	10	5	10	15/10 = 150%
Customs	10	5	20	15/20 = 75%

greater than 100 percent, the process is supply-constrained. Given that there is too little supply, we can expect queues to form. Eventually, those queues will clear because the demand rate of arriving passengers will at some point fall below capacity (otherwise, the queues will just continue to grow, which we know will not happen indefinitely at an airport). But during the times in which the arrival rates of passengers is higher than our capacity, where will the queues form? The answer to this question depends on how we prioritize work.

The escalator has plenty of capacity, so no priority decision needs to be made there. At immigration, there is enough capacity for 10 U.S. citizens and 3 visitors. So 13 passengers may be passed on to luggage handling, but luggage handling can accommodate only 10 passengers. Suppose we give priority to U.S. citizens. In that case, all of the U.S. citizens proceed through luggage handling without interruption, and a queue of visitors will form at the rate of 3 per minute. Of course, there will also be a queue of visitors in front of immigration, as it can handle only 3 per minute while 5 arrive per minute. With this priority scheme, the outflow from this process will be 10 US citizens per minute. However, if we give visitors full priority at luggage handling, then a similar analysis reveals that a queue of U.S. citizens forms in front of luggage handling, and a queue of visitors forms in front of immigration. The outflow is 7 U.S. citizens and 3 visitors.

The operator of the process may complain that the ratio of U.S. citizens to visitors in the outflow (7 to 3) does not match the inflow ratio (2 to 1), even though visitors are given full priority. If we were to insist that those ratios match, then the best we could do is have an outflow of 6 U.S. citizens and 3 visitors—we cannot produce more than 3 visitors per minute given the capacity of immigration, so the 2 to 1 constraint implies that we can "produce" no more than 6 U.S. citizens per minute. Equity surely has a price in this case—we could have an output of 10 passengers per minute, but the equity constraint would limit us to 9 passengers per minute. To improve upon this output while maintaining the equity constraint, we should add more capacity at the bottleneck—immigration for visitors.

3.7 Summary

Figure 3.11 is a summary of the major steps graphically. Exhibits 3.1 and 3.2 summarize the steps required to do the corresponding calculations for a single flow unit and multiple flow units, respectively.

Exhibit 3.1

STEPS FOR BASIC PROCESS ANALYSIS WITH ONE TYPE OF FLOW UNIT

1. Find the capacity of every resource; if there are multiple resources performing the same activity, add their capacities together.
2. The resource with the lowest capacity is called the *bottleneck.* Its capacity determines the capacity of the entire process (*process capacity*).
3. The flow rate is found based on

$$\text{Flow rate} = \text{Minimum } \{\text{Available input, Demand, Process capacity}\}$$

4. We find the utilization of the process as

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}}$$

The utilization of each resource can be found similarly.

Any process analysis should begin with the creation of a process flow diagram. This is especially important for the case of multiple flow units, as their flows are typically more complex.

Next, we need to identify the bottleneck of the process. As long as there exists only one type of flow unit, this is simply the resource with the lowest capacity. However, for more general cases, we need to perform some extra analysis. Specifically, if there is a product mix, we have to compute the requested capacity (workload) at each resource and then compare it to the available capacity. This corresponds to computing the implied utilization, and we identify the bottleneck as the resource with the highest implied utilization.

Finally, once we have found the bottleneck, we can compute a variety of performance measures. As in the previous chapter, we are interested in finding the flow rate. The flow rate also allows us to compute the process utilization as well as the utilization profile across resources. Utilizations, while not necessarily a business goal by themselves, are important measures in many industries, especially capital-intensive industries.

FIGURE 3.11 **Summary of Process Analysis**

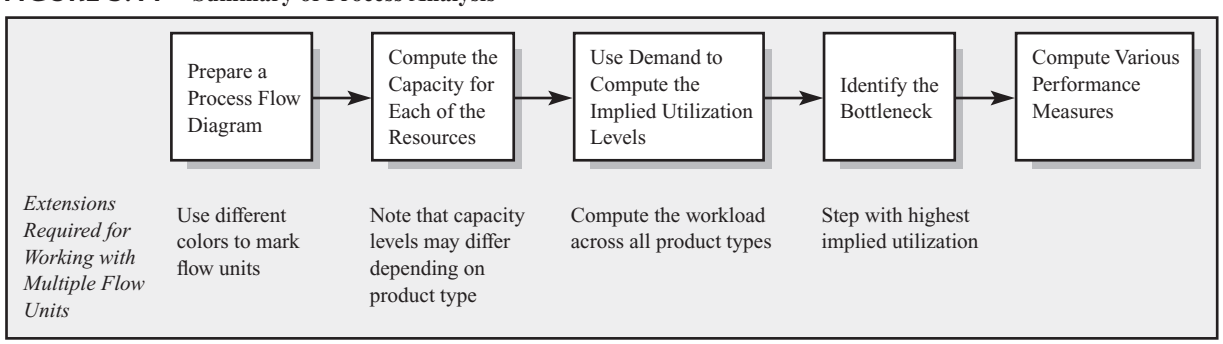


Exhibit 3.2

STEPS FOR BASIC PROCESS ANALYSIS WITH MULTIPLE TYPES OF FLOW UNITS

1. For each resource, compute the number of minutes that the resource can produce; this is 60 [min./hour] × Number of resources within the resource pool.
2. Create a process flow diagram, indicating how the flow units go through the process; use multiple colors to indicate the flow of the different flow units.
3. Create a table indicating how much workload each flow unit is consuming at each resource:
   - The rows of the table correspond to the resources in the process.
   - The columns of the table correspond to the different types of flow units.
   - Each cell of the table should contain one of the following:

   If flow unit does not visit the corresponding resource, 0;
   Otherwise, demand per hour of the corresponding flow unit × processing time.
4. Add up the workload of each resource across all flow units.
5. Compute the implied utilization of each resource as

$$\text{Implied utilization} = \frac{\text{Result of step 4}}{\text{Result of step 1}}$$

The resource with the highest implied utilization is the bottleneck.

The preceding approach is based on Table 3.6; that is, the flow unit is "one minute of work."

3.8 Practice Problems

Q3.1* **(Process Analysis with One Flow Unit)** Consider a process consisting of three resources:

Resource	Processing Time [Min./Unit]	Number of Workers
1	10	2
2	6	1
3	16	3

What is the bottleneck? What is the process capacity? What is the flow rate if demand is eight units per hour? What is the utilization of each resource if demand is eight units per hour?

Q3.2* **(Process Analysis with Multiple Flow Units)** Consider a process consisting of five resources that are operated eight hours per day. The process works on three different products, A, B, and C:

Resource	Number of Workers	Processing Time for A [Min./Unit]	Processing Time for B [Min./Unit]	Processing Time for C [Min./Unit]
1	2	5	5	5
2	2	3	4	5
3	1	15	0	0
4	1	0	3	3
5	2	6	6	6

Demand for the three different products is as follows: product A, 40 units per day; product B, 50 units per day; and product C, 60 units per day.

What is the bottleneck? What is the flow rate for each flow unit assuming that demand must be served in the mix described above (i.e., for every four units of A, there are five units of B and six units of C)?

(* indicates that the solution is at the end of the book)

Q3.3 **(Cranberries)** International Cranberry Uncooperative (ICU) is a competitor to the National Cranberry Cooperative (NCC). At ICU, barrels of cranberries arrive on trucks at a rate of 150 barrels per hour and are processed continuously at a rate of 100 barrels per hour. Trucks arrive at a uniform rate over eight hours, from 6:00 a.m. until 2:00 p.m. Assume the trucks are sufficiently small so that the delivery of cranberries can be treated as a continuous inflow. The first truck arrives at 6:00 a.m. and unloads immediately, so processing begins at 6:00 a.m. The bins at ICU can hold up to 200 barrels of cranberries before overflowing. If a truck arrives and the bins are full, the truck must wait until there is room in the bins.

a. What is the maximum number of barrels of cranberries that are waiting on the trucks at any given time?

b. At what time do the trucks stop waiting?

c. At what time do the bins become empty?

d. ICU is considering using seasonal workers in addition to their regular workforce to help with the processing of cranberries. When the seasonal workers are working, the processing rate increases to 125 barrels per hour. The seasonal workers would start working at 10:00 a.m. and finish working when the trucks stop waiting. At what time would ICU finish processing the cranberries using these seasonal workers?

Q3.4 **(Western Pennsylvania Milk Company)** The Western Pennsylvania Milk Company is producing milk at a fixed rate of 5,000 gallons/hour. The company's clients request 100,000 gallons of milk over the course of one day. This demand is spread out uniformly from 8 a.m. to 6 p.m. If there is no milk available, clients will wait until enough is produced to satisfy their requests.

The company starts producing at 8 a.m. with 25,000 gallons in finished goods inventory. At the end of the day, after all demand has been fulfilled, the plant keeps on producing until the finished goods inventory has been restored to 25,000 gallons.

When answering the following questions, treat trucks/milk as a continuous flow process. Begin by drawing a graph indicating how much milk is in inventory and how much milk is "back-ordered" over the course of the day.

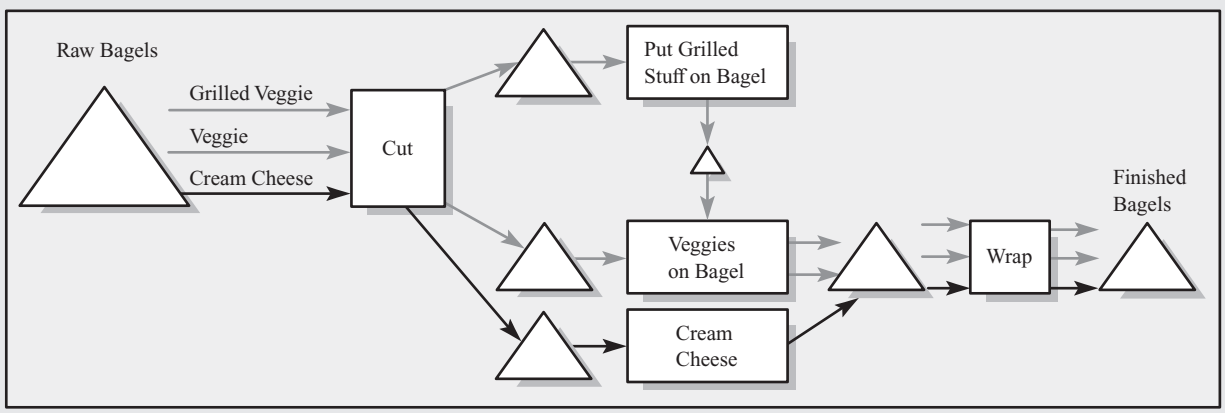
a. At what time during the day will the clients have to start waiting for their requests to be filled?

b. At what time will clients stop waiting?

c. Assume that the milk is picked up in trucks that hold 1,250 gallons each. What is the maximum number of trucks that are waiting?

d. Assume the plant is charged $50 per hour per waiting truck. What are the total waiting time charges on a day?

Q3.5** **(Bagel Store)** Consider a bagel store selling three types of bagels that are produced according to the process flow diagram outlined below. We assume the demand is 180 bagels a day, of which there are 30 grilled veggie, 110 veggie only, and 40 cream cheese. Assume that the workday is 10 hours long and each resource is staffed with one worker.



Moreover, we assume the following Processing times:

	Cut	Grilled Stuff	Veggies	Cream Cheese	Wrap
Processing time	3 [min./bagel]	10 [min./bagel]	5 [min./bagel]	4 [min./bagel]	2 [min./bagel]

Processing times are independent of which bagel type is processed at a resource (for example, cutting a bagel takes the same time for a cream cheese bagel as for a veggie bagel).

a. Where in the process is the bottleneck?

b. How many units can the process produce within one hour, assuming the product mix has to remain constant?

Q3.6 **(Valley Forge Income Tax Advice)** VF is a small accounting firm supporting wealthy individuals in their preparation of annual income tax statements. Every December, VF sends out a short survey to their customers, asking for the information required for preparing the tax statements. Based on 24 years of experience, VF categorizes their cases into the following groups:

- Group 1 (new customers, easy): 15 percent of cases
- Group 2 (new customers, complex): 5 percent of cases
- Group 3 (repeat customers, easy): 50 percent of cases
- Group 4 (repeat customers, complex): 30 percent of cases

Here, "easy" versus "complex" refers to the complexity of the customer's earning situation.

In order to prepare the income tax statement, VF needs to complete the following set of activities. Processing times (and even which activities need to be carried out) depend on which group a tax statement falls into. All of the following processing times are expressed in minutes per income tax statement.

Group	Filing	Initial Meeting	Preparation	Review by Senior Accountant	Writing
1	20	30	120	20	50
2	40	90	300	60	80
3	20	No meeting	80	5	30
4	40	No meeting	200	30	60

The activities are carried out by the following three persons:

- Administrative support person: filing and writing.
- Senior accountant (who is also the owner): initial meeting, review by senior accountant.
- Junior accountant: preparation.

Assume that all three persons work eight hours per day and 20 days a month. For the following questions, assume the product mix as described above. Assume that there are 50 income tax statements arriving each month.

a. Which of the three persons is the bottleneck?

b. What is the (implied) utilization of the senior accountant? The junior accountant? The administrative support person?

c. You have been asked to analyze which of the four product groups is the most profitable. Which factors would influence the answer to this?

d. How would the process capacity of VF change if a new word processing system would reduce the time to write the income tax statements by 50 percent?

Q3.7 **(Car Wash Supply Process)** CC Car Wash specializes in car cleaning services. The services offered by the company, the exact service time, and the resources needed for each of them are described in the table following:

Service	Description	Processing Time	Resource Used
A. Wash	Exterior car washing and drying	10 min.	1 automated washing machine
B. Wax	Exterior car waxing	10 min.	1 automated waxing machine
C. Wheel cleaning	Detailed cleaning of all wheels	7 min.	1 employee
D. Interior cleaning	Detailed cleaning inside the car	20 min.	1 employee

The company offers the following packages to their customers:

- Package 1: Includes only car wash (service A).
- Package 2: Includes car wash and waxing (services A and B).
- Package 3: Car wash, waxing, and wheel cleaning (services A, B, and C).
- Package 4: All four services (A, B, C, and D).

Customers of CC Car Wash visit the station at a constant rate (you can ignore any effects of variability) of 40 customers per day. Of these customers, 40 percent buy Package 1, 15 percent buy Package 2, 15 percent buy Package 3, and 30 percent buy Package 4. The mix does not change over the course of the day. The store operates 12 hours a day.

a. What is the implied utilization of the employee doing the wheel cleaning service?

b. Which resource has the highest implied utilization?

For the next summer, CC Car Wash anticipates an increase in the demand to 80 customers per day. Together with this demand increase, there is expected to be a change in the mix of packages demanded: 30 percent of the customers ask for Package 1, 10 percent for Package 2, 10 percent for Package 3, and 50 percent for Package 4. The company will install an additional washing machine to do service A.

c. What will be the new bottleneck in the process?

d. How many customers a day will not be served? Which customers are going to wait? Explain your reasoning!

Q3.8 **(Starbucks)** After an "all night" study session the day before their last final exam, four students decide to stop for some much-needed coffee at the campus Starbucks. They arrive at 8:30 a.m. and are dismayed to find a rather long line.

Fortunately for the students, a Starbucks executive happens to be in line directly in front of them. From her, they learn the following facts about this Starbucks location:

I. There are three employee types:
   - There is a single cashier who takes all orders, prepares nonbeverage food items, grinds coffee, and pours drip coffee.
   - There is a single frozen drink maker who prepares blended and iced drinks.
   - There is a single espresso drink maker who prepares espressos, lattes, and steamed drinks.

II. There are typically four types of customers:
   - Drip coffee customers order only drip coffee. This requires 20 seconds of the cashier's time to pour the coffee.
   - Blended and iced drink customers order a drink that requires the use of the blender. These drinks take on average 2 minutes of work of the frozen drink maker.
   - Espresso drink customers order a beverage that uses espresso and/or steamed milk. On average, these drinks require 1 minute of work of the espresso drink maker.
   - Ground coffee customers buy one of Starbucks' many varieties of whole bean coffee and have it ground to their specification at the store. This requires a total of 1 minute of the cashier's time (20 seconds to pour the coffee and 40 seconds to grind the whole bean coffee).

III. The customers arrive uniformly at the following rates from 7 a.m. (when the store opens) until 10 a.m. (when the morning rush is over), with no customers arriving after 10 a.m.:
   - Drip coffee customers: 25 per hour.
   - Blended and iced drink customers: 20 per hour.
   - Espresso drink customers: 70 per hour.
   - Ground coffee customers: 5 per hour.

IV. Each customer spends, on average, 20 seconds with the cashier to order and pay.

V. Approximately 25 percent of all customers order food, which requires an additional 20 seconds of the cashier's time per transaction.

While waiting in line, the students reflect on these facts and they answer the following questions:

a. What is the implied utilization of the frozen drink maker?

b. Which resource has the highest implied utilization?

From their conversation with the executive, the students learn that Starbucks is considering a promotion on all scones (half price!), which marketing surveys predict will increase the percentage of customers ordering food to 30 percent (the overall arrival rates of customers will *not* change). However, the executive is worried about how this will affect the waiting times for customers.

c. How do the levels of implied utilization change as a response to this promotion?

Q3.9 **(Paris Airport)** Kim Opim, an enthusiastic student, is on her flight over from Philadelphia (PHL) to Paris. Kim reflects upon how her educational experiences from her operations courses could help explain the long wait time that she experienced before she could enter the departure area of Terminal A at PHL. As an airline representative explained to Kim, there are four types of travelers in Terminal A:

- Experienced short-distance (short-distance international travel destinations are Mexico and various islands in the Atlantic) travelers: These passengers check in online and do not speak with any agent nor do they take any time at the kiosks.
- Experienced long-distance travelers: These passengers spend 3 minutes with an agent.
- Inexperienced short-distance travelers: These passengers spend 2 minutes at a kiosk; however, they do not require the attention of an agent.
- Inexperienced long-distance travelers: These passengers need to talk 5 minutes with an agent.

After a passenger checks in online, or talks with an agent, or uses a kiosk, the passenger must pass through security, where they need 0.5 minutes independent of their type. From historical data, the airport is able to estimate the arrival rates of the different customer types at Terminal A of Philadelphia International:

- Experienced short-distance travelers: 100 per hour
- Experienced long-distance travelers: 80 per hour
- Inexperienced short-distance travelers: 80 per hour
- Inexperienced long-distance travelers: 40 per hour

At this terminal, there are four security check stations, six agents, and three electronic kiosks. Passengers arrive uniformly from 4 p.m. to 8 p.m., with the entire system empty prior to 4 p.m. (the "midafternoon lull") and no customers arrive after 8 p.m. All workers must stay on duty until the last passenger is entirely through the system (e.g., has passed through security).

a. What are the levels of implied utilization at each resource?

b. At what time has the last passenger gone through the system? Note: If passengers of one type have to wait for a resource, passengers that do not require service at the resource can pass by the waiting passengers!

c. Kim, an experienced long-distance traveler, arrived at 6 p.m. at the airport and attempted to move through the check-in process as quickly as she could. How long did she have to wait before she was checked at security?

d. The airline considers showing an educational program that would provide information about the airport's check-in procedures. Passenger surveys indicate that 80 percent of the inexperienced passengers (short or long distance) would subsequently act as experienced passengers (i.e., the new arrival rates would be 164 experienced short-distance, 112 experienced long-distance, 16 inexperienced short-distance, and 8 inexperienced long-distance [passengers/hour]). At what time has the last passenger gone through the system?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 4

Estimating and Reducing Labor Costs

The objective of any process should be to create value (make profits), not to maximize the utilization of every resource involved in the process. In other words, we should not attempt to produce more than what is demanded from the market, or from the resource downstream in the process, just to increase the utilization measure. Yet, the underutilization of a resource, human labor or capital equipment alike, provides opportunities to improve the process. This improvement can take several forms, including

- If we can reduce the excess capacity at some process step, the overall process becomes more efficient (lower cost for the same output).
- If we can use capacity from underutilized process steps to increase the capacity at the bottleneck step, the overall process capacity increases. If the process is capacity-constrained, this leads to a higher flow rate.

In this chapter, we discuss how to achieve such process improvements. Specifically, we discuss the concept of line balancing, which strives to avoid mismatches between what is supplied by one process step and what is demanded from the following process step (referred to as the process step downstream). In this sense, line balancing attempts to match supply and demand within the process itself.

We use Novacruz Inc. to illustrate the concept of line balancing and to introduce a number of more general terms of process analysis. Novacruz is the producer of a high-end kick scooter, known as the Xootr (pronounced "zooter"), displayed in Figure 4.1.

4.1 Analyzing an Assembly Operation

With the increasing popularity of kick scooters in general, and the high-end market segment for kick scooters in particular, Novacruz faced a challenging situation in terms of organizing their production process. While the demand for their product was not much higher than 100 scooters per week in early March 2000, it grew dramatically, soon reaching 1,200 units per week in the fall of 2000. This demand trajectory is illustrated in Figure 4.2.

First consider March 2000, during which Novacruz faced a demand of 125 units per week. At this time, the assembly process was divided between three workers (resources) as illustrated by Figure 4.3.

The three workers performed the following activities. In the first activity, the first 30 of the overall 80 parts are assembled, including the fork, the steer support, and the t-handle.

FIGURE 4.1
The Xootr by Novacruz

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Given the complexity of this assembly operation, it takes about 13 minutes per scooter to complete this activity. We refer to the 13 minutes/unit as the *processing time.* Depending on the context, we will also refer to the processing time as the *activity time* or the *service time* Note that in the current process, each activity is staffed with exactly one worker.

In the second activity, a worker assembles the wheel, the brake, and some other parts related to the steering mechanism. The second worker also assembles the deck. This step is somewhat faster and its processing time is 11 minutes per unit. The scooter is completed by the third worker, who wipes off the product, applies the decals and grip tape, and conducts the final functional test. The processing time is about 8 minutes per unit.

FIGURE 4.2
Lifecycle Demand Trajectory for Xootrs

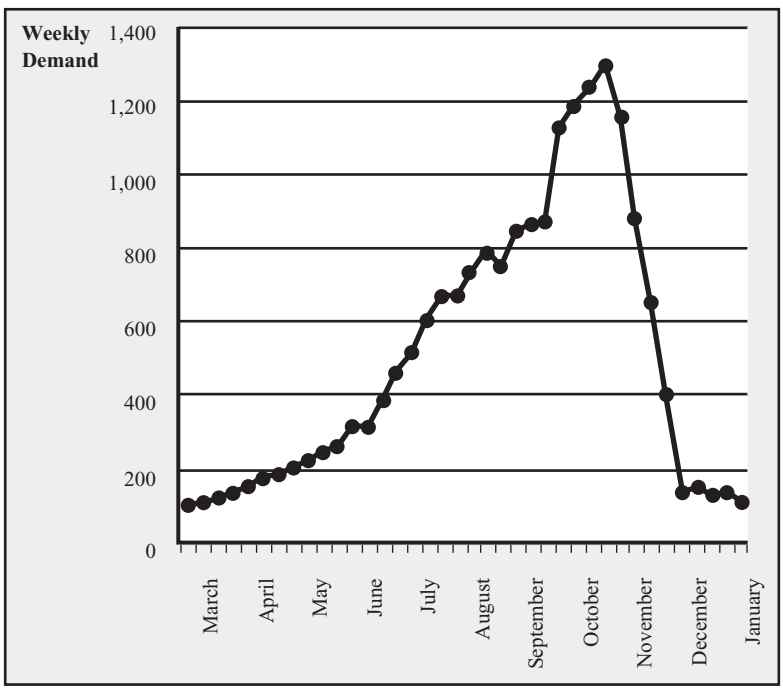

FIGURE 4.3
Current process layout

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To determine the capacity of an individual resource or a group of resources performing the same activity, we write:

$$\text{Capacity} = \frac{\text{Number of resources}}{\text{Processing time}}$$

This is intuitive, as the capacity grows proportionally with the number of workers.

For example, for the first activity, which is performed by one worker, we write:

$$\text{Capacity} = \frac{1}{13 \text{ minutes/scooter}} = 0.0769 \text{ scooter/minute}$$

which we can rewrite as

$$0.0769 \text{ scooter/minute} \times 60 \text{ minutes/hour} = 4.6 \text{ scooters/hour}$$

Similarly, we can compute capacities of the second worker to be 5.45 scooters/hour and of the third worker to be 7.5 scooters/hour.

As we have done in the preceding chapter, we define the bottleneck as the resource with the lowest capacity. In this case, the bottleneck is the first resource, resulting in a process capacity of 4.6 scooters/hour.

4.2 Time to Process a Quantity *X* Starting with an Empty Process

Imagine Novacruz received a very important rush order of 100 scooters, which would be assigned highest priority. Assume further that this order arrives early in the morning and there are no scooters currently in inventory, neither between the resources (work-in-process, WIP) nor in the finished goods inventory (FGI). How long will it take to fulfill this order?

As we are facing a large order of scooters, we will attempt to move as many scooters through the system as possible. Therefore, we are capacity-constrained and the flow rate of the process is determined by the capacity of the bottleneck (one scooter every 13 minutes). The time between the completions of two subsequent flow units is called the *cycle time* of a process and will be defined more formally in the next section.

We cannot simply compute the time to produce 100 units as 100 units/0.0769 unit/minute = 1,300 minutes because that calculation assumes the system is producing at the bottleneck rate, one unit every 13 minutes. However, that is only the case once the system is "up and running." In other words, the first scooter of the day, assuming the system starts the day empty (with no work in process inventory), takes even longer than 13 minutes to complete. How much longer depends on how the line is paced.

The current system is called a *worker-paced* line because each worker is free to work at his or her own pace: if the first worker finishes before the next worker is ready to accept the parts, then the first worker puts the completed work in the inventory between them. Eventually the workers need to conform to the bottleneck rate; otherwise, the inventory before the bottleneck would grow too big for the available space. But that concern is not relevant for the first unit moving through the system, so the time to get the first scooter through the system is 13 + 11 + 8 = 32 minutes. More generally:

$$\text{Time through an empty worker-paced process} = \text{Sum of the processing times}$$

An alternative to the worker-paced process is a machine-paced process as depicted in Figure 4.4. In a machine-paced process, all of the steps must work at the same rate even with the first unit through the system. Hence, if a machine-paced process were used, then the first Xootr would be produced after 3 × 13 minutes, as the conveyor belt has the same speed at all three process steps (there is just one conveyor belt, which has to be paced to the slowest step). More generally,

$$\text{Time through an empty machine-paced process} = \text{Number of resources in sequence} \times \text{Processing time of the bottleneck step}$$

Now return to our worker-paced process. After waiting 32 minutes for the first scooter, it only takes an additional 13 minutes until the second scooter is produced and from then onwards, we obtain an additional scooter every 13 minutes. Thus, scooter 1 is produced after 32 minutes, scooter 2 after 32 + 13 = 45 minutes, scooter 3 after 32 + (2 × 13) = 58 minutes, scooter 4 after 32 + (3 × 13) = 71 minutes, and so on.

More formally, we can write the following formula. The time it takes to finish X units starting with an empty system is

$$\text{Time to finish } X \text{ units starting with an empty system} = \text{Time through an empty process} + \frac{X - 1 \text{ unit}}{\text{Flow rate}}$$

You may wonder whether it is always necessary to be so careful about the difference between the time to complete the first unit and all of the rest of the units. In this case, it is because the number of scooters is relatively small, so each one matters. But

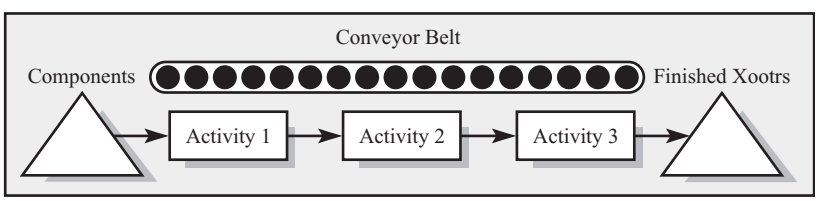


FIGURE 4.4
A Machine-Paced Process Layout
(Note: conveyor belt is only shown for illustration)

Exhibit 4.1

TIME TO PROCESS A QUANTITY *X* STARTING WITH AN EMPTY PROCESS

1. Find the time it takes the flow unit to go through the empty system:
   - In a worker-paced line, this is the sum of the processing times.
   - In a machine-paced line, this is the cycle time × the number of stations.
2. Compute the capacity of the process (see previous methods). Since we are producing *X* units as fast as we can, we are capacity-constrained; thus,

$$\text{Flow rate} = \text{Process capacity}$$

3. Time to finish *X* units

$$= \text{Time through empty process} + \frac{X - 1\text{ unit}}{\text{Flow rate}}$$

Note: If the process is a continuous process, we can use *X* instead.

imagine a continuous-flow process such as a cranberry processing line. Suppose you want to know how long it takes to produce five tons of cranberries. Let's say a cranberry weighs one gram, so five tons equals five million cranberries. Now how long does it take to produce five million cranberries? Strictly speaking, we would look at the time it takes the first berry to flow through the system and then add the time for the residual 4,999,999 berries. However, for all computational purposes, five million minus one is still five million, so we can make our life a little easier by just ignoring this first berry:

$$\text{Time to finish } X \text{ units with a continuous-flow process}$$
$$= \text{Time through an empty process} + \frac{X \text{ units}}{\text{Flow rate}}$$

Exhibit 4.1 summarizes the calculations leading to the time it takes the process to produce *X* units starting with an empty system.

4.3 Labor Content and Idle Time

What is the role of labor cost in the production of the Xootr? Let's look first at how much actual labor is involved in the assembly of the Xootr. Towards this end, we define the *labor content* as the sum of the processing times of the three workers. In this case, we compute a labor content of

$$\begin{aligned}\text{Labor content} &= \text{Sum of processing times with labor}\\ &= 13\text{ minutes/unit} + 11\text{ minutes/unit} + 8\text{ minutes/unit}\\ &= 32\text{ minutes per unit}\end{aligned}$$

These 32 minutes per unit reflect how much labor is invested into the production of one scooter. We could visualize this measure as follows. Let's say there would be a slip of paper attached to a Xootr and each worker would write the amount of time spent working on the Xootr on this slip. The sum of all numbers entered on the slip is the labor content.

Assume that the average hourly rate of the assembly employees is \$12 per hour (and thus \$0.20 per minute). Would the resulting cost of labor then be 32 minutes/unit × \$0.20/minute = \$6.40/unit? The answer is a clear *no*! The reason for this is that the labor content is a measure that takes the perspective of the flow unit but does not reflect any information about how the process is actually operated.

Assume—for illustrative purposes—that we would hire an additional worker for the second activity. As worker 2 is not a constraint on the overall output of the process, this would probably not be a wise thing to do (and that is why we call it an illustrative example). How would the labor content change? Not at all! It would still require the same 32 minutes of labor to produce a scooter. However, we have just increased our daily wages by 33 percent, which should obviously be reflected in our cost of direct labor.

To correctly compute the cost of direct labor, we need to look at two measures:

- The number of scooters produced per unit of time (the flow rate).
- The amount of wages we pay for the same time period.

Above, we found that the process has a capacity of 4.6 scooters an hour, or 161 scooters per week (we assume the process operates 35 hours per week). Given that demand is currently 125 scooters per week (we are demand-constrained), our flow rate is at 125 scooters per week.

Now, we can compute the cost of direct labor as

$$\begin{aligned}\text{Cost of direct labor} &= \frac{\text{Total wages per unit of time}}{\text{Flow rate per unit of time}}\\ &= \frac{\text{Wages per week}}{\text{Scooters produced per week}}\\ &= \frac{3 \times \$12/\text{h} \times 35\,\text{h/week}}{125\ \text{scooters/week}}\\ &= \frac{\$1,260/\text{week}}{125\ \text{scooters/week}} = \$10.08/\text{scooter}\end{aligned}$$

Why is this number so much higher than the number we computed based on the direct labor content? The difference between the two numbers reflects underutilization, or what we will refer to as *idle time.* In this case, there are two sources of idle time:

- The process is never able to produce more than its bottleneck. In this case, this means one scooter every 13 minutes. However, if we consider worker 3, who only takes eight minutes on a scooter, this translates into a 5-minute idle time for every scooter built.
- If the process is demand-constrained, even the bottleneck is not operating at its full capacity and, consequently also exhibits idle time. Given a demand rate of 125 scooters/week, that is, 3.57 scooters/hour or one scooter every 16.8 minutes, all three workers get an extra 3.8 minutes of idle time for every scooter they make.

This reflects the utilization profile and the sources of underutilization that we discussed in Chapter 3 with the Circored process.

Note that this calculation assumes the labor cost is fixed. If it were possible to shorten the workday from the current 7 hours of operations to 5 hours and 25 minutes (25 scooters a day × 1 scooter every 13 minutes), we would eliminate the second type of idle time.

More formally, define the following:

$$\text{Cycle time} = \frac{1}{\text{Flow rate}}$$

Cycle time provides an alternative measure of how fast the process is creating output. As we are producing one scooter every 16.8 minutes, the cycle time is 16.8 minutes. Similar to what we did intuitively above, we can now define the idle time for worker *i* as the following:

$$\text{Idle time for a single worker} = \text{Cycle time} - \text{Processing time of the single worker}$$

Note that this formula assumes that every activity is staffed with exactly one worker. The idle time measures how much unproductive time a worker has for every unit of output produced. These calculations are summarized by Table 4.1.

If we add up the idle time across all workers, we obtain the total idle time that is incurred for every scooter produced:

$$3.8 + 5.8 + 8.8 = 18.4 \text{ minutes/unit}$$

Now, apply the wage rate of \$12 per hour (\$0.20/minute × 18.4 minutes/unit) and, voilà, we obtain exactly the difference between the labor cost we initially expected based on the direct labor content alone (\$6.40 per unit) and the actual cost of direct labor computed above.

As a final measure of process efficiency, we can look at the average labor utilization of the workers involved in the process. We can obtain this number by comparing the labor content with the amount of labor we have to pay for (the labor content and the idle time):

$$\text{Average labor utilization} = \frac{\text{Labor content}}{\text{Labor content} + \text{Sum of idle times across workers}} = \frac{32[\text{minutes per unit}]}{32[\text{minutes per unit}] + 18.4[\text{minutes per unit}]} = 63.5\%$$

TABLE 4.1
Basic Calculations Related to Idle Time

	Worker 1	Worker 2	Worker 3
Processing time	13 minutes/unit	11 minutes/unit	8 minutes/unit
Capacity	$\frac{1}{13}$ unit/minute	$\frac{1}{11}$ unit/minute	$\frac{1}{8}$ unit/minute
	= 4.61 units/hour	= 5.45 units/hour	= 7.5 units/hour
Process capacity	Minimum {4.61 units/h, 5.45 units/h, 7.5 units/h} = 4.61 units/hour		
Flow rate	Demand = 125 units/week = 3.57 units/hour Flow rate = Minimum {demand, process capacity} = 3.57 units/hour		
Cycle time	1/3.57 hours/unit = 16.8 minutes/unit		
Idle time	16.8 minutes/unit − 13 minutes/unit = 3.8 minutes/unit	16.8 minutes/unit − 11 minutes/unit = 5.8 minutes/unit	16.8 minutes/unit − 8 minutes/unit = 8.8 minutes/unit
Utilization	3.57/4.61 = 77%	3.57/5.45 = 65.5%	3.57/7.5 = 47.6%

Exhibit 4.2

SUMMARY OF LABOR COST CALCULATIONS

1. Compute the capacity of all resources; the resource with the lowest capacity is the bottleneck (see previous methods) and determines the process capacity.
2. Compute Flow rate = Min{Available input, Demand, Process capacity}; then compute

$$\text{Cycle time} = \frac{1}{\text{Flow rate}}$$

3. Compute the total wages (across all workers) that are paid per unit of time:

$$\text{Cost of direct labor} = \frac{\text{Total wages}}{\text{Flow rate}}$$

4. Compute the idle time across all workers at resource i

$$\text{Idle time across all workers at resource } i = \text{Cycle time} \times (\text{Number of workers at resource } i) - \text{Processing time at resource } i$$

5. Compute the labor content of the flow unit: this is the sum of all processing times involving direct labor.
6. Add up the idle times across all resources (total idle time); then compute

$$\text{Average labor utilization} = \frac{\text{Labor content}}{\text{Labor content} + \text{Total idle time}}$$

An alternative way to compute the same number is by averaging the utilization level across the three workers:

$$\text{Average labor utilization} = \tfrac{1}{3} \times (\text{Utilization}_1 + \text{Utilization}_2 + \text{Utilization}_3) = 63.4\%$$

where Utilization_i denotes the utilization of the ith worker. Exhibit 4.2 summarizes the calculations related to our analysis of labor costs. It includes the possibility that there are multiple workers performing the same activity.

4.4 Increasing Capacity by Line Balancing

Comparing the utilization levels in Table 4.1 reveals a strong imbalance between workers: while worker 1 is working 77 percent of the time, worker 3 is only active about half of the time (47.6 percent to be exact). Imbalances within a process provide micro-level mismatches between what could be supplied by one step and what is demanded by the following steps. *Line balancing* is the act of reducing such imbalances. It thereby provides the opportunity to

- Increase the efficiency of the process by better utilizing the various resources, in this case labor.
- Increase the capacity of the process (without adding more resources to it) by reallocating either workers from underutilized resources to the bottleneck or work from the bottleneck to underutilized resources.

While based on the present demand rate of 125 units per week and the assumption that all three workers are a fixed cost for 35 hours per week, line balancing would change neither the flow rate (process is demand-constrained) nor the cost of direct labor (assuming the 35 hours per week are fixed); this situation changes with the rapid demand growth experienced by Novacruz.

Consider now a week in May, by which, as indicated by Figure 4.1, the demand for the Xootr had reached a level of 200 units per week. Thus, instead of being demand-constrained, the process now is capacity-constrained, specifically, the process now is constrained by worker 1, who can produce one scooter every 13 minutes, while the market demands scooters at a rate of one scooter every 10.5 minutes (200 units/week/35 hours/week = 5.714 units/hour).

Given that worker 1 is the constraint on the system, all her idle time is now eliminated and her utilization has increased to 100 percent. Yet, workers 2 and 3 still have idle time:

- The flow rate by now has increased to one scooter every 13 minutes or $\frac{1}{13}$ unit per minute (equals $\frac{1}{13} \times 60 \times 35 = 161.5$ scooters per week) based on worker 1.
- Worker 2 has a capacity of one scooter every 11 minutes, that is, $\frac{1}{11}$ unit per minute. Her utilization is thus $\text{Flow rate}/\text{Capacity}_2 = \frac{1}{13}\Big/\frac{1}{11} = \frac{11}{13} = 84.6\%$.
- Worker 3 has a capacity of one scooter every 8 minutes. Her utilization is thus $\frac{1}{13}\Big/\frac{1}{8} = \frac{8}{13} = 61.5\%$.

Note that the increase in demand not only has increased the utilization levels across workers (the average utilization is now $\frac{1}{3} \times (100\% + 84.6\% + 61.5\%) = 82\%$), but also has reduced the cost of direct labor to

$$\begin{aligned}\text{Cost of direct labor} &= \frac{\text{Total wages per unit of time}}{\text{Flow rate per unit of time}}\\ &= \frac{\text{Wages per week}}{\text{Scooters produced per week}}\\ &= \frac{3 \times \$12/\text{hour} \times 35\ \text{hours/week}}{161.5\ \text{scooters/week}}\\ &= \frac{\$1,260/\text{week}}{161.5\ \text{scooters/week}} = \$7.80/\text{scooter}\end{aligned}$$

Now, back to the idea of line balancing. Line balancing attempts to evenly (fairly!) allocate the amount of work that is required to build a scooter across the three process steps.

In an ideal scenario, we could just take the amount of work that goes into building a scooter, which we referred to as the labor content (32 minutes/unit), and split it up evenly between the three workers. Thus, we would achieve a perfect line balance if each worker could take 32/3 minutes/unit; that is, each would have an identical processing time of 10.66 minutes/unit.

Unfortunately, in most processes, it is not possible to divide up the work that evenly. Specifically, the activities underlying a process typically consist of a collection of *tasks* that cannot easily be broken up. A closer analysis of the three activities in our case reveals the task structure shown in Table 4.2.

For example, consider the last task of worker 1 (assemble handle cap), which takes 118 seconds per unit. These 118 seconds per unit of work can only be moved to another worker in their entirety. Moreover, we cannot move this task around freely, as it obviously would not be feasible to move the "assemble handle cap" task to after the "seal carton" task.

TABLE 4.2
Task Durations

Worker	Tasks	Task Duration [seconds/unit]
Worker 1	Prepare cable	30
	Move cable	25
	Assemble washer	100
	Apply fork, threading cable end	66
	Assemble socket head screws	114
	Steer pin nut	49
	Brake shoe, spring, pivot bolt	66
	Insert front wheel	100
	Insert axle bolt	30
	Tighten axle bolt	43
	Tighten brake pivot bolt	51
	Assemble handle cap	118
		Total: 792
Worker 2	Assemble brake lever and cable	110
	Trim and cap cable	59
	Place first rib	33
	Insert axles and cleats	96
	Insert rear wheel	135
	Place second rib and deck	84
	Apply grip tape	56
	Insert deck fasteners	75
		Total: 648
Worker 3	Inspect and wipe off	95
	Apply decal and sticker	20
	Insert in bag	43
	Assemble carton	114
	Insert Xootr and manual	94
	Seal carton	84
		Total: 450

However, we could move the 118 seconds per unit from worker 1 to worker 2. In this case, worker 1 would now have an processing time of 674 seconds per unit and worker 2 (who would become the new bottleneck) would have an processing time of 766 seconds per unit. The overall process capacity is increased, we would produce more scooters, and the average labor utilization would move closer to 100 percent.

But can we do better? Within the scope of this book, we only consider cases where the sequence of tasks is given. Line balancing becomes more complicated if we can resequence some of the tasks. For example, there exists no technical reason why the second to last task of worker 2 (apply grip tape) could not be switched with the subsequent task (insert deck fasteners). There exist simple algorithms and heuristics that support line balancing in such more complex settings. Yet, their discussion would derail us from our focus on managerial issues.

But even if we restrict ourselves to line balancing solutions that keep the sequence of tasks unchanged, we can further improve upon the 766-second cycle time we outlined above. Remember that the "gold standard" of line balancing, the even distribution of the labor content across all resources, suggested an processing time of 10.66 minutes per unit, or 640 seconds per unit.

Moving the "assemble handle cap" task from worker 1 to worker 2 was clearly a substantial step in that direction. However, worker 2 has now 126 seconds per unit (766 seconds/unit − 640 seconds/unit) more than what would be a balanced workload. This situation

can be improved if we take the worker's last two tasks (apply grip tape, insert deck fasteners) and move the corresponding 56 + 75 seconds/unit = 131 seconds/unit to worker 3.

The new processing times would be as follows:

- Worker 1: 674 seconds per unit (792 − 118 seconds/unit).
- Worker 2: 635 seconds per unit (648 + 118 − 56 − 75 seconds/unit).
- Worker 3: 581 seconds per unit (450 + 56 + 75 seconds/unit).

Are they optimal? No! We can repeat similar calculations and further move work from worker 1 to worker 2 (tighten brake pivot bolt, 51 seconds per unit) and from worker 2 to worker 3 (place second rib and deck, 84 seconds per unit). The resulting (final) processing times are now

- Worker 1: 623 seconds per unit (674 − 51 seconds/unit).
- Worker 2: 602 seconds per unit (635 + 51 − 84 seconds/unit).
- Worker 3: 665 seconds per unit (581 + 84 seconds/unit).

To make sure we have not "lost" any work on the way, we can add up the three new processing times and obtain the same labor content (1,890 seconds per unit) as before. The resulting labor utilization would be improved to

$$\begin{aligned}\text{Average labor utilization} &= \text{Labor content} / (\text{Labor content} + \text{Total idle time}) \\ &= 1{,}890 / (1{,}890 + 42 + 63 + 0) = 94.7\%\end{aligned}$$

The process improvement we have implemented based on line balancing is sizeable in its economic impact. Based on the new bottleneck (worker 3), we see that we can produce one Xootr every 665 seconds, thereby having a process capacity of $\frac{1}{665}$ units/second × 3,600 seconds/hour × 35 hours/week = 189.5 units per week. Thus, compared to the unbalanced line (161.5 units per week), we have increased process capacity (and flow rate) by 17 percent (28 units) without having increased our weekly spending rate on labor. Moreover, we have reduced the cost of direct labor to $6.65/unit.

Figure 4.5 summarizes the idea of line balancing by contrasting cycle time and task allocation of the unbalanced line (before) and the balanced line (after).

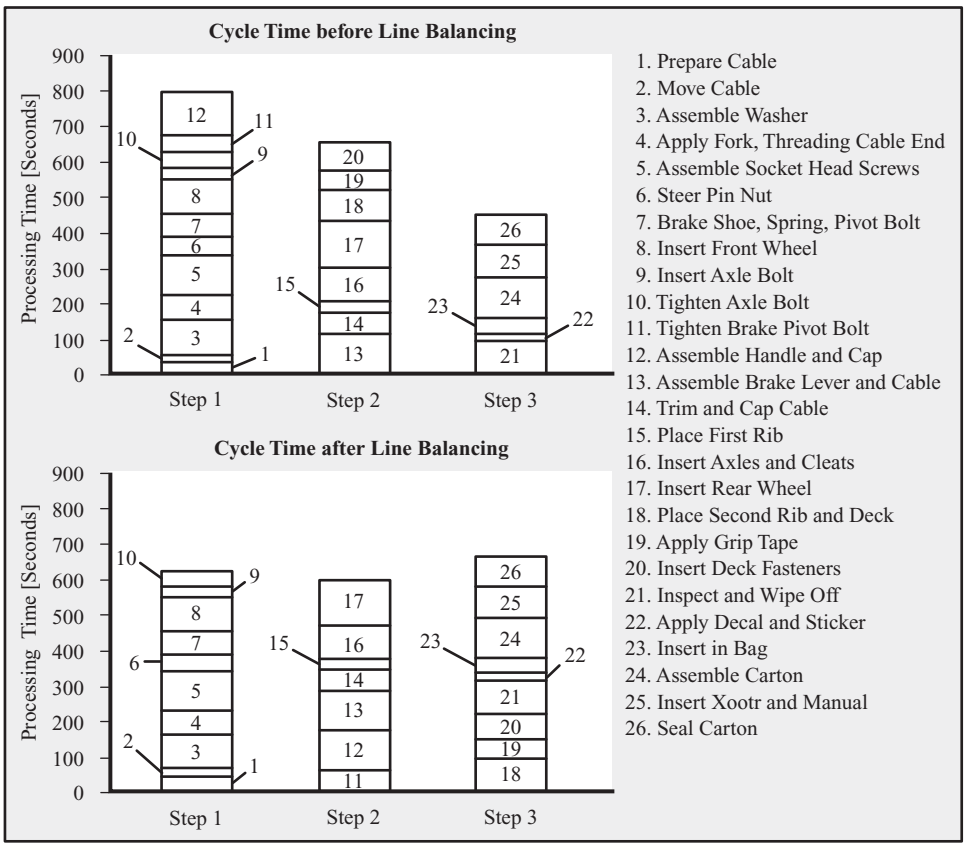
4.5 Scale Up to Higher Volume

As indicated by Figure 4.2, demand for the Xootr increased dramatically within the next six months and, by July, had reached a level of 700 units per week. Thus, in order to maintain a reasonable match between supply and demand, Novacruz had to increase its process capacity (supply) further.

To increase process capacity for a worker-paced line, in this case from 189.5 units per week (see balanced line with three workers above) to 700 units per week, additional workers are needed. While the fundamental steps involved in building a Xootr remain unchanged, we have several options to lay out the new, high-volume process:

- Using the exact same layout and staffing plan, we could replicate the—now balanced—process and add another (and another, . . .) worker-paced line with three workers each.
- We could assign additional workers to the three process steps, which would increase the capacity of the steps and hence lead to a higher overall process capacity.
- We could divide up the work currently performed by three workers, thereby increasing the specialization of each step (and thus reducing processing times and hence increasing capacity).

FIGURE 4.5
Graphical Illustration of Line Balance



We will quickly go through the computations for all three approaches. The corresponding process flow diagrams are summarized in Figure 4.6.

Increasing Capacity by Replicating the Line

As the capacity of the entire operation grows linearly with the number of replications, we could simply add three replications of the process to obtain a new total capacity of 4×189.5 units/week $= 758$ units per week.

The advantage of this approach is that it would allow the organization to benefit from the knowledge it has gathered from their initial process layout. The downside of this approach is that it keeps the ratio of workers across the three process steps constant (in total, four people do step 1, four at step 2, and four at step 3), while this might not necessarily be the most efficient way of allocating workers to assembly tasks (it keeps the ratio between workers at each step fixed).

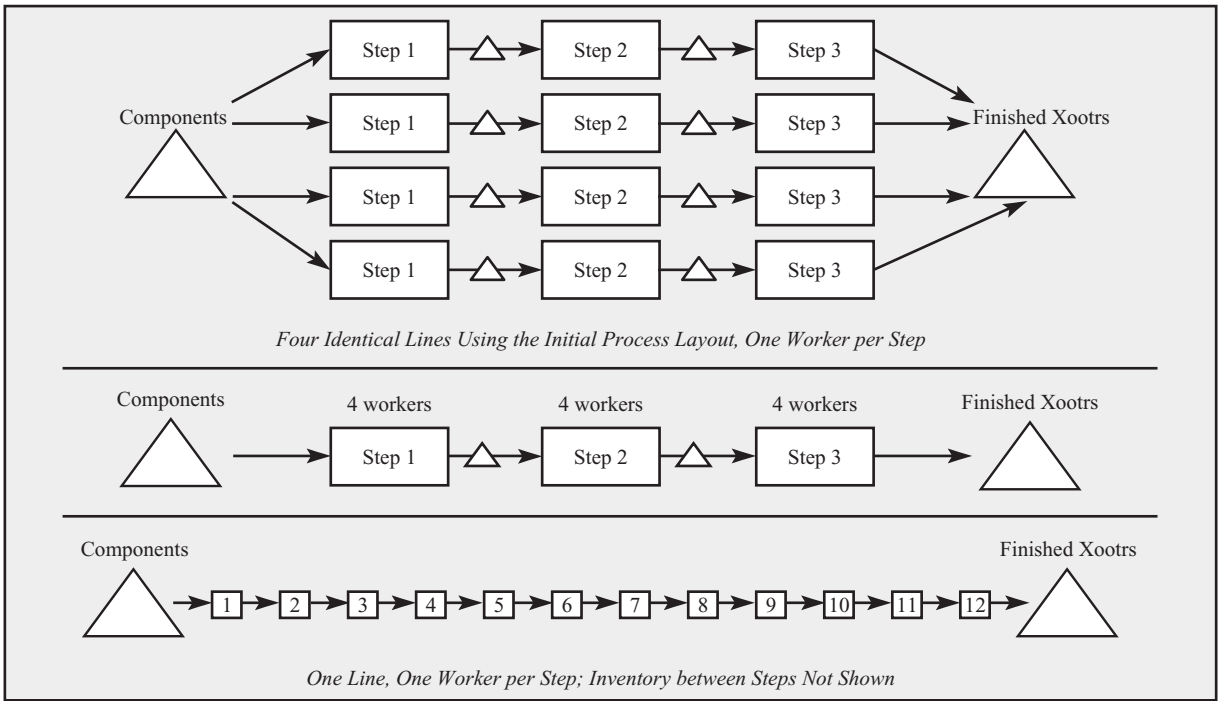
Alternatively, we could just add two replications and obtain a process capacity of 568.5 units per week and make up for the remaining 131.5 units ($700 - 568.5$ units/week) by adding overtime. Given that the 131.5 units to be produced in overtime would be spread over three lines, each line would have to produce $131.53/3 = 43.84$ units per week corresponding to 8.1 hours of overtime per week (43.83 units/week/5.41 units/hour).

Under the assumption that we could use overtime, the average labor utilization would remain unchanged at 94.7 percent.

Increasing Capacity by Selectively Adding Workers

While the first approach assumed the number of workers at each process step to be the same, such a staffing might not necessarily be optimal. Specifically, we observe that (after the

FIGURE 4.6 Three Process Layouts for High-Volume Production



rebalancing) the third step is the bottleneck (processing time of 665 seconds per unit). Thus, we feel tempted to add over-proportionally more workers to this step than to the first two.

Given that we defined the capacity at each resource as the number of workers divided by the corresponding processing time, we can write the following:

$$\text{Requested capacity} = \frac{\text{Number of workers}}{\text{Activity time}}$$

For step 1, this calculation yields (700 units per week at 35 hours per week is 0.00555 unit per second):

$$0.00555 \text{ unit/second} = \frac{\text{Number of workers}}{623 \text{ seconds per unit}}$$

Thus, the number of workers required to meet the current demand is $0.00555 \times 623 = 3.46$ workers. Given that we cannot hire half a worker (and ignoring overtime for the moment), this means we have to hire four workers at step 1. In the same way, we find that we need to hire 3.34 workers at step 2 and 3.69 workers at step 3.

The fact that we need to hire a total of four workers for each of the three steps reflects the good balance that we have achieved above. If we would do a similar computation based on the initial numbers (792,648,450 seconds/unit for workers 1, 2, and 3 respectively; see Table 3.2), we would obtain the following:

- At step 1, we would hire 0.00555 unit/second = Number of workers/792 seconds/unit; therefore, Number of workers = 4.4.
- At step 2, we would hire 0.00555 unit/second = Number of workers/648 seconds/unit; therefore, Number of workers = 3.6.
- At step 3, we would hire 0.00555 unit/second = Number of workers/450 seconds/unit; therefore, Number of workers = 2.5.

Thus, we observe that a staffing that allocates extra resources to activities with longer processing times (5 workers for step 1 versus 4 for step 2 and 3 for step 3) provides an alternative way of line balancing.

Note also that if we had just replicated the unbalanced line, we would have had to add four replications as opposed to the three replications of the balanced line (we need five times step 1). Thus, line balancing, which at the level of the individual worker might look like "hair-splitting," debating about every second of worker time, at the aggregate level can achieve very substantial savings in direct labor cost.

At several places throughout the book, we will discuss the fundamental ideas of the Toyota Production System, of which line balancing is an important element. In the spirit of the Toyota Production System, idle time is considered as waste (*muda*) and therefore should be eliminated from the process to the extent possible.

Increasing Capacity by Further Specializing Tasks

Unlike the previous two approaches to increase capacity, the third approach fundamentally alters the way the individual tasks are assigned to workers. As we noted in our discussion of line balancing, we can think of each activity as a set of individual tasks. Thus, if we increase the level of specialization of workers and now have each worker only be responsible for one or two tasks (as opposed to previously an activity consisting of 5 to 10 tasks), we would be able to reduce processing time and thereby increase the capacity of the line.

Specifically, we begin our analysis by determining a targeted cycle time based on demand: in this case, we want to produce 700 units per week, which means 20 scooters per hour or one scooter every three minutes. How many workers does it take to produce one Xootr every three minutes?

The answer to this question is actually rather complicated. The reason for this complication is as follows. We cannot compute the capacity of an individual worker without knowing which tasks this worker will be in charge of. At the same time, we cannot assign tasks to workers, as we do not know how many workers we have.

To break this circularity, we start our analysis with the staffing we have obtained under the previous approaches, that is, 12 workers for the entire line. Table 4.3 shows how we can assign the tasks required to build a Xootr across these 12 workers.

Following this approach, the amount of work an individual worker needs to master is reduced to a maximum of 180 seconds. We refer to this number as the *span of control.* Given that this span of control is much smaller than under the previous approaches (665 seconds), workers will be able to perform their tasks with significantly less training. Workers are also likely to improve upon their processing times more quickly as specialization can increase the rate of learning.

The downside of this approach is its negative effect on labor utilization. Consider what has happened to labor utilization:

$$\text{Average labor utilization} = \frac{\text{Labor content}}{\text{Labor content} + \text{Sum of idle time}}$$

$$= \frac{1890}{1{,}890 + 25 + 0 + 65 + 7 + 11 + 11 + 51 + 45 + 40 + 10 + 3 + 2} = 87.5\%$$

Note that average labor utilization was 94.7 percent (after balancing) with three workers. Thus, specialization (smaller spans of control) makes line balancing substantially more complicated. This is illustrated by Figure 4.7.

The reason for this decrease in labor utilization, and thus the poorer line balance, can be found in the granularity of the tasks. Since it is not possible to break up the individual tasks further, moving a task from one worker to the next becomes relatively more significant. For example, when we balanced the three-worker process, moving a 51-second-per-unit

TABLE 4.3
Processing times and Task Allocation under Increased Specialization

Worker	Tasks	Task Duration [seconds/unit]
Worker 1	Prepare cable	30
	Move cable	25
	Assemble washer	100
		Total: 155
Worker 2	Apply fork, threading cable end	66
	Assemble socket head screws	114
		Total: 180
Worker 3	Steer pin nut	49
	Brake shoe, spring, pivot bolt	66
		Total: 115
Worker 4	Insert front wheel	100
	Insert axle bolt	30
	Tighten axle bolt	43
		Total: 173
Worker 5	Tighten brake pivot bolt	51
	Assemble handle cap	118
		Total: 169
Worker 6	Assemble brake lever and cable	110
	Trim and cap cable	59
		Total: 169
Worker 7	Place first rib	33
	Insert axles and cleats	96
		Total: 129
Worker 8	Insert rear wheel	135
		Total: 135
Worker 9	Place second rib and deck	84
	Apply grip tape	56
		Total: 140
Worker 10	Insert deck fasteners	75
	Inspect and wipe off	95
		Total: 170
Worker 11	Apply decal and sticker	20
	Insert in bag	43
	Assemble carton	114
		Total: 177
Worker 12	Insert Xootr and manual	94
	Seal carton	84
		Total: 178
	Total labor content	1,890

task to another step accounted for just 8 percent of the step's work (674 seconds per unit). In a 12-step process, however, moving the same 51-second-per-unit task is now relative to a 169-second-per-unit workload for the step, thereby accounting for 30 percent of work. For this reason, it is difficult to further improve the allocation of tasks to workers relative to what is shown in Figure 4.7.

The observation that line balancing becomes harder with an increase in specialization can best be understood if we "turn this reasoning on its head": line balancing becomes

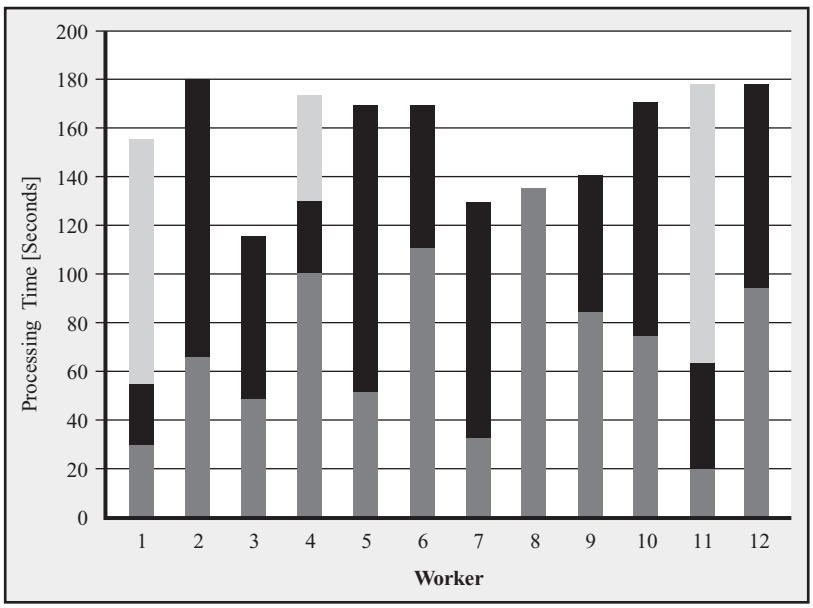


FIGURE 4.7
Line Balance in a Highly Specialized Line
(Different shades represent different tasks)

easier with a decrease in specialization. To see this, consider the case of having one single worker do all the tasks in the process. The corresponding labor utilization would be 100 percent (assuming there is enough demand to keep at least one worker busy), as, by definition, this one person also would be the bottleneck.

The idea of having one resource perform all activities of the process is referred to as a work cell. The process flow diagram of a work cell is illustrated by Figure 4.8. Since the processing time at a work cell with one worker is the same as the labor content, we would have a capacity per work cell of $\frac{1}{1{,}890}$ unit per second = 1.9048 units per hour, or 66.67 units per week. Already 11 work cells would be able to fulfill the demand of 700 Xootrs per week. In other words, the improved balance that comes with a work cell would allow us to further improve efficiency.

Again, the downside of this approach is that it requires one worker to master a span of control of over 30 minutes, which requires a highly trained operator. Moreover, Novacruz found that working with the 12-person line and the corresponding increase in specialization led to a substantial reduction in processing times.

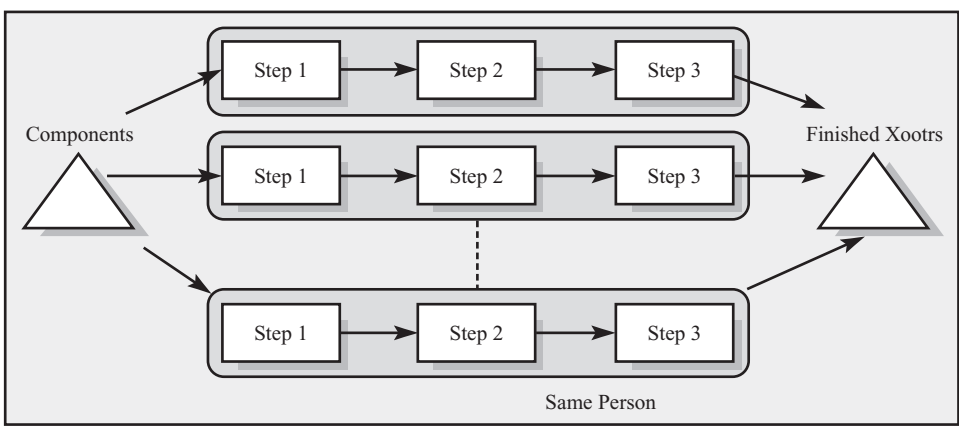


FIGURE 4.8
Parallel Work Cells
(Only three work cells are shown)

4.6 Summary

In this chapter, we introduced the concept of line balancing. Line balancing attempts to eliminate idle time from the process and thereby increase labor utilization. At first sight, line balancing seems to belong in the same category as "hair-splitting" and "penny-counting." However, it is important to understand the managerial role that line balancing plays in operations. Specifically, it is important to understand the following three managerial benefits:

- First of all, while it is always more tempting to talk about dollars rather than pennies, pennies do matter in many industries. Consider, for example, the computer industry. All PC manufacturers purchase from the same pool of suppliers of processors, disk drives, optical devices, and so forth. Thus, while the $10 of labor cost in a computer might seem small relative to the purchase price of the computer, those $10 are under our managerial control, while most of the other costs are dictated by the market environment.
- Second, in the spirit of the Toyota Production System (TPS), idle time is waste and thereby constitutes what in TPS is known as *muda.* The problem with *muda*/idle time is that it not only adds to the production costs, but has the potential to hide many other problems. For example, a worker might use idle time to finish or rework a task that she could not complete during the allocated processing time. While this does not lead to a direct, out-of-pocket cost, it avoids the root cause of the problem, which, when it surfaces, can be fixed.
- Third, while the $10 labor cost in the assembly operation of a PC manufacturer discussed above might seem like a low number, there is much more labor cost involved in the PC than $10. What appears as procurement cost for the PC maker is to some extent labor cost for the suppliers of the PC maker. If we "roll up" all operations throughout the value chain leading to a PC, we find that the cost of labor is rather substantial. This idea is illustrated in Figure 4.9 for the case of the automotive industry: while for a company like an automotive company assembly labor costs seem to be only a small element of costs, the 70 percent of costs that are procurement costs themselves include assembly labor costs from suppliers, subsuppliers, and so forth. If we look at all costs in the value chain (from an automotive company to their fifth-tier supplier), we see that about a quarter of costs in the automotive supply chain are a result of labor costs. A consequence of this observation is that it is not enough to improve our own operations

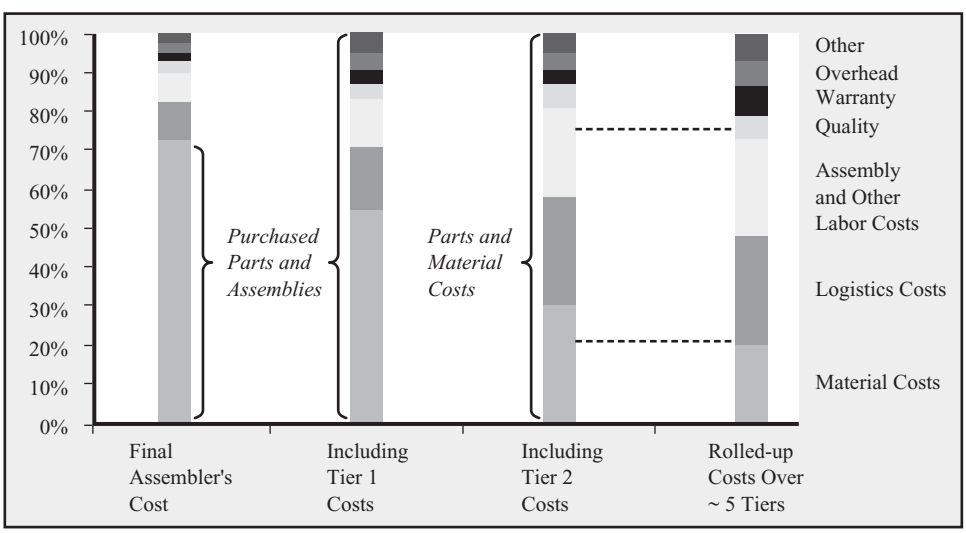


FIGURE 4.9
Sources of Cost in the Supply Chain

Source: Whitney 2004.

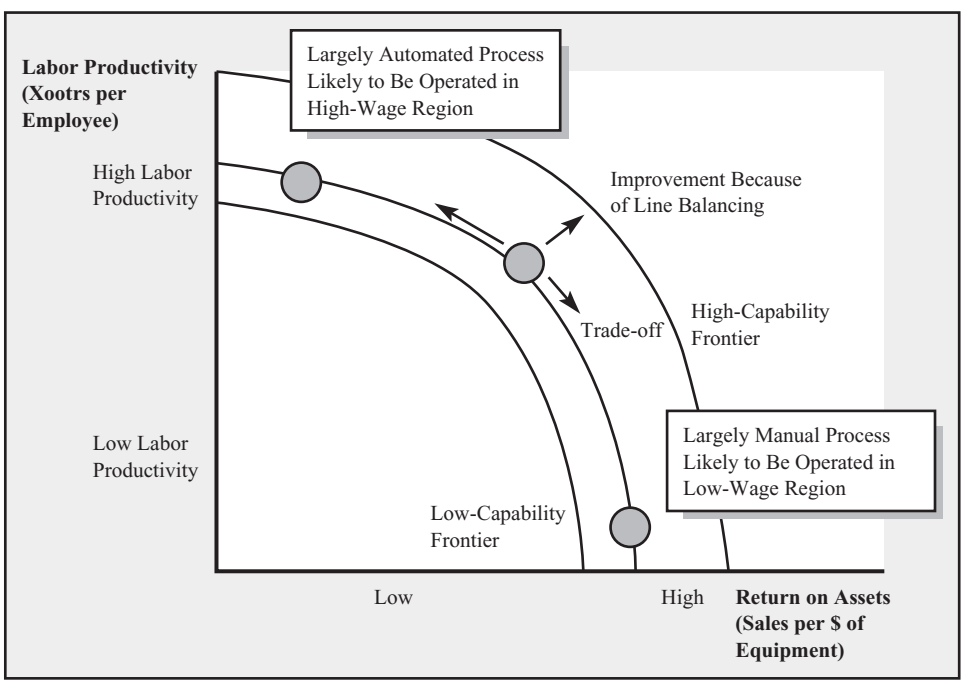
internally, but to spread such improvements throughout the supplier network, as this is where the biggest improvement opportunities are hidden. This concept of supplier development is another fundamental concept of the Toyota Production System.

In addition to these three factors, line balancing also illustrates an important—and from a managerial perspective very attractive—property of operations management. Line balancing improves per-unit labor cost (productivity) and does not require any financial investments in assets! To improve labor productivity, we would typically attempt to automate parts of the assembly, which would lower the per-unit labor cost, but at the same time require a higher investment of capital. Such an approach would be most likely if we operated in a high-wage location such as Germany or France. In contrast, we could try to operate the process with little or no automation but have a lot of labor time invested in the process. Such an approach would be more likely if we moved the process to a low-wage location such as China or Taiwan.

This tension is illustrated by Figure 4.10. The horizontal axis of Figure 4.10. shows the return on the assets tied up in the manufacturing process. High returns are desirable, which could be achieved by using little automation and a lot of labor. The vertical axis shows the productivity of labor, which would be maximized if the process were highly automated. As can be seen in Figure 4.10, there exists a tension (trade-off) between the dimensions, visible in the form of an efficient frontier. Thus, changes with respect to the level of automation would move the process up or down the frontier. One dimension is traded against the other.

In contrast, the effect of line balancing in the context of Figure 4.10 is very different. Line balancing improves labor productivity without any additional investment. To the extent that line balancing allows the firm to eliminate some currently underutilized resources using production equipment, line balancing also reduces the required assets. Thus, what from a strategic perspective seems like a simple, one-dimensional positioning problem along the technology frontier now has an additional dimension. Rather than simply taking the current process as given and finding a good strategic position, the firm should attempt to improve its process capability and improve along both performance dimensions simultaneously.

FIGURE 4.10
Trade-off between Labor Productivity and Capital Investment



4.7 Further Reading

Bartholdi and Eisenstein (1996) develop the concept of a bucket brigade, which corresponds to a line operation that is self-balancing. In this concept, workers move between stations and follow relatively simple decision rules that determine which task should be performed next.

Whitney (2004) presents a systematic approach to design and production of mechanical assemblies. This book introduces mechanical and economic models of assemblies and assembly automation. The book takes a system view of assembly, including the notion of product architecture, feature-based design, computer models of assemblies, analysis of mechanical constraint, assembly sequence analysis, tolerances, system-level design for assembly and JIT methods, and economics of assembly automation.

4.8 Practice Problems

Q4.1* **(Empty System, Labor Utilization)** Consider a process consisting of three resources in a worker-paced line and a wage rate of $10 per hour. Assume there is unlimited demand for the product.

Resource	Processing time (minutes)	Number of Workers
1	10	2
2	6	1
3	16	3

a. How long does it take the process to produce 100 units starting with an empty system?

b. What is the average labor content?

c. What is the average labor utilization?

d. What is the cost of direct labor?

Q4.2** **(Assign Tasks to Workers)** Consider the following six tasks that must be assigned to four workers on a conveyor-paced assembly line (i.e., a machine-paced line flow). Each worker must perform at least one task.

	Time to Complete Task (seconds)
Task 1	30
Task 2	25
Task 3	35
Task 4	40
Task 5	15
Task 6	30

The current conveyor-paced assembly line configuration assigns the workers in the following way:

- Worker 1: Task 1
- Worker 2: Task 2
- Worker 3: Tasks 3, 4
- Worker 4: Tasks 5, 6

a. What is the capacity of the current line?

b. Now assume that tasks are allocated to maximize capacity of the line, subject to the conditions that (1) a worker can only perform two adjacent operations and (2) all tasks need to be done in their numerical order. What is the capacity of this line now?

c. Now assume that tasks are allocated to maximize capacity of the line and that tasks can be performed in any order. What is the maximum capacity that can be achieved?

Q4.3 **(PowerToys)** PowerToys Inc. produces a small remote-controlled toy truck on a conveyor belt with nine stations. Each station has, under the current process layout, one worker assigned to it. Stations and processing times are summarized in the following table:

(* indicates that the solution is at the end of the book)

Station	Task	Processing Times (seconds)
1	Mount battery units	75
2	Insert remote control receiver	85
3	Insert chip	90
4	Mount front axle	65
5	Mount back axle	70
6	Install electric motor	55
7	Connect motor to battery unit	80
8	Connect motor to rear axle	65
9	Mount plastic body	80

a. What is the bottleneck in this process?

b. What is the capacity, in toy trucks per hour, of the assembly line?

c. What is the direct labor cost for the toy truck with the current process if each worker receives $15/hour, expressed in dollars per toy truck?

d. What would be the direct labor cost for the toy truck if work would be organized in a work cell, that is, one worker performs all tasks? Assume that the processing times would remain unchanged (i.e., there are no specialization gains).

e. What is the utilization of the worker in station 2?

Because of a drastically reduced forecast, the plant management has decided to cut staffing from nine to six workers per shift. Assume that (i) the nine tasks in the preceding table cannot be divided; (ii) the nine tasks are assigned to the six workers in the most efficient way possible; and (iii) if one worker is in charge of two tasks, the tasks have to be adjacent (i.e., one worker cannot work on tasks 1 and 3).

f. How would you assign the nine tasks to the six workers?

g. What is the new capacity of the line (in toy trucks per hour)?

Q4.4 **(12 Tasks to 4 Workers)** Consider the following tasks that must be assigned to four workers on a conveyor-paced assembly line (i.e., a machine-paced line flow). Each worker must perform at least one task. There is unlimited demand.

	Time to Complete Task (seconds)
Task 1	30
Task 2	25
Task 3	15
Task 4	20
Task 5	15
Task 6	20
Task 7	50
Task 8	15
Task 9	20
Task 10	25
Task 11	15
Task 12	20

The current conveyor-paced assembly-line configuration assigns the workers in the following way:

- Worker 1: Tasks 1, 2, 3
- Worker 2: Tasks 4, 5, 6
- Worker 3: Tasks 7, 8, 9
- Worker 4: Tasks 10, 11, 12

a. What is the capacity of the current line?

b. What is the direct labor content?

c. What is the average labor utilization (do not consider any transient effects such as the line being emptied before breaks or shift changes)?

d. How long would it take to produce 100 units, starting with an empty system?

The firm is hiring a fifth worker. Assume that tasks are allocated to the five workers to maximize capacity of the line, subject to the conditions that (i) a worker can only perform adjacent operations and (ii) all tasks need to be done in their numerical order.

e. What is the capacity of this line now?

Again, assume the firm has hired a fifth worker. Assume further that tasks are allocated to maximize capacity of the line and that tasks can be performed in any order.

f. What is the maximum capacity that can be achieved?

g. What is the minimum number of workers that could produce at an hourly rate of 72 units? Assume the tasks can be allocated to workers as described in the beginning (i.e., tasks cannot be done in any order).

Q4.5** **(Geneva Watch)** The Geneva Watch Corporation manufactures watches on a conveyor belt with six stations. One worker stands at each station and performs the following tasks:

Station	Tasks	Processing Time (seconds)
A: Preparation 1	Heat-stake lens to bezel	14
	Inspect bezel	26
	Clean switch holes	10
	Install set switch in bezel	18
	Total time for A	68
B: Preparation 2	Check switch travel	23
	Clean inside bezel	12
	Install module in bezel	25
	Total time for B	60
C: Battery installation	Install battery clip on module	20
	Heat-stake battery clip on module	15
	Install 2 batteries in module	22
	Check switch	13
	Total time for C	70
D: Band installation	Install band	45
	Inspect band	13
	Total time for D	58
E: Packaging preparation	Cosmetic inspection	20
	Final test	55
	Total time for E	75
F: Watch packaging	Place watch and cuff in display box	20
	Place cover in display box base	14
	Place owner's manual, box into tub	30
	Total time for F	64

These six workers begin their workday at 8:00 a.m. and work steadily until 4:00 p.m. At 4:00, no new watch parts are introduced into station A and the conveyor belt continues until all of the work-in-process inventory has been processed and leaves station F. Thus, each morning the workers begin with an empty system.

a. What is the bottleneck in this process?

b. What is the capacity, in watches per hour, of the assembly line (ignore the time it takes for the first watch to come off the line)?

c. What is the direct labor content for the processes on this conveyor belt?

d. What is the utilization of the worker in station B (ignore the time it takes for the first watch to come off the line)?

e. How many minutes of idle time will the worker in station C have in one hour (ignore the time it takes for the first watch to come off the line)?

f. What time will it be (within one minute) when the assembly line has processed 193 watches on any given day?

Q4.6 **(Yoggo Soft Drink)** A small, privately owned Asian company is producing a private-label soft drink, Yoggo. A machine-paced line puts the soft drinks into plastic bottles and then packages the bottles into boxes holding 10 bottles each. The machine-paced line is comprised of the following four steps: (1) the bottling machine takes 1 second to fill a bottle, (2) the lid machine takes 3 seconds to cover the bottle with a lid, (3) a labeling machine takes 5 seconds to apply a label to a bottle, and (4) the packaging machine takes 4 seconds to place a bottle into a box. When a box has been filled with 10 bottles, a worker tending the packaging machine removes the filled box and replaces it with an empty box. Assume that the time for the worker to remove a filled box and replace it with an empty box is negligible and hence does not affect the capacity of the line. At step 3 there are two labeling machines that each process alternating bottles, that is, the first machine processes bottles 1, 3, 5, . . . and the second machine processes bottles 2, 4, 6, . . . Problem data are summarized in the table following.

Process Step	Number of Machines	Seconds per Bottle
Bottling	1	1
Applying a lid	1	3
Labeling	2	5
Packaging	1	4

a. What is the process capacity (bottles/hour) for the machine-paced line?

b. What is the bottleneck in the process?

c. If one more identical labeling machine is added to the process, how much is the increase in the process capacity going to be (in terms of bottles/hour)?

d. What is the implied utilization of the packaging machine if the demand rate is 60 boxes/hour? Recall that a box consists of 10 bottles.

Q4.7 **(Atlas Inc.)** Atlas Inc. is a toy bicycle manufacturing company producing a five-inch small version of the bike that Lance Armstrong rode to win his first Tour de France. The assembly line at Atlas Inc. consists of seven work stations, each performing a single step. Stations and processing times are summarized here:

- Step 1 (30 sec.): The plastic tube for the frame is cut to size.
- Step 2 (20 sec.): The tube is put together.
- Step 3 (35 sec.): The frame is glued together.
- Step 4 (25 sec.): The frame is cleaned.
- Step 5 (30 sec.): Paint is sprayed onto the frame.
- Step 6 (45 sec.): Wheels are assembled.
- Step 7 (40 sec.): All other parts are assembled to the frame.

Under the current process layout, workers are allocated to the stations as shown here:

- Worker 1: Steps 1, 2
- Worker 2: Steps 3, 4
- Worker 3: Step 5
- Worker 4: Step 6
- Worker 5: Step 7

a. What is the bottleneck in this process?

b. What is the capacity of this assembly line, in finished units/hour?

c. What is the utilization of Worker 4, ignoring the production of the first and last units?

d. How long does it take to finish production of 100 units, starting with an empty process?

e. What is the average labor utilization of the workers, ignoring the production of the first and last units?

f. Assume the workers are paid $15 per hour. What is the cost of direct labor for the bicycle?

g. Based on recommendations of a consultant, Atlas Inc. decides to reallocate the tasks among the workers to achieve maximum process capacity. Assume that if a worker is in charge of two tasks, then the tasks have to be adjacent to each other. Also, assume that the sequence of steps cannot be changed. What is the maximum possible capacity, in units per hour, that can be achieved by this reallocation?

h. Again, assume a wage rate of $15 per hour. What would be the cost of direct labor if one single worker would perform all seven steps? You can ignore benefits of specialization, set-up times, or quality problems.

i. On account of a reduced demand forecast, management has decided to let go of one worker. If work is to be allocated among the four workers such that (i) the tasks can't be divided, (ii) if one worker is in charge of two tasks, the tasks have to be adjacent, (iii) the tasks are assigned in the most efficient way and (iv) each step can only be carried out by one worker, what is the new capacity of the line (in finished units/hour)?

Q4.8 **(Glove Design Challenge)** A manufacturer of women's designer gloves has employed a team of students to redesign her manufacturing unit. They gathered the following information. The manufacturing process consists of four activities: (1) fabric cutting; (2) dyeing; (3) stitching, done by specially designed machines; and (4) packaging. Processing times are shown below. Gloves are moved between activities by a conveyor belt that paces the flow of work (machine-paced line).

Process Step	Number of Machines	Minutes per Glove
Cutting	1	2
Dyeing	1	4
Stitching	1	3
Packaging	1	5

a. What is the process capacity in gloves/hour?

b. Which one of the following statements is true?

i. The capacity of the process increases by reducing the dyeing time.

ii. If stitching time increases to 5 min./glove, the capacity of the process remains unchanged, but "time through an empty machine-paced process" increases.

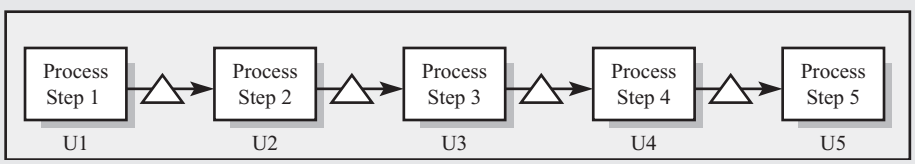
iii. By reducing packaging time, the process capacity increases.

iv. By reducing cutting time, the capacity of the process increases.

c. What is the implied utilization of the packaging machine if the demand rate is 10 gloves/hour?

d. What is the flow time for a glove?

Q4.9 **(Worker-Paced Line)**



The accompanying diagram depicts a five-step, worker-paced headphone manufacturing plant. The headphones are meant to be used with iPods and DVD players. Step 1 involves a worker bending a metal strip into an arc shape. In step 2, the metal arc is fitted with

a plastic sleeve. In step 3, the headphones are fitted at the end of the metal and plastic strips. In step 4, the wires are soldered into the headphones. Step 5 involves a specially designed packaging unit. After the plant has been operational for a couple of hours, the manager inspects the plant. He is particularly interested in cutting labor costs. He observes the following. The process is capacity constrained and the entire process produces 36 units in one hour. U1 through U5 denote the utilization at steps 1 through 5 respectively. Currently, there is a single worker at each step and the utilizations are as follows: U1 = 4/30, U2 = 4/15, U3 = 4/5, U4 = 1, U5 = 2/5.

Answer the following questions based on the given data and information.

a. What is the capacity of step 5?
b. Which step is the bottleneck?
c. Which process step has the highest capacity?
d. If the wage rate is $36 per hour per person, what is the direct labor cost per unit?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 5

Project Management

In the previous chapters, we established the process view of the organization.[1] Processes are all about repetition—we don't perform an operation once, we perform it over and over again. This process management view fits many, if not most, operations problems well. Mining and production plants, back offices of insurances or banks, hospitals, and call centers are all about repetition, and many flow units journey through the corresponding processes on a daily basis.

There are, however, a number of operations for which the repetition-based approach of process management is less appropriate. Consider, for example, a major construction project, the development of a new product, or the planning of a wedding party. In these situations, your primary concern is about planning the completion of one flow unit, and typically, you would like to see this completion to happen sooner rather than later.

Whether you care about the completion of one or many flow units often depends on which role you play in an operation. While most of us think about one wedding (at a time) and thus should think of a wedding event as a project, a wedding planner organizes numerous weddings and thus should think of weddings as flow units in a process. Similarly, a developer working on the launch of a new product or the construction worker building a new office complex are likely to think about their work as a project, while many echelons up in the organization, the vice president of product development or the owner of a real estate development company think about these projects as flow units in a big process.

We define a *project* as a temporary (and thus nonrepetitive) operation. Projects have a limited time frame, have one or more specific objectives, a temporary organizational structure, and thus often are operated in a more ad-hoc, improvised management style. In this chapter, you will learn the basics of project management, including:

- Mapping out the various activities that need to be completed as part of the project.
- Computing the completion time of the project based on the critical path.
- Accelerating a project to achieve an earlier completion time.
- Understanding the types of uncertainty a project faces and how to deal with them.

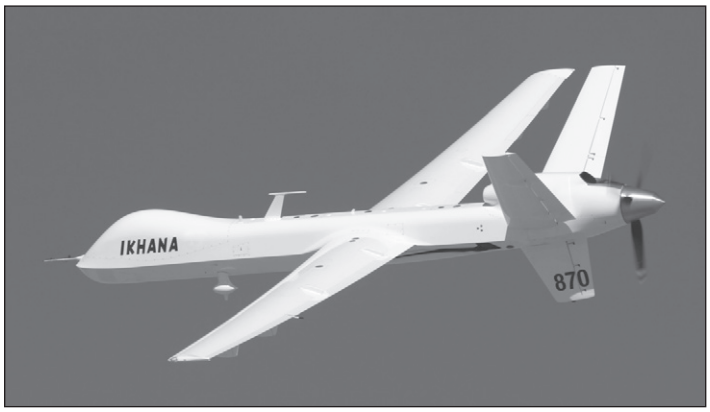
5.1 Motivating Example

Unmanned aerial vehicles (UAVs) are aircrafts that are flown without a human being on board. They are either controlled remotely or have built-in navigation intelligence to

[1] The authors gratefully acknowledge the help of Christoph Loch and Stylios Kavadias, whose case study on the Dragonfly UAV is the basis for the motivating example in this chapter.

FIGURE 5.1
UAV Offered by Boeing

Unmanned aerial vehicles (UAVs)



determine their direction. Most of their applications lie in the military arena, but UAVs can also be used for scientific exploration or search-and-rescue operations (see Figure 5.1).

We use the example of the development of a UAV to illustrate several tools and techniques of project management. In particular, we look at the decision situation of a developer who has just completed a prototype UAV and now is putting together a more detailed proposal for commercial development (see Kavadias, Loch, and De Meyer for further details. The authors gratefully acknowledge the help of Christoph Loch and Stylios Kavadias, whose case study on the Dragonfly UAV is the basis for the chapter). Table 5.1 lists the activities that need to be done to complete the proposal. Note that this entirely captures the work required for the proposal, not the actual development itself.

A quick (and rather naïve) view of Table 5.1 is that the total time to complete the proposal will be $9 + 3 + 11 + 7 + 8 + 6 + 21 + 10 + 15 + 5 = 95$ days. Alternatively, one might (equally naively) claim, the proposal development should take 21 days, the duration of the longest activity.

Both of these views omit an important aspect of the nature of project management. Some, but not all, of the activities are dependent on each other. For example, activity A_3 (aerodynamics analysis) requires the completion of activity A_2 (prepare and discuss surface models). Such dependencies are also referred to as *precedence relationships.* They can be summarized in a *dependency matrix* as shown in Table 5.2. In the dependency matrix, each

TABLE 5.1
Activities for the UAV Proposal Development

Activity	Description	Expected Duration (days)
A_1	Prepare preliminary functional and operability requirements, and create preliminary design configuration	9
A_2	Prepare and discuss surface models	3
A_3	Perform aerodynamics analysis and evaluation	11
A_4	Create initial structural geometry, and prepare notes for finite element structural simulation	7
A_5	Develop structural design conditions	8
A_6	Perform weights and inertia analyses	6
A_7	Perform structure and compatibility analyses and evaluation	21
A_8	Develop balanced free-body diagrams and external applied loads	10
A_9	Establish internal load distributions, evaluate structural strength stiffness; preliminary manufacturing planning and analysis	15
A_{10}	Prepare proposal	5

TABLE 5.2 Dependency Matrix for the UAV

		Information-Providing Activity (Upstream)									
		A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Information-Receiving Activity (Downstream)	A_1										
	A_2	X									
	A_3		X								
	A_4		X								
	A_5				X						
	A_6				X						
	A_7			X			X				
	A_8			X		X	X				
	A_9								X		
	A_{10}							X		X	

column represents an activity that provides information, and each row indicates an activity that receives information. An entry in column *i* and row *j* suggests that the activity in the i-th column (A_i) provides information to the activity in the j-th row (A_j). We also say that A_i precedes A_j or that A_j is dependent of A_i. Dependent activities require information or physical outputs from the input providing activities. The dependency matrix implicitly suggests a sequencing of the activities and thus dictates the flow of the project. The project will start with activity A_1, because it does not have any input providing activities. It will end with activity A_{10}. Similar to process flow terminology, people often refer to a preceding activity as "upstream" and the dependent activity as "downstream."

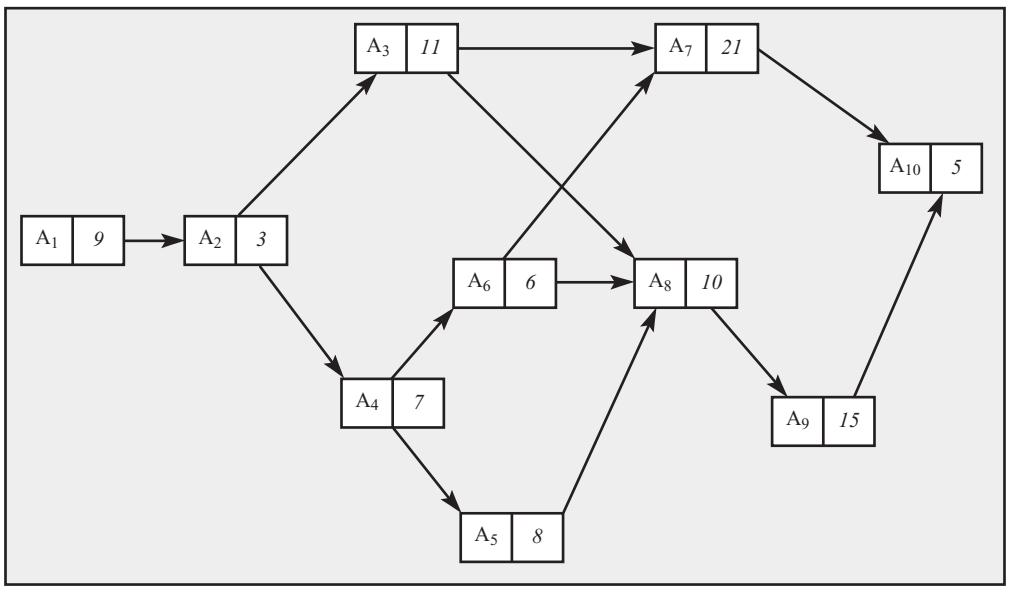
5.2 Critical Path Method

There exist multiple approaches to represent project information as displayed in Tables 5.1 and 5.2. In the *activity-on-node (AON) representation,* nodes correspond to project activities and arrows correspond to precedence relationships (with an arrow going from the input providing activity to the corresponding dependent activity). In this chapter, we focus on the AON representation because it is similar to the process flow diagrams that we discuss in the other chapters of this book.

To create an AON representation of a project, we start with the activity that requires no input, in our case that is activity A_1. We then work our way through the dependency matrix, mimicking the evolution of the project:

1. We create a node in the form of a box for the activity, including its name as well as its expected duration.
2. After creating the node for the activity, we consider the activity as done. Thus, all information provided by the activity to its dependent activities is now available. We can draw a line through the corresponding column, and draw an arrow out of the activity for each dependency (for each "X").
3. Next, we look for any other activity in the dependency matrix that has all its information providing activities completed and go back to step 1 until we have worked ourselves to the last activity.

FIGURE 5.2 **Activity on node (AON) representation of the UAV project. Left part of the box is the activity name; right part is the activity duration**



If we repeatedly execute these three steps, we obtain a graph as shown in Figure 5.2. This graph provides a practical and visual way to illustrate the evolution of the project. It resembles the process flow diagram introduced in Chapter 3.

5.3 Computing Project Completion Time

Despite the similarity between process flow diagram and the AON representation, we should remember the fundamental difference between process management and project management. In process management, we directed our attention to the resource that had the lowest capacity, the bottleneck. If each activity in the process flow diagram was staffed by one worker (or machine), the bottleneck was the activity with the longest activity time.

What matters for the completion time of the project, however, is not the individual activity times, but the completion time of the entire project. This completion time requires ALL activities to be completed. In fact, we will see that in the UAV project, the activity with the longest duration (A_7) will not constrain the duration of the overall project.

So, how long then will the project in Figure 5.2 take? This turns out to be a tricky question. It is intuitive that the project can be carried out in less than $9+3+11+7+8+6+21+10+15+5=95$ days (the sum of the activity times). Some activities can be carried out in parallel and so the 10 activities do not create a 10-person relay race. On the other hand, the degree to which we can execute the activities in parallel is limited by the dependency matrix. For example, activity A_3 requires the completion of activity A_2, which, in turn, requires the completion of activity A_1. Things get even more convoluted as we consider activity A_7. For it to be complete, A_3 and A_6 have to be complete. A_3, in turn, requires completion of A_2 and A_1, while A_6 requires completion of A_4, which, once again, requires completion of A_2 and A_1. What a mess!

To correctly compute the completion time of the project, a more structured approach is needed. This approach is based on considering all possible paths through the network in Figure 5.2. A path is a sequence of nodes (activities) and (directional) arrows. For example, the sequence A_1, A_2, A_3, A_7, A_{10} is a path. Every path can be assigned a duration by simply

adding up the durations of the activities that constitute the path. The duration of the path A_1, A_2, A_3, A_7, A_{10} is $9+3+11+21+5=49$ days.

The number of paths through the AON representation depends on the shape of the dependency matrix. In the easiest case, every activity would just have one information-providing activity and one dependent activity. In such a (relay race) project, the dependency matrix had just one entry per row and one entry per column. The duration of the project would be the sum of the activity times. Every time one activity provides information to multiple activities, the number of paths is increased.

In the UAV project and its project graph shown in Figure 5.2, we can identify the following paths connecting the first activity (A_1) with the last activity (A_{10}):

$$A_1-A_2-A_3-A_7-A_{10} \text{ with a duration of } 9+3+11+21+5=49 \text{ days}$$

$$A_1-A_2-A_3-A_8-A_9-A_{10} \text{ with a duration of } 9+3+11+10+15+5=53 \text{ days}$$

$$A_1-A_2-A_4-A_6-A_7-A_{10} \text{ with a duration of } 9+3+7+6+21+5=51 \text{ days}$$

$$A_1-A_2-A_4-A_6-A_8-A_9-A_{10} \text{ with a duration of } 9+3+7+6+10+15+5=55 \text{ days}$$

$$A_1-A_2-A_4-A_5-A_8-A_9-A_{10} \text{ with a duration of } 9+3+7+8+10+15+5=57 \text{ days}$$

The path with the longest duration is called the critical path. Its duration determines the duration of the overall project. In our case, the critical path is $A_1-A_2-A_4-A_5-A_8-A_9-A_{10}$ and the resulting project duration is 57 days. Note that A_7, the activity with the longest duration, is not on the *critical path.*

5.4 Finding the Critical Path and Creating a Gantt Chart

The exercise of identifying every possible path through the project graph along with its duration is a rather tedious exercise. The more activities and the more dependency relationships we have, the greater the number of paths we have to evaluate before we find the one we truly care about, the *critical path.*

Fortunately, there is a simpler way to compute the project duration. The idea behind this easier way is to compute the earliest possible start time for each activity. For each activity, we can find the *earliest start time (EST)* by looking at the earliest time all information providing activities have been completed. The earliest start time of the first activity is time zero. The *earliest completion time (ECT)* of an activity is the earliest start time plus the activity duration. We then work our way through the project graph, activity by activity, starting from the first activity and going all the way to the last.

More formally, we can define the following algorithm to compute the earliest completion time of the project. The approach is similar to our method of coming up with the graphical representation of the project graph:

1. Start with the activity that has no information-providing activity and label that activity as the start. The earliest start time of that activity is defined as 0. The earliest completion time is the duration of this activity.
2. Identify all activities that can be initiated at this point (i.e., have all information-providing activities complete). For a given such activity i, compute the earliest start time as:
$$EST(A_i) = Max\{ECT(A_j)\}, \text{ where } A_j \text{ are all activities providing input to } A_i$$
3. Compute the earliest completion time of A_i as
$$ECT(A_i) = EST(A_i) + Duration(A_i)$$
4. Consider activity i as completed, and identify any further activities that now can be initiated. Go back to step 2.

TABLE 5.3
Computing the Completion Time of a Project (table is created row by row, starting with the first activity)

Activity	Earliest Start Time (EST)	Expected Duration (days)	Earliest Completion Time (ECT)
A_1	0	9	9
A_2	$ECT(A_1) = 9$	3	12
A_3	$ECT(A_2) = 12$	11	23
A_4	$ECT(A_2) = 12$	7	19
A_5	$ECT(A_4) = 19$	8	27
A_6	$ECT(A_4) = 19$	6	25
A_7	$Max\{ECT(A_3),ECT(A_6)\} = Max\{23,25\} = 25$	21	46
A_8	$Max\{ECT(A_3), ECT(A_5),ECT(A_6)\} = Max\{23, 27, 25\} = 27$	10	37
A_9	$ECT(A_8) = 37$	15	52
A_{10}	$ECT(A_9) = 52$	5	57

This algorithm is illustrated in Table 5.3. The table is created from the top to the bottom, one activity at a time. As you construct a given row i, you have to ask yourself, "What activities provide information to i? What activities does i depend on?" You can see this by reading row i in the dependency matrix, or you can see this in the project graph.

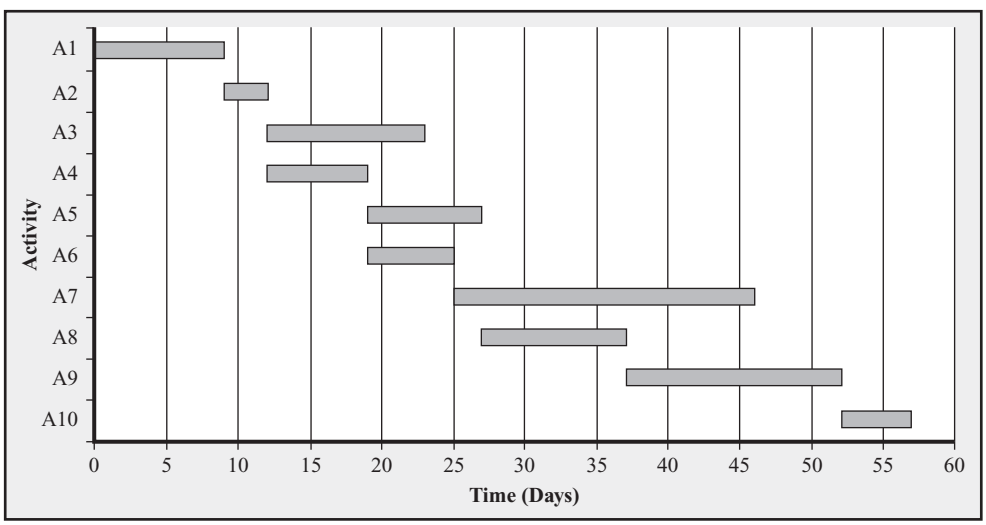
Based on the earliest start and earliest completion time, we can create a Gantt chart for the project. The *Gantt chart* is basically a timeline with the activities included as bars. Gantt charts are probably the most commonly used visualization for project time lines. Note that unlike the AON representation, the Gantt chart itself does not capture the dependencies of the activities. Based on the previously explained computations of the earliest start and completion times, we have already ensured that activities only get initiated when all required information is available.

The Gantt chart for the UAV project is shown in Figure 5.3.

5.5 Computing Slack Time

It lies in the nature of the critical path that any delay in activities on the critical path will immediately cause a delay in the overall project. For example, a one-day delay in activity A_9 will automatically delay the overall project by one day. However, this is not true for

FIGURE 5.3
Gantt Chart for the UAV Project



activities that are not part of the critical path. We can delay activity A_7 even by several days (six to be exact) without affecting the overall completion of the project. In other words, activity A_7 has some built in "wiggle room." The technical term for this wiggle room is *slack time.* It is the amount of time an activity can be delayed without affecting the overall completion time of the project.

The slack time of an activity is determined based on an additional set of calculations known as the late start schedule. So far, we have computed the earliest start time (EST) and earliest completion time (ECT) of each activity by going through the project from beginning to end. We now compute the *latest start time (LST)* and *latest completion time (LCT)* for each activity such that the project still completes on time. We do this by beginning with the last activity and working our way backward through the project until we reach the beginning. Thus, we start with the last activity (A_{10}) and end with the first activity (A_1).

So, let's start with the last activity. Assuming we want to complete the project as early as possible, we define the LCT of the last activity as being the same as its ECT:

$$\text{LCT(Last activity)} = \text{ECT(Last activity)}$$
$$\text{LCT}(A_{10}) = \text{ECT}(A_{10}) = 57$$

There exist some cases in which an early completion is not desired—instead, there exists a target time at which the project should be complete. In this case, we can define the LCT of the last activity as the target date.

The latest start time of the last activity is simply the latest completion time minus the duration of the last activity:

$$\text{LST(Last activity)} = \text{LCT(Last activity)} - \text{Duration(Last activity)}$$
$$\text{LST}(A_{10}) = \text{LCT}(A_{10}) - 5 = 57 - 5 = 52$$

More generally, we define the LCT for an activity as the smallest (earliest) LST value of all activities that are depending on it and the LST as the LCT minus the duration. Consider activity A_9, which only has A_{10} as a dependent activity. Thus, we can define:

$$\text{LCT}(A_9) = \text{LST}(A_{10}) = 52$$
$$\text{LST}(A_9) = \text{LCT}(A_9) - \text{Duration}(A_9) = 52 - 15 = 37$$

In the same manner, we compute:

$$\text{LCT}(A_8) = \text{LST}(A_9) = 37$$
$$\text{LST}(A_8) = \text{LCT}(A_8) - \text{Duration}(A_8) = 37 - 10 = 27$$

Next, consider activity A_7, the activity we previously observed to have some slack time.

$$\text{LCT}(A_7) = \text{LST}(A_{10}) = 52$$
$$\text{LST}(A_7) = \text{LCT}(A_7) - \text{Duration}(A_7) = 52 - 21 = 31$$

Note the difference between the earliest start time of A_7, which was 25, and the latest start time of A7, which we just found to be 31. In other words, we can delay the start of A_7 by six days without affecting the overall completion time of the project.

Based on this observation, we define the slack of an activity as:

$$\text{Slack time} = \text{Latest start time} - \text{Earliest start time}$$

In the same way, we can compute the other information of the late schedule. This information is shown in Table 5.4. Note that the columns LST and LCT are computed by going backward through the project graph; thus, we start with the rows at the bottom of the table and work our way up. As expected, the slack time of all activities on the critical path is zero.

TABLE 5.4
Computation of Slack Time

Activity	EST	Duration	ECT	LCT	LST = LCT − Duration	Slack = LST − EST
A_1	0	9	9	$LST(A_2) = 9$	$9 - 9 = 0$	0
A_2	9	3	12	$Min\{LST(A_3),LST(A_4)\} = Min\{16,12\} = 12$	$12 - 3 = 9$	0
A_3	12	11	23	$Min\{LST(A_7),LST(A_8)\} = Min\{31,27\} = 27$	$27 - 11 = 16$	$27 - 23 = 4$
A_4	12	7	19	$Min\{LST(A_5),LST(A_6)\} = Min\{19,21\} = 19$	$19 - 7 = 12$	0
A_5	19	8	27	$LST(A_8) = 27$	$27 - 8 = 19$	0
A_6	19	6	25	$Min\{LST(A_7),LST(A_8)\} = Min\{31,27\} = 27$	$27 - 6 = 21$	$27 - 25 = 2$
A_7	25	21	46	$LST(A_{10}) = 52$	$52 - 21 = 31$	$52 - 46 = 6$
A_8	27	10	37	$LST(A_9) = 37$	$37 - 10 = 27$	0
A_9	37	15	52	$LST(A_{10}) = 52$	$52 - 15 = 37$	0
A_{10}	52	5	57	57	$57 - 5 = 52$	0

What is the benefit of knowing how much slack time there is associated with an activity? The main benefit from knowing the slack time information is as follows:

- *Potentially delay the start of the activity:* To the extent that we can delay the start of an activity without delaying the overall project, we might prefer a later start over an earlier start. Because activities are often associated with direct expenses, simple discounted cash flow calculations suggest that the start times be delayed wherever possible.
- *Accommodate the availability of resources:* Internal or external resources might not always be available when we need them. Slack time provides us with a way to adjust our schedule (as shown in the Gantt chart) without compromising the completion time of the overall project.

Exhibit 5.1 summarizes the steps to plan the time line of a project and to identify the critical path as well as the slack times of the activities. Based on this information, we can

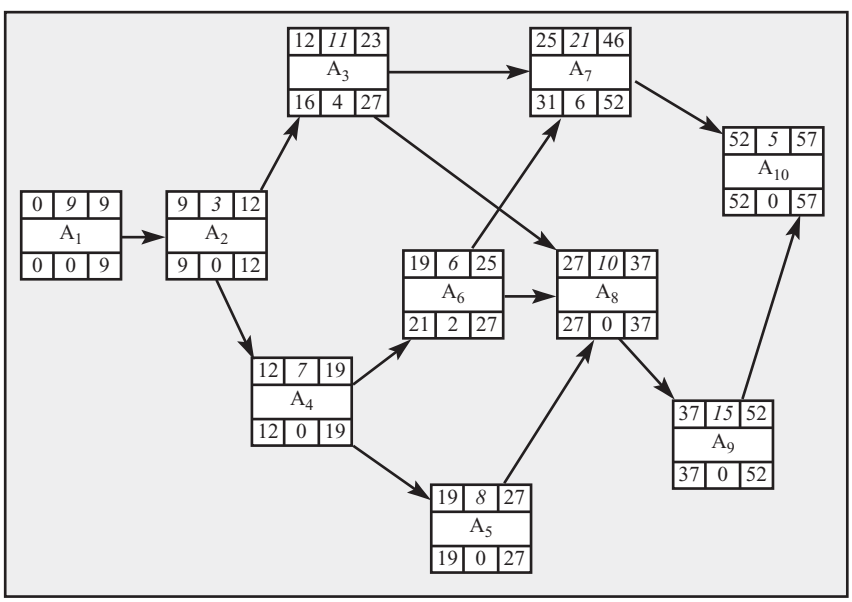


FIGURE 5.4
Augmented Project Graph. The top row includes the earliest start time, the duration, and the earliest completion time. The middle row is the activity name. The bottom row is the latest start time, the slack, and the latest completion time.

Exhibit 5.1

SUMMARY OF CALCULATIONS FOR A CRITICAL PATH ANALYSIS

1. Identify all activities that constitute the project.
2. Determine the dependencies among the activities by either creating a dependency matrix or by creating the project graph. Make sure there exists no circularity in the dependencies (i.e., the dependency matrix only has entries to the lower left of the diagonal and the project graph does not contain any loops).
3. Compute the earliest start time (EST) and the earliest completion time (ECT) by working forward through the project graph (from start to end).

$$EST(A_i) = Max\{ECT(A_j)\}, \text{ where } A_j \text{ are all activities providing input to } A_i$$

$$ECT(A_i) = EST(A_i) + Duration(A_i)$$

4. Compute the latest start time (LST) and the latest completion time (LCT) by working backward through the project graph (from end to start)

$$LCT(A_i) = Min\{LST(A_j)\}, \text{ where } A_j \text{ are all activities receiving input from } A_i$$

$$LST(A_i) = LCT(A_i) - Duration(A_i)$$

5. Compute the slack of an activity as

$$Slack(A_i) = LST(A_i) - EST(A_i)$$

6. Create the critical path by highlighting all activities with zero slack.

augment the initial project graph and present all information we computed for each activity in a graphical format, similar to what is similar to what is shown in Figure 5.2. This representation, as shown in Figure 5.4., is the output of many commercial software packages dealing with project management as well as a set of consulting tools.

Note that all of these computations assume that there exists no uncertainty in the activity durations (and dependencies). Uncertainty is the subject of the next section.

5.6 Dealing with Uncertainty

Given our definition of projects as temporary operations that deal with nonroutine work, projects often face a significant amount of uncertainty at their outset. Incorporating this uncertainty into the project plan is thus a central concern of project management.

How much uncertainty a project is exposed to depends on the nature of a project and its environment. Launching a new entrepreneurial venture is likely to be associated with more uncertainty than the construction of a residential building. We find it helpful to distinguish among four project management frameworks that we present in increasing order of the level of uncertainty they are suited for.

Random Activity Times

So far, we have behaved as if all activity times in the project were deterministic—that is, they could be predicted with certainty. However, it lies in the nature of many project activities that their duration can vary considerably. Often, project managers are asked to come up with a best-case, an expected-case, and a worst-case scenario for the duration of each activity.

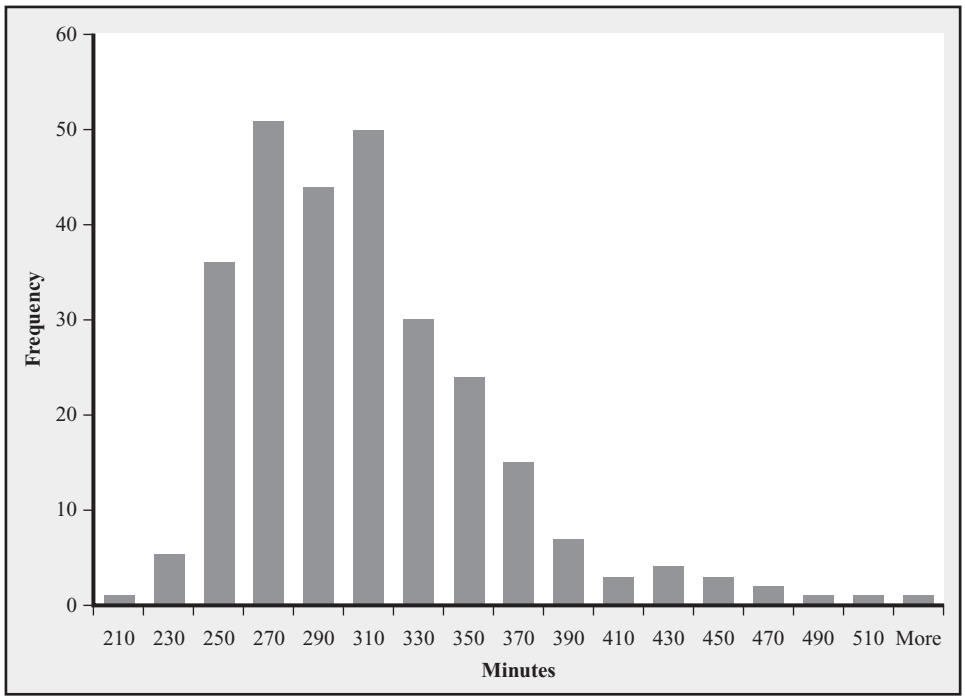


FIGURE 5.5
Procedure Durations in the operating room for Open Heart Surgery

(Data taken from Olivares et al.)

With that information in mind, it is possible to compute the variance of an activity time as well as the probability of meeting a certain due date. This is similar to the logic of uncertain activity times in waiting models that we explore in Chapter 8. Figure 5.5 shows the activity durations for a sample of cardiac surgeries in the operating room of a large hospital. We observe that there exists a considerable amount of procedure variation. Moreover, we observe that the distribution is not symmetric: activity durations that are more than double the mean duration can happen—the distribution has a "long tail."

When facing uncertainty in the activity time durations, it is important to understand that uncertainty in activity duration is a bad thing because it, on average, will lead to a later completion time of the project. It is a misconception that uncertainties in activity times will cancel each other out, just as the statement, "Put your head in the freezer and your feet in the oven and the average temperature you are exposed to is just about right," makes little sense. In a similar manner, variation in activity duration will not cancel out. When some activities are completed early and others are completed late, the overall impact on the project duration is almost always undesirable.

To see this, consider the simple project graph displayed in Figure 5.6. On the left side of the figure, we have a project with deterministic activity times. Given the activity durations of 5 days for A_1, 4 days for A_2, and 6 days for A_3, as well as the dependency structure shown by the project graph, the critical path of this project is $A_1 - A_3$ and the completion time is 11. Now, consider the activity times on the ride side of the figure. A_1 now has a

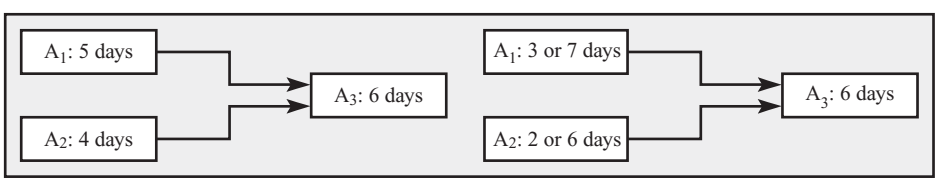


FIGURE 5.6
Simple Example of a Project with Uncertainty in the Activity Duration

TABLE 5.5
Example Calculations for a Small Project with Three Activities (based on Figure 5.6)

Scenario	Probability	Explanation	Start of A_3	Completion
A_1 late and A_2 late	0.25	A_1 would take 7 days (during which time, the 6 days of A_2 will also be completed)	7	13
A_1 early, A_2 late	0.25	A_2 would take 6 days (during which time, the 3 days of A_1 would also be completed)	6	12
A_1 late, A_2 early	0.25	A_1 would take 7 days (during which time, the 2 days of A_2 would also be completed)	7	13
A_1 early and A_2 early	0.25	A_1 would take 3 days (during which time, the 2 days of A_2 would also be completed)	3	9

completion time of 3 days with a 50% probability and 7 days with a 50% probability and A_2 has a completion time of 2 days with a 50% probability and 6 days with a 50% probability.

Note that in expectation (on average) the completion times of A_1 and A_2 have not changed. But the expected completion time of the project has. To see this, consider the calculations displayed in Table 5.5.

Observe that the expected completion time is:

$$0.25 \times 13 \text{ days} + 0.25 \times 12 \text{ days} + 0.25 \times 13 \text{ days} + 0.25 \times 9 \text{ days} = 11.75 \text{ days}$$

almost one day (0.75 day, to be exact) longer than in the deterministic base case. Note that this relies on a rather optimistic assumption in the case that both activities are completed early: we implicitly assume that A_3 has the flexibility of starting earlier than planned, when both A_1 and A_2 are completed early. If we cannot benefit from the early completion of activities, the overall penalty we incur from uncertainty would be even higher.

In the first three scenarios, we are slower than the deterministic completion time of 11 days. Only in the last case are we faster. Thus, we are not just exposed to the risk of the project running later than in the deterministic case, but we will be running later on average.

The reason for this effect is that the critical path of the project can potentially shift. In other words, an activity not on the critical path might delay the overall project because of a longer than expected duration. While A_2 was not on the critical path in the deterministic base case, we saw that it was holding up the project (and thus was on the critical path) in the second scenario analyzed earlier. Unfortunately, many books and software packages ignore this effect and pretend that the variance of the overall project duration can be directly computed from the variances of the activity times on the critical path. This is simply incorrect—a rigorous evaluation of the overall project duration almost always requires some Monte Carlo simulation.

Beyond avoiding this simple, yet very common mistake, correctly estimating the duration of the activity is a challenge. Estimates of activity durations are often inflated, especially when working with internal resources: because nobody on the team wants to be blamed for potential schedule overruns, it is common to quote excessively long estimates of activity durations (the estimates are "padded"). This is especially common if there exists no threat of substitution for a resource, as is common with resources internal to the organization (e.g., the IT department). Resources simply declare that it takes 10 days to complete the activity, even if their true forecast for the completion is 5 days. After all, what would be the incentive for the resource to commit to an aggressive schedule? Once the project gets under way, the schedule looks very tight. However, if one truly observes the execution of the project, most activities make little progress, and the corresponding

resources are either idle or working on other projects, even if they are associated with the critical path. Obtaining honest (unbiased) activity durations is thus essential. One technique is to compare actual activity durations with their forecasts, similar to what we discuss in Chapter 12.

However, estimates of activity durations can also be underestimated, especially when working with external resources: if contractors for a project are asked to submit a time estimate, they have a substantial incentive to underestimate the project completion time because this increases their likelihood of being selected for the project. Once on the job, however, they know that they cannot be easily kicked out of the project should their activity run late. For example, consider the OR data from Figure 5.5 discussed earlier. If we compare the actual time taken in the OR with the time estimates made initially when the OR was booked, it turns out that, on average, procedures take 10 percent longer than initially forecasted. The reason for this is that doctors often want to get a particular slot on the OR schedule—and they know that they are more likely to get a slot in the near future if their procedure time is short. However, they also know that once they have started the procedure, there exists no way to penalize them for a schedule overrun. With this in mind, they simply promise overly optimistic activity durations. Again, obtaining unbiased activity durations is important. Project contracts, and especially late completion penalties, are also an instrument to consider when working with external parties.

Potential Iteration/Rework Loops

The previously introduced dependency matrix (see Table 5.2) had an important property—all dependencies were on the lower left of the diagonal. In other words, there existed a one-way path from the beginning of the project to the end.

In practice, however, projects often require iteration. In fact, the previously discussed UAV project commonly (in about 3 out of 10 cases) iterates between activities A_4 and A_9. Such iterations are typical for product development and innovation projects where problem solving can be a more organic, iterative process. It is often referred to as rework.

In general, such *rework loops* are more likely to happen in high-uncertainty environments. For example, a development team for an Internet platform might want to adjust its business plan after having launched a beta prototype, creating a rework loop. In contrast, we hope that the architect in charge of a major construction project does not want to revisit her drawings after the first tenants moved into the building. Consequently, project planning tools such as Gantt charts and the critical path method are more valuable for low-uncertainty projects, and they can provide a false sense of planning accuracy when applied in high-uncertainty environments.

Several tools exist for modeling and analyzing projects with iteration. We restrict ourselves to the main insight from this line of research. The presence of iteration loops typically dominates the effect of uncertain activity duration. In other words, when faced with the potential of some activities taking longer than expected and an unexpected iteration requiring reworking one or multiple previously completed activities, a project manager should focus on the threat of the iteration because it has a stronger effect on the overall completion time.

Decision Tree/Milestones/Exit Option

The previous two types of uncertainty reflected the question, "When will the project be completed?" Activities might take a little longer (uncertain activity times) or sometimes might even have to be repeated (rework loops), but in the end, we always complete the project.

Often, however, a more fundamental question is of essence to the project manager: "Will we complete this project at all, or should we terminate the project?" Such uncertainty is common in many innovation settings, include venture capital–funded projects or pharmaceutical

research and development (R&D). For example, only a small fraction of R&D projects that enter phase 1 clinical trials will be launched in the market. More than 80 percent of the projects will be canceled along the way.

Project management techniques as reviewed earlier are inappropriate for handling this type of uncertainty. The threat of terminating the project because of new market data (market uncertainty) or new technical data (technological uncertainty) looms so large that it trumps the previously discussed types of uncertainty.

Decision trees map out the potential scenarios that can occur once the uncertainty is resolved and the potential set of actions that can be taken in each scenario. A key insight that can be derived from such models is the observation that it is often substantially cheaper to exit a project early, instead of letting costs escalate and then exiting the project later on at higher costs. The project management implication of this is that it is very desirable to move activities that resolve this type of uncertainty (feasibility studies, market research) to the early part of the project.

Unknown Unknowns

When Christopher Columbus set out to find a new way to sail to India, he (most likely) did not set up a project plan. Even for modern time explorers, be it in sailing or in business, there exist situations where the amount of uncertainty we face is simply too large to make any careful planning process meaningful. In such settings, we face so much uncertainty that we don't even know what we don't know. We face *unknown unknowns*, also referred to as *unk-unks.*

It lies in the nature of many high-uncertainty projects that they will not be completed. In that sense, a timely abandonment often is the goal as it avoids an escalation in costs. Often, a useful exercise is to simply list all variables in the project that are currently not known and to look for activities that would help resolve these unknowns. At any moment in time, the project manager should then attempt to spend as little as possible to learn enough to decide whether or not to move forward with the project. This technique, also referred to as *discovery-driven planning,* will help resolve some uncertainties and potentially identify new ones.

Exhibit 5.2 summarizes these levels of project uncertainty. The main point is that different project management tools apply to different projects, depending on the amount of uncertainty they face. It is neither advisable to use a high-uncertainty tool (such as decision trees) for a low-uncertainty project (why would you want to evaluate an exit option every day in an ongoing construction project?) nor vice versa (why try to find and optimize the critical path if you do not even know if you are in business next quarter?).

5.7 How to Accelerate Projects

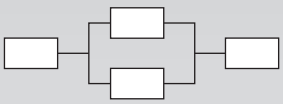
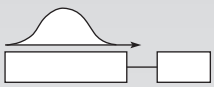
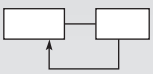
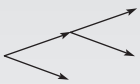
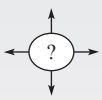
Project managers typically pursue a combination of three objectives: project completion time, project cost (budget), and the quality of the accomplished work. Sometimes, these objectives are in conflict with another. This then creates a trade-off among the three dimensions, similar to what we have seen in other chapters of this book (e.g., the trade-off between call center responsiveness and efficiency in Chapter 1).

Consider the development of the UAV discussed earlier. Most likely, more time would allow the developers to put together an even more convincing proposal. Similarly, if budget would not be a constraint, it might be possible to outsource at least some work, which, if it shortens the duration of a critical path activity, would lead to an earlier project completion time.

Beyond trading off one goal against another goal, we can also try to "break the trade-off" and just be smarter about how we manage the project. The following provides a set of inexpensive

Exhibit 5.2

SUMMARY OF DIFFERENT UNCERTAINTY LEVELS IN A PROJECT

Certainty 	Low uncertainty project such as construction projects or routine development projects	Calculate critical path use slack to optimize timing
Uncertainty in activity duration 	Projects with minor uncertainties about activity durations and or resource availability	Monte Carlo Analysis–watch for changes in critical path
Potential iteration 	Potentially iterative projects that include one or multiple rework loops	Rework loops
Potential termination 	Multiple scenarios exist, one or more of them require termination of the project	Decision trees
Unk-Unks 	High levels of uncertainty and a dynamic environment; chaos	Discovery driven planning

actions a project manager can take to accelerate the completion time of the project without necessarily sacrificing the quality of the accomplished work or the project budget:

- *Start the project early:* The last day before the project's due date is typically a day of stress and busyness. In contrast, the first day of the project is typically characterized by little action. This effect is similar to the "term paper syndrome" well familiar to most students. It reflects human optimism and overconfidence in the ability to complete work in the future. At the risk of stating the obvious—a day at the beginning of the project is equally long as a day at the end of the project—why do little or no work on the former and jam all the work into the latter?
- *Manage the project scope:* One of the most common causes of delay in projects is that the amount of work that is part of the project changes over the course of the project. Features are added and engineering change orders requested. If such changes occur late in the project, they often cause significant project delays and budget overruns for relatively little increased quality. For this reason, it is advisable to finalize the scope of the project early on.
- *Crash activities:* Often, an increase in spending allows for a faster completion time of a project. Contractors are willing to work overtime for a premium, and expensive equipment might help further shorten activity duration. However, the reverse is not always true. Projects that take excessively long are not necessarily cheaper. Because typically there are some fixed costs associated with a project, a project that drags on forever might actually be also very expensive.
- *Overlap critical path activities:* A central assumption underlying the dependency matrix shown in Table 5.2 has been that an activity that is dependent on an information-providing activity needs to wait until that activity is completed. However, it is often possible to allow the dependent activity to start early, relying on preliminary information

from the information-providing activity. For example, it seems plausible that the activity "Building design" should be completed before starting the activity "Building construction." However, does this imply that all of the design has to be completed? Or, maybe, would it be possible to begin digging the foundation of the building while the designers are still finalizing the shape of the windows? By identifying the exact dependencies among activities, it is often possible to provide the dependent activity with a head start.

5.8 Literature/ Further Reading

Loch et al. (2006) provide a comprehensive framework of managing projects with uncertainty. The authors use many illustrative examples and target experienced project managers as their audience.

Terwiesch and Ulrich (2009) deal with far-horizon innovation projects as well as multiple challenges associated with financial evaluations of innovation projects.

Ulrich and Eppinger (2011) is the classic textbook for product development and includes an easy-to-follow introductory chapter on project management and project organization.

5.9 Practice Problems

Q5.1* **(Venture Fair)** In order to participate at a venture fair, Team TerraZ is preparing a project plan for their new-product offering. The team plans to spend 3 days on ideation. Once ideation is complete, the team aims to interview 20 potential customers (6 days) and to engage in a careful analysis of competing products (12 days). Following the customer interviews, the team expects to spend 10 days on careful user observation and 4 days on sending out e-mail surveys. These two activities are independent from each other, but both require that the interviews be completed. With the input from the customer observation and the e-mail surveys, the team then plans to spend 5 days on putting together the target specifications for the product. This activity also requires the analysis of competing products to be complete.

After the target specifications, the team aims to create a product design, which will take 10 days. With the product design complete, they plan to get price quotes (6 days) and build a prototype (4 days) that they then want to test out with some customers (5 days). Once the prototype has been tested and the price quotes are in, they can put together their information material for the venture fair (3 days).

a. Create a dependency matrix for the activities described, and build a project graph.

b. Find the critical path. What is the latest time the team can start working, assuming the venture fair is scheduled for April 18?

	1	2	3	4	5	6	7	8	9	10
1										
2	X									
3	X									
4		X								
5		X								
6			X							
7					X					
8						X				
9							X	X		
10				X					X	

Activity	1	2	3	4	5	6	7	8	9	10
Time	5	4	8	5	1	12	4	7	6	2

Q5.2 **(10 Activities)** Consider the dependency matrix and the activity durations provided above.

a. Build a project graph, visually depicting the evolution of this project.

b. Find the critical path. What is the earliest time that the project can be completed?

c. For each activity, compute the late start, the late completion, and the slack time.

Q5.3** **(Graduation Party)** Thierry, Ute, and Vishal are getting ready for their last period in the MBA program. Following the final exams, they intend to throw a big party with many of their friends from back home. Presently, they have identified the following set of activities that need to be completed. They decide to not spend any work on preparing the party until all final exams are over. Moreover, they aim to spend a 3-day beach vacation as early as possible, but not before all party planning activities are completed.

On June 10, they will enter the final exam week, which will take 5 days. They then want to arrange for live music (which will take 5 days), evaluate a number of potential party sites (6 days), and prepare a guest list, which includes inviting their friends and receiving the RSVPs (7 days). They want to visit their two most promising party sites, which they expect to take 4 days. However, this can only be done once they have completed the list of party sites. Once they have finished the guest list and received the RSVPs, they want to book hotel rooms for their friends and create a customized T-shirt with their names on it as well as the name of the guest. Hotel room reservation (3 days) and T-shirt creation (6 days) are independent from each other, but both of them require the guest list to be complete. Once they have picked the party site, they want to have a meeting on site with an event planner, which they expect to take 4 days. And then, once all work is completed, they plan to take off to the beach.

a. Create a dependency matrix for the activities described.

b. Build a project graph, visually depicting the evolution of this project.

c. Find the critical path. What is the earliest time that the three can go to the beach?

d. For each activity, compute the late start, the late completion, and the slack time.

Q5.4 **(Three Activities with Uncertainty)** A small project consists of three activities: A, B, and C. To start activity C, both activities A and B need to be complete. Activity A takes 3 days with a probability of 50 percent and 5 days with a probability of 50 percent, and so does Activity B. Activity C takes 1 day. What is the expected completion time of the project?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 6

The Link between Operations and Finance

To the reader new to the area of operations management, the previous chapters might have appeared more technical than expected.[1] After all, most of the performance measures we used were concepts such as balancing the line to increase labor utilization, reducing inventories, improving flow time, and so on. But WHY do we have to worry about these measures? Do they really matter to our job? Or, asked differently, what is the objective of all this?

The objective of most incorporated organizations is to create economic value. Those who have money invested in the enterprise want to see a return on their money—a return that exceeds the return that they would get if they invested their money differently, for example, in a bond, a savings account, or a competing organization. Economic value is created whenever the return on invested capital (ROIC) in a corporation exceeds the cost of capital (the weighted average cost of capital, WACC, is an important concept from the field of corporate finance). This is visible in the basic value equation:

$$\text{Economic value created} = \text{Invested capital} \times (\text{ROIC} - \text{WACC})$$

Since the cost of capital cannot be changed easily in the short term, our focus here is on the return on invested capital. More details about corporate valuation can be found in Koller, Goedhart, and Wessels (2005).

In this chapter, we show the link between the operational variables we have discussed previously (and that are discussed throughout this book) and ROIC. This is an ambitious task. In many organizations, not to mention business school courses, the topics of operations management and corporate finance are rather remote from each other.

Given this fundamental disconnect, managers and consultants often struggle with questions such as "What performance measures should we track?"; "How do operational performance measures impact the bottom line performance?"; or "How do we go about improving processes to achieve various operational performance improvements, including cost savings, lead-time reduction, or increases in product variety?"

[1] The authors thank Stephen Doig and Taylor Randall for their input to this chapter. They are especially grateful to Paul Downs for providing them with detailed data about his company.

The objective of this chapter is to provide readers with a set of tools that support them in analyzing the operational performance of a company and to guide them in increasing the overall value of the firm by improving its operations. We will do this in three steps. First, we introduce the ROIC tree, also known as the KPI tree (KPI stands for key performance indicators). Second, we show how to value operational improvement opportunities, that is, predicting by how much the ROIC improves if we improve our process along some of the operational measures defined elsewhere in the book. Third, we provide examples of KPI trees and look at how we can read financial statements to get a sense of the operational performance of a firm. The first two steps will be illustrated using the case of a small Pennsylvania furniture company, Paul Downs Cabinetmakers.

6.1 Paul Downs Cabinetmakers

Paul Downs started making furniture in 1986 in a small shop in Manayunk, Pennsylvania. (Manayunk, pronounced "Man-ee-yunk," is a hip neighborhood in Philadelphia.) Over the years, his business outgrew four shops and is now operating in a 33,000-square-foot facility in Bridgeport, Pennsylvania. The company focuses on high-end, residential furniture. Figure 6.1(a) shows one of their most popular dining table models.

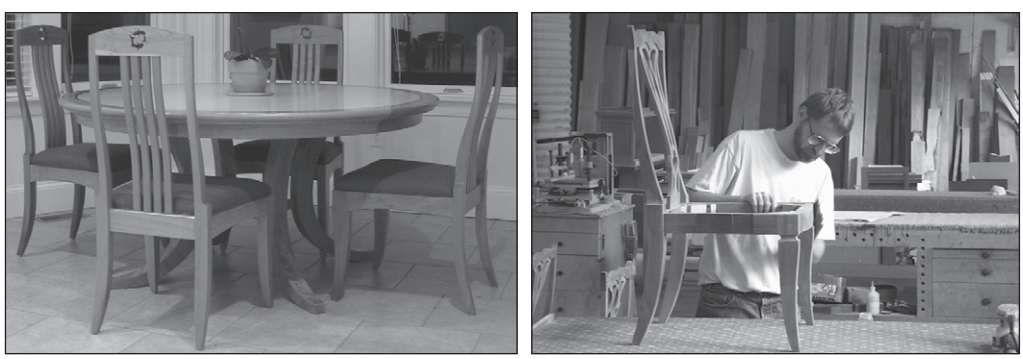
Paul Downs' production facility includes machines and other wood-processing equipment valued at about $450,000. There is an annual depreciation associated with the machines (reflecting the duration of their useful life) of $80,000. Rents for the showroom and the factory amount to roughly $150,000 per year. Other indirect costs for the company are about $100,000 per year for marketing related expenses, $180,000 for management and administration, and $60,000 for a highly skilled worker in charge of finishing furniture and conducting a quality inspection.

The company has two major types of inventory. There is about $20,000 tied up in raw materials. This is wood that is purchased from suppliers in large order quantities (see Chapter 7 for further details on order quantities). When purchasing wood, Paul Downs needs to pay his suppliers roughly one month in advance of receiving the shipment. There is also about $50,000 of work-in-process inventory. This corresponds to furniture that is in the process of being completed.

Furniture production, especially in the high-end segment, is a very manual process and requires a highly skilled workforce. Paul employs 12 cabinetmakers (see Figure 6.1(b)), many of whom have been with his company for more than a decade. The cabinetmakers

FIGURE 6.1 **Finished Product and Work in Progress from Paul Downs' Production Facility**

Source: Paul Downs



(a) (b)

work about 220 days in a year (on average about 8 hours per day). The typical wage rate for a cabinetmaker is $20 per hour.

To finish a typical piece of furniture, a worker needs about 40 hours. This corresponds to our previous concept of an processing time. The work is organized in work cells. Instead of having the cabinetmakers specialize in one aspect of furniture making (e.g., cutting, sanding, or polishing), a cabinetmaker handles a job from beginning to end. Of their overall number of hours worked, cabinetmakers spend about 15 percent of their time building fixtures and setting up machines (more on setup times in the Chapter 7). Given the modern production equipment, a good part of this includes programming computer-controlled machines. Since the cabinetmakers are organized in work cells, it would be too expensive to equip each cell with all wood-working equipment; instead, the cabinetmakers share the most expensive tools. This leads to an occasional delay if multiple cabinetmakers need access to the same unit of equipment at the same time. Consequently, cabinetmakers spend about 10 percent of their time waiting for a particular resource to become available.

From a design perspective, a typical piece of furniture requires about 30 kg of wood. In addition to this wood, about 25 percent additional wood is needed to account for scrap losses, primarily in the cutting steps of a job. Wood costs about $10 per kg.

Purchasing high-end furniture is not cheap—customers pay about $3,000 for a dining table like the one shown in Figure 6.1(a). Typically, customers are expected to pay 50 percent of the price as a down payment. They then receive their furniture about three months later. This delay reflects the custom nature of the end product as well as the fact that Paul Downs's facility at the moment is fully utilized, that is, there is more demand than what can be processed by the factory.

6.2 Building an ROIC Tree

As the owner of the firm, Paul Downs is primarily interested in creating economic value and thus in increasing the ROIC of his firm. The problem with respect to increasing ROIC is that ROIC, in and of itself, is not a lever that is under direct managerial control. It can be computed at the end of a quarter or a year, but while a manager might go to work in the morning thinking, "Today, I will increase my ROIC by 5 percent," it is not at all clear how to achieve that objective. The idea behind building an ROIC tree is to cascade the high-level financial metric into its key operational ingredients, thereby revealing the levers a manager can use to improve ROIC. To use a metaphor from the sciences, to understand how a biological cell works, we need to explain the behavior of its component molecules.

Let's begin by writing down our overall goal, the ROIC:

$$\text{ROIC} = \frac{\text{Return}}{\text{Invested capital}}$$

Now, let's do a simple algebraic manipulation and write

$$\text{ROIC} = \frac{\text{Return}}{\text{Invested capital}} = \frac{\text{Return}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Invested capital}}$$

The first ratio, Return/Revenue, is the company's margin. The second ratio, Revenue/Invested capital, is called the company's capital turns. Note that it resembles the measure of inventory turns that we introduced in Chapter 2. This simple, though elegant, way of decomposing the ROIC into margin and asset turns is often referred to as the DuPont model. DuPont was among the pioneers introducing financial performance measures to its business units.

Companies and industries differ widely with respect to how they achieve a specific ROIC. Some industries are asset-intensive: the capital turns are low, but their margins are significant. Others require little capital. Such industries are typically easier to enter for new competitors, leading to relatively thin margins.

Now, back to Paul Downs. As advisors to Paul, we can now help him improve his business by saying: "Paul, to improve your ROIC, you need to either increase your margin or turn your assets faster . . ." It is unlikely that this advice would ensure our future career as management consultants.

Nevertheless, let's keep pushing the same logic further and now decompose margin and asset turns into their drivers. Consider margin first. Based on standard accounting logic, we can write the Return (profits) of the firm as

$$\text{Return} = \text{Revenue} - \text{Fixed costs} - \text{Production volume} \times \text{Variable costs}$$

Because this is not an accounting book, and to be consistent with our definitions throughout the book, let us use "Flow rate" instead of "Production volume." Given the above equation, and keeping in mind that Revenue = Flow rate × Price, we can rewrite the previous equation by dividing both sides by Revenue, which yields

$$\begin{aligned}\frac{\text{Return}}{\text{Revenue}} &= \frac{\text{Revenue}}{\text{Revenue}} - \frac{\text{Fixed costs}}{\text{Revenue}} - \frac{\text{Flow rate} \times \text{Variable costs}}{\text{Revenue}} \\ &= 1 - \frac{\text{Fixed costs}}{\text{Flow rate} \times \text{Price}} - \frac{\text{Flow rate} \times \text{Variable costs}}{\text{Flow rate} \times \text{Price}} \\ &= 1 - \frac{\text{Fixed costs}}{\text{Flow rate} \times \text{Price}} - \frac{\text{Variable costs}}{\text{Price}}\end{aligned}$$

Using a similar logic as we used for margin, we can write asset turns as

$$\frac{\text{Revenue}}{\text{Invested capital}} = \frac{\text{Flow rate} \times \text{Price}}{\text{Invested capital}}$$

Our overall ROIC equation can now be written as

$$\text{ROIC} = \left[1 - \frac{\text{Fixed costs}}{\text{Flow rate} \times \text{Price}} - \frac{\text{Variable costs}}{\text{Price}}\right] \times \frac{\text{Flow rate} \times \text{Price}}{\text{Invested capital}}$$

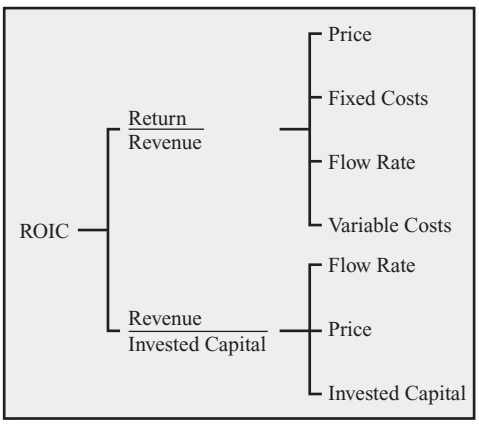
Because ultimately, we want to be able to express the ROIC as a function of its atomic ingredients such as wage rates, processing times, idle times, and so forth, we need to continue this process further. To avoid an explosion of mathematical equations, we prefer to write them in tree forms (see Figure 6.2).

Now, consider the four variables that we discovered as drivers of margins in greater detail: Flow rate, Fixed costs, Variable costs, and Price.

To focus our analysis on the operations aspects of this case, we assume Price has already been established—in other words, we do not consider Price to be one of our potential levers. Of course, we could take our operations-focused analysis and modify it appropriately to conduct a similar marketing-focused analysis that concentrates on the pricing decision. In general though, we caution the reader not to "build a machine with too many moving parts"—especially at the start of a project, looking at an operation in detail, one simply needs to make some assumptions. Otherwise, one runs the risk of getting lost in the complexity.

Next, consider the variable costs. In our example, the variable costs are driven primarily by the consumption of wood. In some cases, one also could consider the cost of labor as a variable cost (especially if workers get paid part of their salary on a piece-rate basis).

FIGURE 6.2
ROIC Tree



Yet, in our case, the number of cabinetmakers, as well as their hourly wages, is given and thus constitutes a fixed cost. Focusing on wood expenses, we can write the variable costs of a piece of furniture as

$$\begin{aligned}\text{Variable cost} &= \text{Price of wood} \times \text{Wood per table} \\ &= \text{Price of wood} \times (\text{Wood in final table} + \text{Cutting loss})\end{aligned}$$

Now, let us turn our attention to Flow rate. Recall from our earlier definition that

$$\text{Flow rate} = \text{Min}\{\text{Demand, Process capacity}\}$$

Because we assume that there is enough demand at the moment, Flow rate is determined by Process capacity. But what determines capacity in this case? The main constraint on this operation is the work of the cabinetmakers. The number of units of furniture that we can produce per year depends on

- The number of available worker hours, which is determined by the number of cabinetmakers multiplied by the hours each cabinetmaker works per year.
- The time a worker needs for a piece of furniture, which is determined by the amount of time it takes a cabinetmaker to wait for a machine to become available, the time to set up the machine, and the actual time to do the work.

Figure 6.3 summarizes these calculations in tree format. The figure also shows how we can make the tree more informative by adding the corresponding mathematical symbols into it.

FIGURE 6.3
The Drivers of Process Capacity

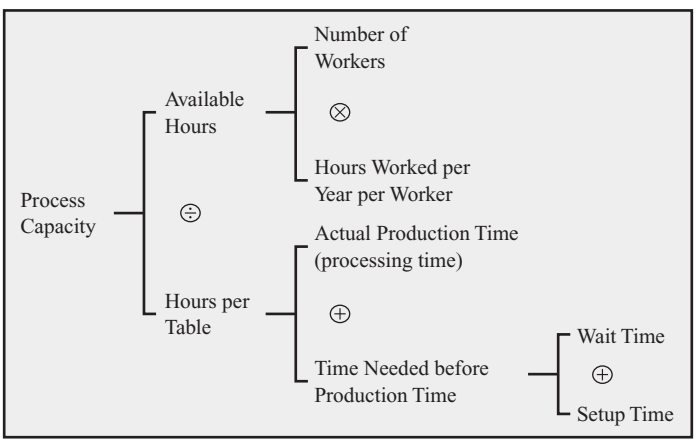
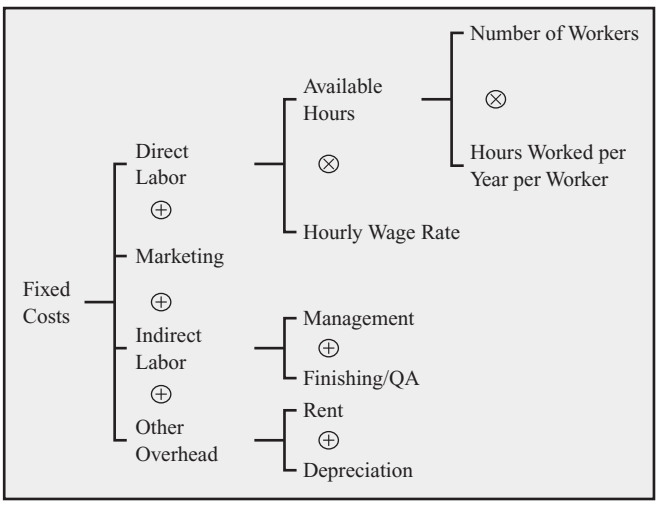


FIGURE 6.4
ROIC Tree for Fixed Costs



Finally, let us consider the Fixed costs. They include expenses for marketing, the labor expenses for overhead (inspection, administration), rent, depreciation, and the cost of the workforce. Figure 6.4 summarizes the main components.

It should be noted that one should be very careful how to measure depreciation. It is important to distinguish between the loss of value of a machine (e.g., a reduction in its useful life) and the depreciation as it is calculated for tax purposes. Following the standard practice of valuation and corporate finance, our emphasis is on the former view of depreciation. Note further that we do not include taxes in our analysis here (i.e., we compute the pre-tax ROIC).

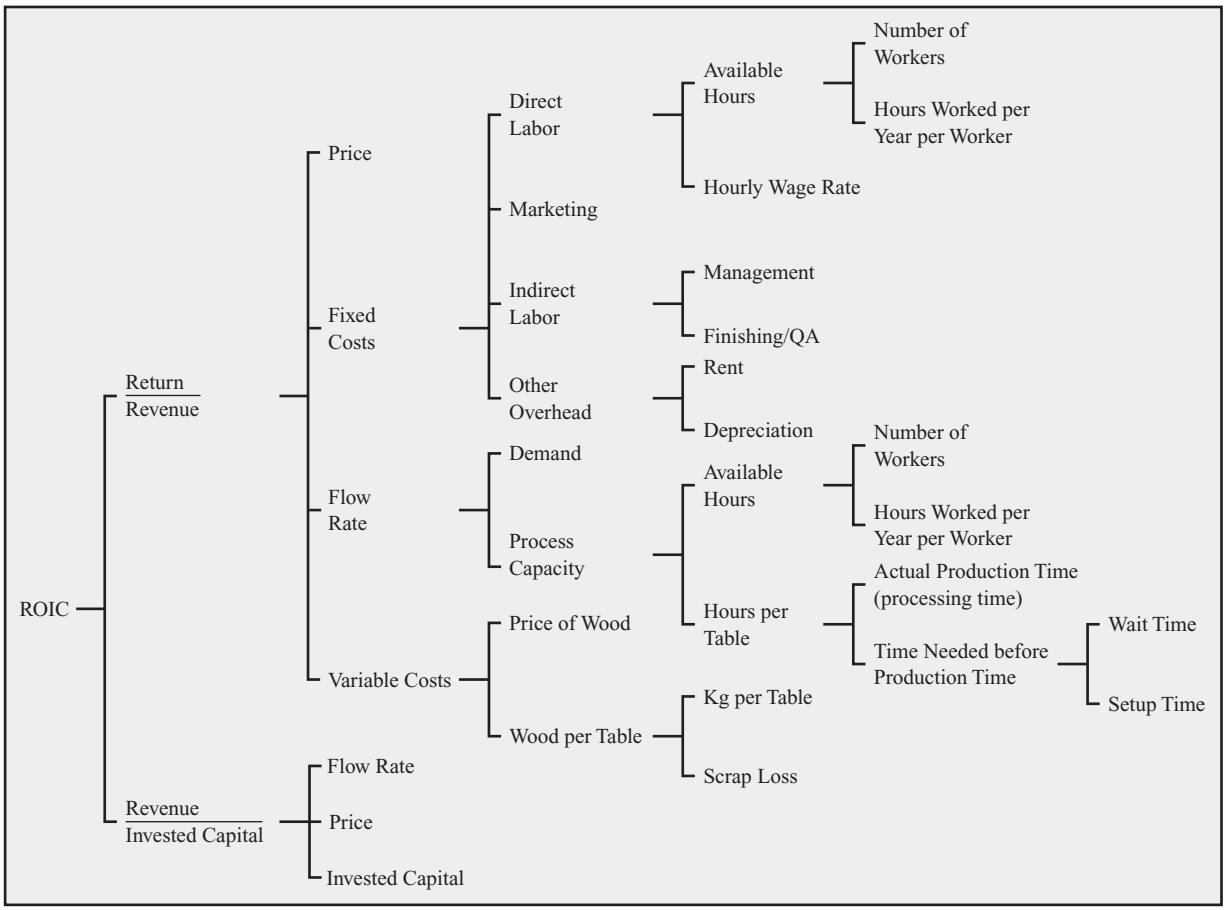
Combining our previous work, we now can extend Figure 6.2 to a more complete picture of ROIC drivers as shown in Figure 6.5. Note that, based on this extended tree, we now have achieved an important part of our objective for this chapter—we have created a direct linkage between the ROIC and "down-to-earth" operational variables such as idle time, setup time, processing time, and flow rate.

To complete our ROIC tree, we now need to turn to the asset-turn branch of the tree and explore it to the same level of detail as we have explored the margin branch. Because we can take the Flow rate (and the Price) from our previous analysis, what is left to be done is a refined analysis of the invested capital. Capital is invested in plant, property, and equipment (PP&E) as well as in three forms of working capital:

- Inventory includes the inventory of raw materials (wood) as well as all work-in-process inventory (WIP), that is, a pile of semi-finished pieces of furniture.
- Prepayments to suppliers include money that we have sent to our suppliers but for which we have not received the associated shipment of raw materials.
- Any money we are waiting to receive from our customers for products that we have already shipped to them. While in most businesses this part of the balance sheet requires an investment in capital, the situation, in our case, is much more favorable. As customers pay us a down payment well in advance of receiving their furniture, this line item actually corresponds to an inexpensive form of cash. For this reason, we should label this item "unearned revenues" so as not to upset any of the accountants in our readership.

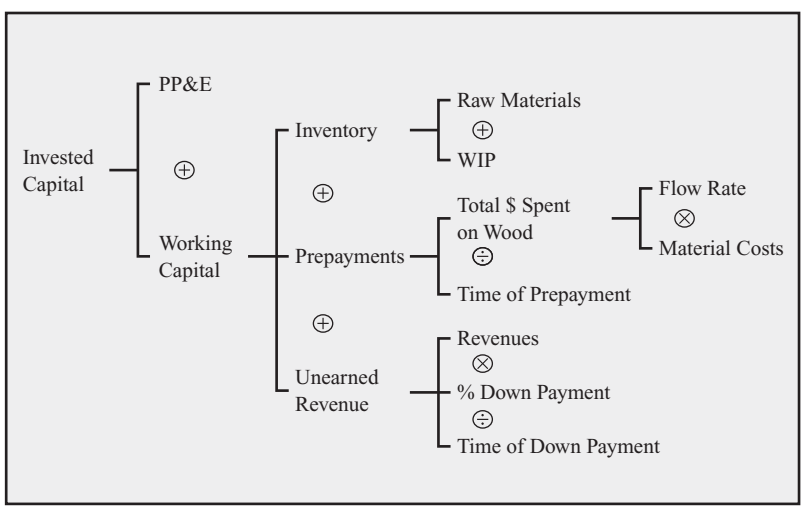
Figure 6.6 summarizes the components in invested capital in tree format. When we compute the amount of money that we need to invest in accounts payable, we first need to find out how much money we spend on wood purchasing every year. Because we have to pay our supplier one month in advance, at any given point, we have one-twelfth of the yearly payment tied up as capital. A similar logic applies to the unearned revenues.

FIGURE 6.5 **Expanded ROIC Tree**



This completes the development of the ROIC tree. We now have expressed our key financial performance measure, ROIC, as a function of detailed operational variables. We have explained the behavior of the cell by looking at its molecules and ultimately at its atoms.

FIGURE 6.6
ROIC Tree for Invested Capital



6.3 Valuing Operational Improvements

Understanding the link between processing times, wage rates, and other operational variables, and ROIC is certainly a useful motivation to illustrate that these variables are worthwhile studying—they are a nice teaser in a book chapter. But are they also useful in practice? What is the benefit of all this work?

The key benefit of the calculations defined above is that we can now assign a value tag to each of the operational levers that we potentially might pull to improve our operations. As the owner, manager, or consultant of the company, one can do many things to improve the ROIC such as

- Cut wages.
- Change the design so that the work required to make a piece of furniture is reduced.
- Reduce the time workers spend waiting for a machine.
- Reduce the setup times.
- Change the payment terms with the supplier, and so on.

But which of these actions are worth pursuing? All of them are likely to come along with some cost, and at the very minimum they will require management time and attention. So, which ones pay back these costs? Or, put differently, where is the juice worth the squeeze?

We thus want to find out how a change in one of the operational variables leads to a change in ROIC. This can require a lot of tedious calculations, so it is best to conduct this analysis using Excel. Figure 6.7 shows our full tree in spreadsheet format. It also populates the tree with numbers, creating a complete picture of the operations of the furniture we make.

Note that one variable might occur at multiple locations in such a spreadsheet model. Consider, for example, the variable Flow Rate in our furniture example. Flow rate shows up in the revenue part of the tree. It also shows up as part of the material costs. And, finally, it also shows up in the working capital calculation as downpayments depend on revenue (and thus flow rate) as well as material costs depend on flow rate. Thus, when building the spread sheet, it is important to keep all these usages of one variables connected, i.e., driven by the same cell. Put differently, an increase in flow rate is not just giving us more revenues. It also creates more material costs, it adds working capital by increasing the downpayments, and it reduces our working capital by adding to the prepaid expenses

Once we are equipped with such a spreadsheet model, we can easily find the impact of an operational variable by changing the corresponding cell and observing the change in the cell corresponding to the ROIC.

Before we do this, let's develop some intuition. What will happen if we reduce the setup times by five percentage points (from 15 percent to 10 percent)? Of course, shorter setup times are a good thing and we expect the ROIC to improve. Put differently, if somebody offered us to reduce setup times for free, we would happily take him or her up on the offer.

The crucial question is thus: by how much will the ROIC improve? What will happen to the root of our tree (ROIC) if we wiggle it at one of its leaves (setup time)? Will the ROIC change by 1 percent? More? Or less?

It is hard to answer such a question based on intuition. When asked to make a guess without a formal analytical model, most people we know argue along the following line: "There are many variables that influence ROIC. So, changing one of them by five percentage points will have an effect substantially smaller than a five-percentage-point ROIC improvement." This logic is in line with the tree metaphor: if you wiggle a tree at any one of its leaves, you do not expect big movements at its roots.

Table 6.1 shows that this argument does not hold. In fact, this guess is well off the mark. A five-percentage-point change in setup times leads in our example to an 18.8-percentage-point improvement in ROIC (i.e., it raises ROIC from the base case of 12.3 percent to 31.1 percent).

FIGURE 6.7 **ROIC Tree in Excel**

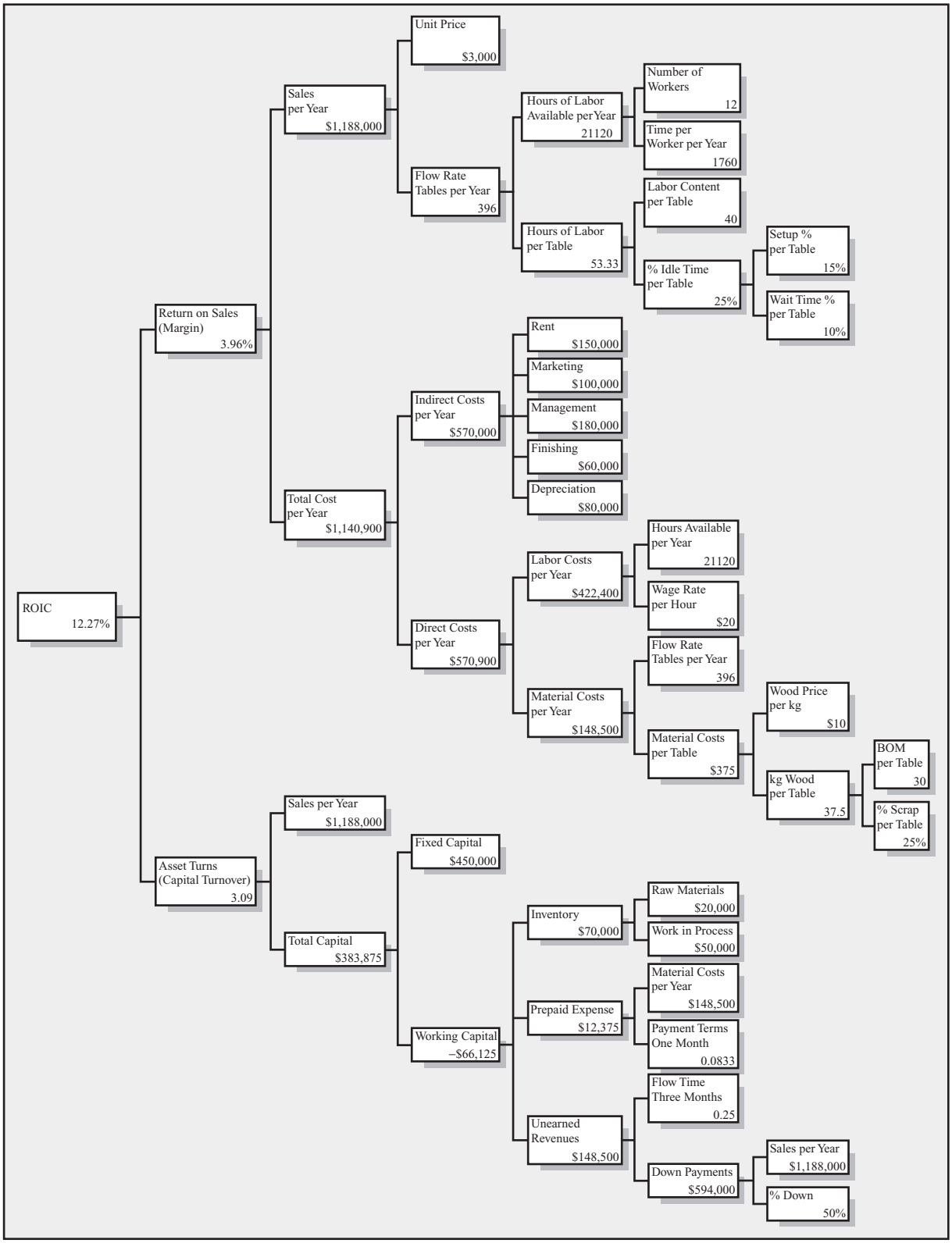


TABLE 6.1 ROIC after the Improvement

Scenario	Base Case	$1/hr Lower Wages	5 Percent Shorter Setups	$10k per Year Lower Rent	2 hr/Table Lower Labor Content	5 Percent Lower Scrap Rate
ROIC [%]	12.3	17.7	31.1	14.8	27.0	13.8

What looked like a small and, at least from a financial perspective, unimportant variable turns out to be a key driver of financial performance. When an operational variable behaves this way, we refer to it as an operational value driver.

A couple of observations are helpful to better understand the role of setup times as an operational value driver.

- If we take a second look at our ROIC tree (see Figure 6.7), we see that setup times drive ROIC in multiple ways. Setup time is a driver of margins, the upper branch of the tree, as shorter setups allow us to produce more and hence to spread out the fixed costs over more units. Moreover, setup times also impact asset turns—we get more revenues out of the same capital investment because setup times influence sales-per-year, which is a component of asset turns.
- This analysis is based on the assumption that there exists enough demand to support a 26-unit increase in sales (the new flow rate would be 422). If the company had been constrained by demand, it is easy to see that shorter setup times would have (marginally) improved ROIC only if we could have used our productivity improvement to reduce the number of cabinetmakers.
- We have considered a one-third reduction in setup times (from 15 percent to 10 percent). As we will discuss in Chapter 7, such a reduction in setup times is indeed feasible and plausible.

A second look at Table 6.1 reveals that process improvements that yield a higher flow rate (lower setup times and lower labor content) are having the biggest impact on ROIC. Figure 6.8 illustrates this logic.

Independent of flow rate, we have to pay $992,400 per year for fixed costs, including the salaries for the cabinetmakers as well as the other items discussed in Figure 6.4. Once these fixed costs are covered (i.e., we exceed the break-even volume), every additional unit of flow rate leads to a $2,625 ($3,000 price minus $375 for wood consumption) increase in profit. As can be seen by the shaded area in Figure 6.8, the small increase in flow rate

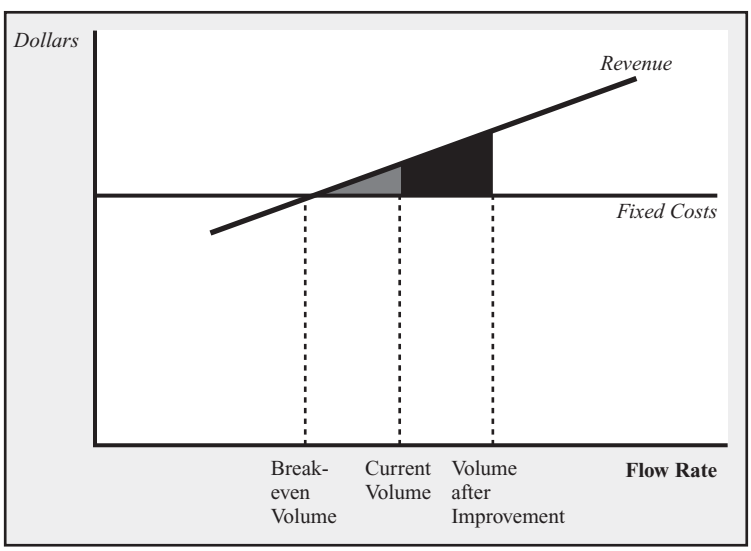


FIGURE 6.8
Fixed Costs versus Variable Costs

Exhibit 6.1

HOW TO CREATE AN ROIC TREE

1. Start with the objective (ROIC) on one side of the tree.
2. Decompose a variable into its components.
   - Example: ROIC = Income/Invested Capital
   - Relationships of variables can be a + b, a − b, a/b, or a × b.
3. Decide which branches of the tree have an impact and are important.
   - What are the main cost drivers (80/20 rule)?
   - What are the strategic levers of the company?
   - Which inputs are most likely to change?
4. Expand important branches (return to step 2).
5. End with measures that can be tied to operational strategy.
6. Populate the tree with actual numbers.
7. Reflect on the tree to see if it makes sense.
   - Benchmark performance.
   - Perform sensitivity analysis.

leads to a big increase in profits. This logic is true for all high fixed-cost operations such as hotels, airlines, and many other services. Chapter 16 will discuss this effect further.

Exhibit 6.1 summarizes the key steps of building an ROIC tree and evaluating potential operational improvements. Such a tree is a powerful starting point for consultants entering a new engagement looking at their client's operations, for a general manager who wants to have a comprehensive understanding of what drives value in his/her business, and for private equity investors that intend to quickly increase the value of a firm by fixing parts of its operation.

6.4 Analyzing Operations Based on Financial Data

In the previous section, we have looked at a relatively small business and built an ROIC tree that was grounded in a detailed understanding of the company's operations. Alternatively, we can start the analysis based on publicly available data (most often, this would be the case for larger firms). In this section, we use the example of the airline industry to illustrate the usefulness of the ROIC tree method.

The first step in our analysis is to identify firms in an industry that have demonstrated and sustained superior financial performance. In the case of the U.S. airline industry, the prime candidate for a success story is clearly Southwest Airlines.

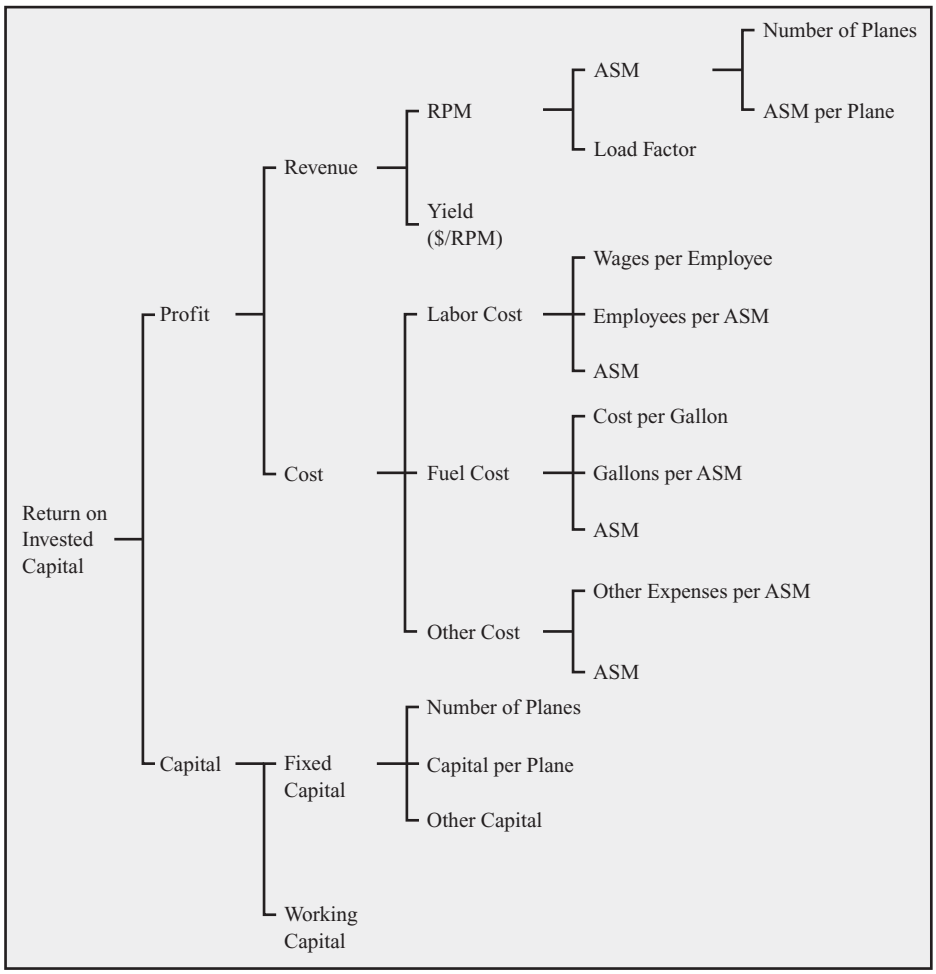
Second, we build an ROIC tree as we did in the Paul Downs case. When analyzing an airline, the following bits of airline vocabulary are helpful:

- Instead of thinking of an airline selling tickets, it is easier to think of an airline selling *revenue passenger miles* (RPMs). An RPM corresponds to transporting a paying customer for one mile. A flight from Philadelphia to Boston, for example, with 200 passengers would correspond to 447 miles × 200 paying passengers = 89,400 RPMs. By focusing on RPM, we avoid some of the problems associated with comparisons between airlines that have different route structures. Furthermore, as we will see, variable costs for an airline are generally tied to the number of miles flown, so it is also convenient to express revenue on a per-mile basis.

- The capacity of an airline is determined by the number and the sizes of its aircrafts. This leads to a measure known as the *available seat miles* (ASMs). One ASM corresponds to one airline seat (with or without a passenger in it) flying for one mile.
- Airlines only make money if they can turn their ASMs into RPMs: a seat with a paying customer is good; an empty seat is not. The ratio RPM/ASM is called the *load factor*—it strongly resembles our definition of utilization as it looks at how many revenue passenger miles the airline creates relative to how much it could create if every seat were filled. Clearly, the load factor must always be less than one—other than small infants sitting on a parent's lap, airlines do not allow two paying customers to occupy the same seat.

Figure 6.9 summarizes a simplified version of an ROIC tree for an airline. There exist, of course, many more levels of details that could be analyzed, including aspects of fleet age and composition, size of flight crews, the percentage of flying time of an aircraft, and so on. But since we are growing this tree from the left to the right, any additional level of detail could be simply tagged on to our analysis in Figure 6.9.

FIGURE 6.9
ROIC Tree for a Generic Airline
(Profit corresponds to pretax income)



As a third step, we want to explore why the financially high-performing firm is doing better than its peers. A good diagnostic tool toward this end is the following method we call productivity ratios. We can write productivity as

$$\text{Productivity} = \frac{\text{Revenue}}{\text{Cost}}$$

and we can write labor productivity as

$$\text{Labor productivity} = \frac{\text{Revenue}}{\text{Labor cost}}$$

and see that Southwest's labor is substantially more productive than US Airways' labor. The Southwest labor productivity ratio is 3.31, which is almost 40 percent higher than the one for US Airways. The following calculations are illustrated with data from the year 2000. We use this old data for two reasons. First, from 1998 to 2000, Southwest managed to double its market capitalization—a financial performance that none of its competitors even came close to. So, clearly, in the eyes of Wall Street, Southwest did something right. Second, following the terrorist attacks of September 2001, the airline industry entered a long period of bankruptcies and restructuring processes, which makes reading the financial statements during these periods somewhat more complicated. At the end of the chapter, we will provide additional data for the more recent years.

But where does the productivity advantage come from? Are Southwest employees serving more customers? Do they make less money? From the ratio alone, we cannot tell. For this reason, we will rewrite the productivity measure as follows:

$$\text{Productivity} = \frac{\text{Revenue}}{\text{Cost}} = \frac{\text{Revenue}}{\text{Flow rate}} \times \frac{\text{Flow rate}}{\text{Resource}} \times \frac{\text{Resource}}{\text{Cost}}$$

Or, applied to labor productivity in airlines:

$$\text{Labor productivity} = \frac{\text{Revenue}}{\text{Labor cost}} = \underbrace{\frac{\text{Revenue}}{\text{RPM}}}_{\text{Yield}} \times \underbrace{\frac{\text{RPM}}{\text{ASM}} \times \frac{\text{ASM}}{\text{Employees}}}_{\text{Efficiency}} \times \underbrace{\frac{\text{Employees}}{\text{Labor costs}}}_{\text{Cost}}$$

It is helpful to break up this expanded productivity calculation into three pieces:

- Yields: the operational yield (Revenue/Flow rate) measures how much money the firm can squeeze out of its output, the flow rate. This measure is largely driven by the firm's pricing power.
- Efficiency: the transformation efficiency (Flow rate/Resource) measures how many resources we need to support the flow rate. This number is determined by how we utilize our resources. It captures the resource utilization (in our case, the load factor) as well as the inherent processing times at each resource (how many available seat miles can a single employee serve?).
- Cost: the cost of resources (Resource/Cost) measures how much of a resource we can get per $1 spent. The reciprocal of this measure is simply the cost of that resource, for example, the average yearly salary of an employee.

Now, let's see what these productivity ratios tell us about Southwest's source of higher labor productivity. Table 6.2 summarizes our results.

The results of our diagnostics confirm US Airways' superior pricing power. Unlike the low-fare carrier Southwest, US Airways gets almost 50 percent more money for every

TABLE 6.2 Comparison between US Airways and Southwest

Airline	Operational Yield [$/RPM]	Load Factor [%]	ASM per Employee	Number of Employees/Million US$ of Labor Costs	Overall Labor Productivity
US Airways	0.197	0.70	0.37	47.35	2.43
Southwest	0.135	0.69	0.53	67.01	3.31

Note: The 47.35 in the second column from the right can also be expressed as US Airways' average wage: $1,000,000/47.35 = $21,119 is the quarterly wage rate for an employee.

passenger mile. Interestingly, both firms operate at roughly the same load factor. However, Southwest (more than!) offsets its pricing disadvantage with the last two ratios:

- A Southwest employee is able to support 50 percent more ASMs compared to a US Airways employee (0.53 as opposed to 0.37)
- While being paid about a 50 percent lower salary (for the same money you pay 47 US Airways employees, you can hire 67 Southwest employees). Note that this number has changed substantially since 2001. In fact, now, Southwest employees earn substantially higher wages than their counterparts at US Airways—wages have moved in the direction of productivity differences.

As a fourth and final step of our analysis, we can now look at how much money US Airways would save if it could imitate one or several of the productivity ratios from Southwest. For example, we can ask the following two questions:

- How much money would US Airways save if it could support as many ASMs with an employee as Southwest does?
- How much money would US Airways save if it paid Southwest wages?

Figure 6.10 summarizes the cost-saving opportunities. Consider the potential savings that US Airways would obtain from paying its employees Southwest wages first. US

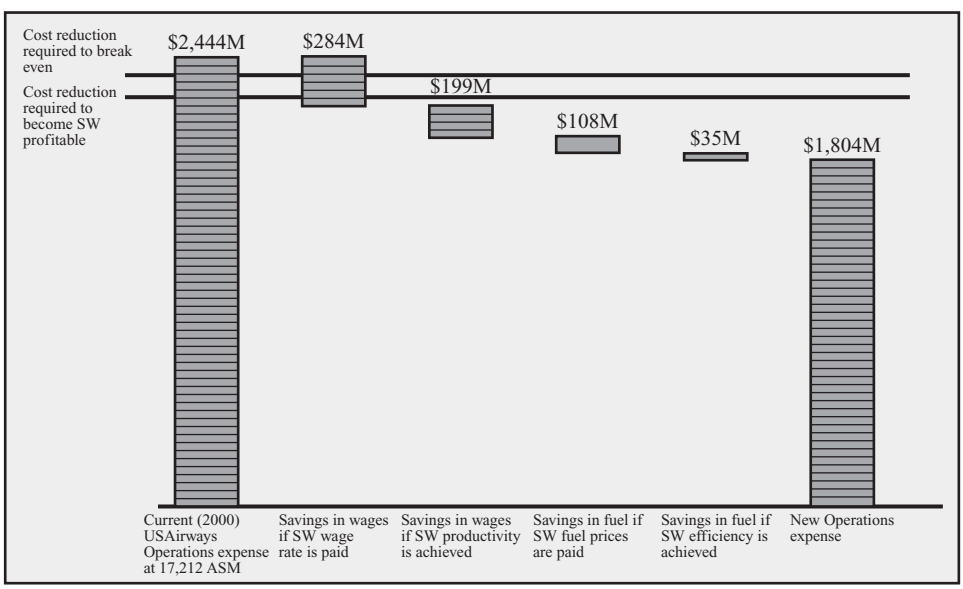


FIGURE 6.10
Potential US Airways Cost Savings

Airways has 45,833 employees on its payroll. The average salary was $21,120 per quarter (see also Table 6.2) compared to $14,922 for the average Southwest employee. If we paid US Airways employees Southwest wages, we would hence save

$$45,833 \times (\$21,120 - \$14,922) = \$284,072,934$$

Next, consider the savings potential if we could have a US Airways employee achieve the same level of productivity as a Southwest employee. It takes US Airways 45,833 employees to service 17,212 ASM, translating to 17,212/45,833 = 0.37 ASM per employee. As we saw in Table 6.2, Southwest is able to service 0.53 ASM per employee. So,

$$17,212 \text{ ASM}/(0.53 \text{ ASM/employee}) = 32,475 \text{ employees}$$

is the number of workers that US Airways would need if it achieved Southwest's labor productivity. This leads to a possible head-count reduction of 13,358 employees (45,833 − 32,475). Given the average US Airways salary of $21,120 per quarter, this corresponds to a quarterly savings opportunity of

$$\$21,120 \text{ per employee} \times 13,358 \text{ employees} = \$282,120,960$$

Note that the savings we would obtain from such an increase in productivity are not savings "on top of" the savings potential reflecting an adjustment of US Airways wages to the Southwest level. In other words, the total (combined) savings from the adjustment in labor cost and the increase in productivity would not be $284,072,934 + $282,120,960. The reason for this is simply that once we have cut salaries, the savings that we get from reducing the number of workers to reflect productivity gains are smaller (they would be the new, lower salary of $14,922 per employee per quarter instead of the higher salary of $21,120).

So, if we want to compute the additional savings we obtain from a productivity increase assuming that the US Airways wages already have been adjusted to the Southwest level, we compute

$$\$14,922 \text{ per employee} \times 13,358 \text{ employees} = \$199,328,076$$

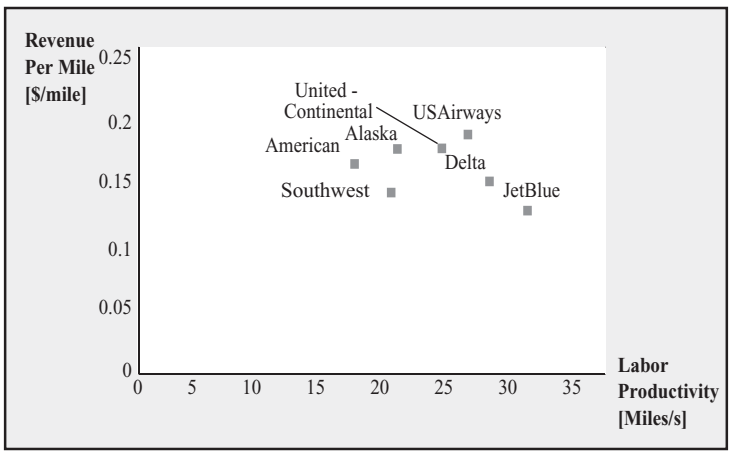
Figure 6.10 also shows and analyzes another productivity advantage of Southwest—the company's ability to procure cheap jet fuel (which is a result of clever—or lucky—investments in fuel hedges).

US Airways would save $108M if it could purchase jet fuel at Southwest's purchasing conditions. And if, on top of it, it could match Southwest's fuel efficiency, it would save another $35M.

Unlike the analysis that we did in the Paul Downs case, the approach summarized in Figure 6.10 is much more of a top-down analysis. Before entering the details of the operations (what does Southwest do differently that their labor can support more ASMs), we start out our analysis by broadly exploring the size of various opportunities.

In general, the top-down approach is most useful when analyzing competing organizations or when there simply are limited data available about operational details, thus, when the analysis happens from "the outside in." In contrast, it should be emphasized that for the management within an operation, the level of granularity that we have outlined in Figure 6.10 is vastly insufficient. Sizing the opportunity is only the first step; the real challenge then is to cascade the labor productivity disadvantage further, all the way into the

FIGURE 6.11
The Airline Industry in 2010



minutes it takes to board an airplane, the durations of the worker breaks, the number of employees at a gate, and so on.

As far as the airline industry is concerned, it has been interesting to notice what has happened between 2000 and 2006. In 2000, the average US Airways salary was 41 percent higher than that of the average Southwest employee. By 2006, Southwest salaries had caught up and even exceeded the 2000 salaries of US Airways. US Airways salaries, on the other hand, had decreased to the 2000 level of Southwest! While salaries changed dramatically, the productivity advantage of Southwest did not. In fact, in 2006, Southwest continued to have a 50 percent higher labor productivity than US Airways.

By 2010, the industry had changed even further, as is illustrated in Figure 6.11. On the vertical dimension, the graph shows the amount of money the average passenger was paying for one mile of air travel on the various carriers. This amount is expressed relative to the industry average. On the horizontal dimension, we show how many passenger miles an airline can generate with one dollar of labor cost, again, relative to the industry average. All data is based on FY2010. Note that the Southwest advantage from 10 years prior has disappeared. While each employee, on average, served more passengers compared to other airlines, Southwest employees were paid substantially above industry average (see preceding discussion), leading to an overall labor productivity that is below industry average. However, because of lower fuel costs/higher fuel efficiency as well as lower other expenses (such as landing fees, commissions, sales and marketing expenses), Southwest still turned substantial profits, despite paying its employees well. As we discuss in Chapter 19, a new player managed to disrupt the industry, JetBlue became the "new Southwest."

6.5 Summary

In this chapter, we have provided a link between the operations of a company and its financial performance. This link can be studied at the micro level, as we have done in the Paul Downs case, or it can be done starting with the financial statements, as we have done in the airline case. Either way, operational variables are key drivers of a company's financial performance. Value creation takes place in the operations of a company and so, to increase the economic value of a company, a detailed analysis of operations is a must.

6.6 Further Reading

Koller, Goedhart, and Wessels (2005) is an excellent book on topics related to valuation and corporate finance. Compared to most other finance books, it is very hands-on and does not shy away from the operational details of business.

Cannon, Randall, and Terwiesch (2007) study the empirical relationship between operational variables and future financial performance in the airline industry.

6.7 Practice Problems

Q6.1* **(Crazy Cab)** Crazy Cab is a small taxi cab company operating in a large metropolitan area. The company operates 20 cabs. Each cab is worth about $20k. The metropolitan area also requires that each cab carry a medallion (a type of license). Medallions are currently traded at $50k. Cab drivers make $8 per hour and are available for every time of the day. The average cab is used for 40 trips per day. The average trip is three miles in length. Passengers have to pay $2 as a fixed fee and $2 per mile they are transported. Fuel and other costs, such as maintenance, are $0.20 per mile. The cab drives about 40 percent of the distance without a paying passenger in it (e.g. returning from a drop-off location, picking up a passenger, etc.)

a. Draw an ROIC tree for the cab company.
b. Populate the tree with numbers. Make assumptions to explore operational variables in as much detail as possible (e.g., assumptions about gas prices, gas consumption, etc.).
c. Which of the variables would you classify as operational value drivers?
d. Analyze the labor efficiency and the efficiency of using the fleet of cabs using productivity ratios.

Q6.2** **(Penne Pesto)** Penne Pesto is a small restaurant in the financial district of San Francisco. Customers order from a variety of pasta dishes. The restaurant has 50 seats and is always full during the four hours in the evening. It is not possible to make reservations at Penne; most guests show up spontaneously on their way home from work. If there is no available seat, guests simply move on to another place.

On average, a guest spends 50 minutes in the restaurant, which includes 5 minutes until the guest is seated and the waiter has taken the order, an additional 10 minutes until the food is served, 30 minutes to eat, and 5 minutes to handle the check-out (including waiting for the check, paying, and leaving). It takes the restaurant another 10 minutes to clean the table and have it be ready for the next guests (of which there are always plenty). The average guest leaves $20 at Penne, including food, drink, and tip (all tips are collected by the restaurant, employees get a fixed salary).

The restaurant has 10 waiters and 10 kitchen employees, each earning $90 per evening (including any preparation, the 4 hours the restaurant is open, and clean-up). The average order costs $5.50 in materials, including $4.50 for the food and $1 for the average drink. In addition to labor costs, fixed costs for the restaurant include $500 per day of rent and $500 per day for other overhead costs.

The restaurant is open 365 days in the year and is full to the last seat even on weekends and holidays. There is about $200,000 of capital tied up in the restaurant, largely consisting of furniture, decoration, and equipment.

a. How many guests will the restaurant serve in one evening?
b. What is the Return on Invested Capital for the owner of the restaurant?
c. Assume that you could improve the productivity of the kitchen employees and free up one person who would be helping to clean up the table. This would reduce the clean-up to 5 minutes instead of 10 minutes. What would be the new ROIC?
d. What would be the new ROIC if overhead charges could be reduced by $100 per day?

Q6.3 **(Philly Air)** PhillyAir Inc. offers low cost air travel between Philadelphia and Atlantic City. Philly Air's invested capital is $5,000,000, corresponding to the investment in the two planes the company owns. Each of the two planes can carry 50 passengers. Each plane does 12 daily trips from Philadelphia to Atlantic City and 12 from Atlantic City to Philadelphia. The price is $100 for each one-way ticket. The current load factor is 70 percent (ie., 35 seats are sold on the average flight). The annual cost of operating the service and

running the business is $60,000,000 (including all costs, such as labor, fuel, marketing, gate fees, landing fees, maintenance, etc). The company operates 365 days a year.

a. Draw an ROIC (return on invested capital) tree for the company that incorporates all of the above information.
b. What is the current ROIC?
c. What is the minimum load factor at which the company breaks even?
d. What load factor would the company have to achieve so that it obtained a 10 percentage-point increase in the ROIC (e.g. an ROIC increasing from 5 percent to 15 percent)?

Q6.4 **(Oscar's Office Building)** Oscar is considering getting into the real estate business. He's looking at buying an existing office building for $1.8 million in cash. He wants to estimate what his return on invested capital (ROIC) will be on an annual basis. The building has 14,000 square feet of rentable space. He'd like to set the rent at $4.00 per square foot per month. However, he knows that demand depends on price. He estimates that the percentage of the building he can fill roughly follows the equation:

$$\%\ \text{Occupied} = 2 - 0.3 \times \text{Rent}$$

(rent is in dollars per square foot per month)

So, at $4.00, Oscar thinks he can fill about 80 percent of the office space.

Oscar considers two categories of costs: variable costs, which are a function of the square feet occupied, and fixed costs. Fixed costs will be $8,000 per month and include such items as insurance, maintenance, and security. Variable costs cover such things as electricity and heat and run $1.25 per month for each square foot occupied.

a. Draw an ROIC (return on invested capital) tree for the company.
b. What is the ROIC?
c. What would be the new ROIC be if Oscar decides to charge rent of $5.00 per square foot per month?

Q6.5 **(OPIM Bus Inc.)** OPIM Bus Inc. offers low-cost bus transportation between Philadelphia and Bryn Mawr. The invested capital is $500,000, corresponding to the investment in the two vehicles it owns. Each of the two buses can carry 50 passengers. Each bus does 12 daily trips from Philadelphia to Bryn Mawr and 12 from Bryn Mawr to Philadelphia. The price is $10 for each one-way ticket. The current load factor is 70 percent (i.e., 35 seats are sold on average). The annual cost of operating the service and running the business is $6 million. The company operates 365 days a year.

a. Draw an ROIC (return on invested capital) tree for the company.
b. What is the current ROIC?
c. What is the minimum load factor at which the company breaks even?
d. What load factor would the company have to achieve so that it obtained a 10 percentage-point increase in the ROIC (e.g. an ROIC increasing from 5 percent to 15 percent)?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 7

Batching and Other Flow Interruptions: Setup Times and the Economic Order Quantity Model

Up to this point, we were working under the assumption that during every X units of time, one flow unit would enter the process and one flow unit would leave the process. We defined X as the process cycle time. In the scooter example of Chapter 4, we established a cycle time of three minutes in conjunction with Table 4.3, allowing us to fulfill demand of 700 scooters per week.

In an ideal process, a cycle time of three minutes would imply that every resource receives one flow unit as an input each three-minute interval and creates one flow unit of output each three-minute interval. Such a smooth and constant flow of units is the dream of any operations manager, yet it is rarely feasible in practice. There are several reasons for why the smooth process flow is interrupted, the most important ones being setups and variability in processing times or quality levels. The focus of this chapter is on setups, which are an important characteristic of batch-flow operations. Problems related to variability are discussed in Chapters 8 and 9. And quality problems are discussed in Chapter 10.

To discuss setups, we return to the Xootr production process. In particular, we consider the computer numerically controlled (CNC) milling machine which is responsible for making two types of parts on each Xootr—the steer support and two ribs (see Figure 7.1). The steer support attaches the Xootr's deck to the steering column, and the ribs help the deck support the weight of the rider. Once the milling machine starts producing one of these parts, it can produce them reasonably quickly. However, a considerable setup time, or changeover time, is needed before the production of each part type can begin. Our primary objective is to understand how setups like these influence the three basic performance measures of a process: inventory, flow rate, and flow time.

FIGURE 7.1 **Milling Machine (left) and Steer Support Parts (right)**

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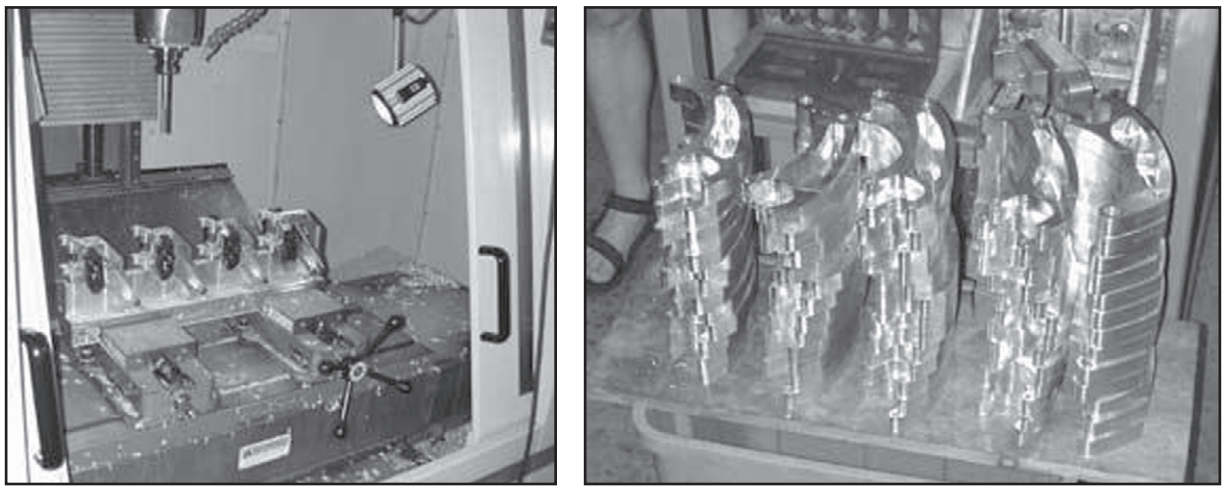


7.1 The Impact of Setups on Capacity

To evaluate the capacity of the milling machine, we need some more information. Specifically, once set up to produce a part, the milling machine can produce steer supports at the rate of one per minute and can produce ribs at the rate of two per minute. Recall, each Xootr needs one steer support and two ribs. Furthermore, one hour is needed to set up the milling machine to start producing steer supports and one hour is also needed to begin producing ribs. Although no parts are produced during those setup times, it is not quite correct to say that nothing is happening during those times either. The milling machine operator is busy calibrating the milling machine so that it can produce the desired part.

It makes intuitive sense that the following production process should be used with these two parts: set up the machine to make steer supports, make some steer supports, set up the machine to make ribs, make some ribs, and finally, repeat this sequence of setups and production runs. We call this repeating sequence a *production cycle*: one production cycle occurs immediately after another, and all productions cycles "look the same" in the sense that they have the same setups and production runs.

We call this a batch production process because parts are made in batches. Although it may be apparent by what is meant by a "batch", it is useful to provide a precise definition:

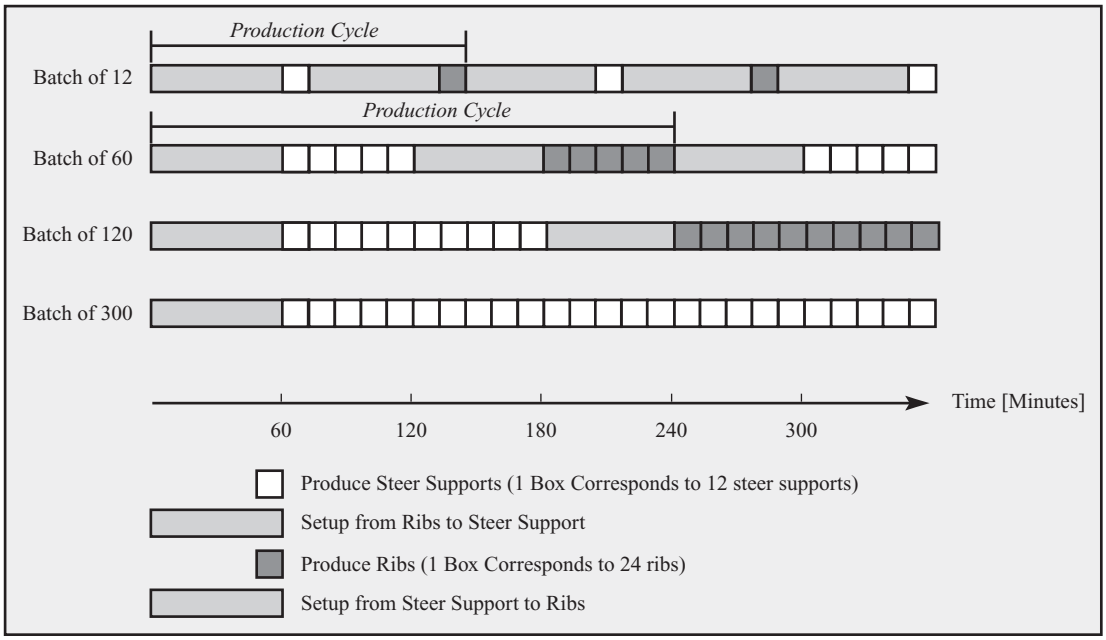
A *batch* is a collection of flow units.

Throughout our analysis, we assume that batches are produced in succession. That is, once the production of one batch is completed, the production of the next batch begins and all batches contain the same number and type of flow unit.

Given that a batch is a collection of flow units, we need to define our flow unit in the case of the Xootr. Each Xootr needs one steer support and two ribs, so let's say the flow unit is a "component set" and each component set is composed of those three parts. Hence, each production cycle produces a batch of component sets.

One might ask why we did not define the flow unit to be one of the two types of parts. For example, we could call the steering supports made in a production run a batch of steering supports. However, our interest is not specifically on the capacity to make steering supports or ribs in isolation. We care about the capacity for component sets because one component

FIGURE 7.2 The Impact of Setup Times on Capacity



set is needed for each Xootr. Thus, for the purpose of this analysis, it makes more sense to define the flow unit as a component set and to think in terms of a batch of component sets.

Because no output is produced while the resource is in setup mode, it is fairly intuitive that frequent setups lead to lower capacity. To understand how setups reduce the capacity of a process, consider Figure 7.2. The impact of setups on capacity is fairly intuitive. As nothing is produced at a resource during setup, the more frequently a resource is set up, the lower its capacity. As discussed above, the milling machine underlying the example of Figure 7.2 has the following processing times/setup times:

- It takes one minute to produce one steer support unit (of which there is one per Xootr).
- It takes 60 minutes to change over the milling machine from producing steer supports to producing ribs (setup time).
- It takes 0.5 minute to produce one rib; because there are two ribs in a Xootr, this translates to one minute/per component set.
- Finally, it takes another 60 minutes to change over the milling machine back to producing steer supports.

Now consider the impact that varying the batch size has on capacity. Recall that we defined capacity as the maximum flow rate at which a process can operate. If we produce in small batches of 12 component sets per batch, we spend a total of two hours of setup time (one hour to set up the production for steer supports and one hour to set up the production of ribs) for every 12 component sets we produce. These two hours of setup time are lost from regular production.

The capacity of the resource can be increased by increasing the batch size. If the machine is set up every 60 units, the capacity-reducing impact of setup can be spread out over 60 units. This results in a higher capacity for the milling machine. Specifically, for a batch size of 60, the milling machine could produce at 0.25 component set per minute. Table 7.1 summarizes the capacity calculations for batch sizes of 12, 60, 120, and 300.

TABLE 7.1
The Impact of Setups on Capacity

Batch Size	Time to Complete One Batch [minutes]	Capacity [units/minute]
12	60 minutes (set up steering support) + 12 minutes (produce steering supports) + 60 minutes (set up ribs) + 12 minutes (produce ribs) 144 minutes	12/144 = 0.0833
60	60 minutes (set up steering support) + 60 minutes (produce steering supports) + 60 minutes (set up ribs) + 60 minutes (produce ribs) 240 minutes	60/240 = 0.25
120	60 minutes (set up steering support) + 120 minutes (produce steering supports) + 60 minutes (set up ribs) + 120 minutes (produce ribs) 360 minutes	120/360 = 0.333
300	60 minutes (set up steering support) + 300 minutes (produce steering supports) + 60 minutes (set up ribs) + 300 minutes (produce ribs) 720 minutes	300/720 = 0.4166

Generalizing the computations in Table 7.1, we can compute the capacity of a resource with setups as a function of the batch size:

$$\text{Capacity given batch size} = \frac{\text{Batch size}}{\text{Setup time} + \text{Batch size} \times \text{Processing time}}$$

Basically, the above equation is spreading the "unproductive" setup time over the members of a batch. To use the equation, we need to be precise about what we mean by batch size, the setup time, and processing time:

- The batch size is the number of flow units that are produced in one "cycle" (i.e., before the process repeats itself, see Figure 7.2).
- The setup time includes all setups in the production of the batch. In this case, this includes $S = 60$ minutes + 60 minutes = 120 minutes. It can also include any other nonproducing time associated with the production of the batch. For example, if the production of each batch requires a 10-minute worker break, then that would be included. Other "setup times" can include scheduled maintenance or forced idled time (time in which literally nothing is happening with the machine—it is neither producing nor being prepped to produce).
- The processing time includes all production time that is needed to produce one complete flow unit of output at the milling machine. In this case, this includes 1 minute/unit for the steer support as well as two times 0.5 minute/unit for the two ribs. The processing time is thus $p = 1$ minute/unit + 2 × 0.5 minute/unit = 2 minutes/unit. Notice that the processing time is 2 minutes even though no single component set is actually produced over a single period of 2 minutes of length. Due to setups, the processing time for a component set is divided over two periods of one minute each, and those two periods can be separated by a considerable amount of time. Nevertheless, from

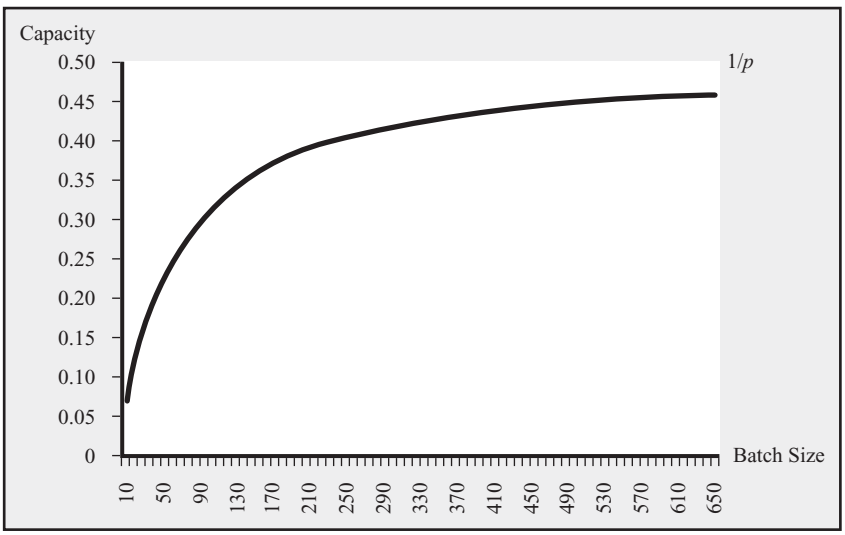


FIGURE 7.3
Capacity as a Function of the Batch Size

the perspective of calculating the capacity of the milling machine when operated with a given batch size, it does not matter whether each component set is produced over a continuous period of time or disjointed periods of time. All that matters is that a total of 2 minutes is needed for each component set.

Given these definitions, say we operate with a batch size of 100 units. Our capacity in this case would be

$$\begin{aligned}\text{Capacity (for } B = 100) &= \frac{\text{Batch size}}{\text{Setup time} + \text{Batch size} \times \text{Processing time}} \\ &= \frac{100 \text{ units}}{120 \text{ minutes} + 100 \text{ units} \times 2 \text{ minutes/unit}} \\ &= 0.3125 \text{ unit/minute}\end{aligned}$$

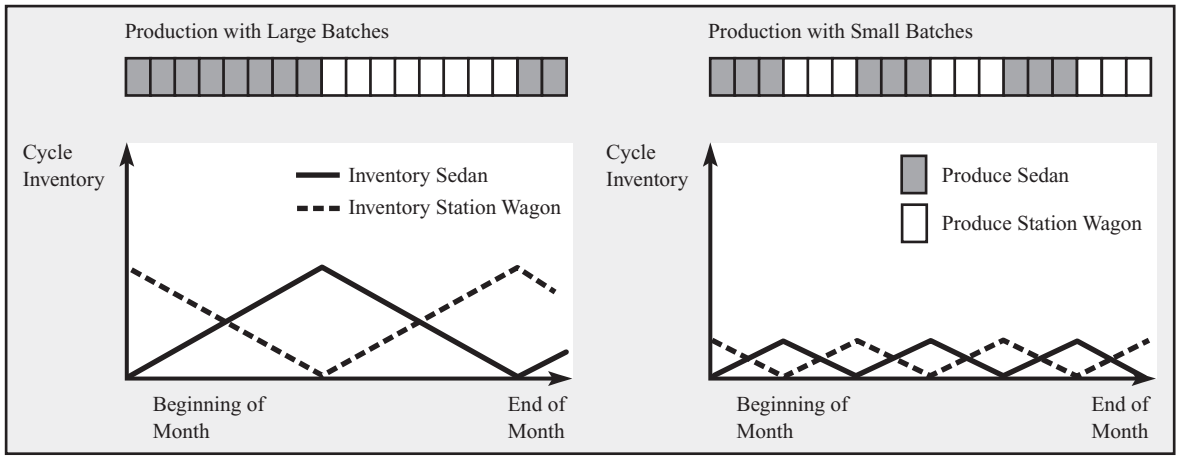
No matter how large a batch size we choose, we will never be able to produce faster than one unit every p units of time. Thus, $1/p$ can be thought of as the maximum capacity the process can achieve. This is illustrated in Figure 7.3.

7.2 Interaction between Batching and Inventory

Given the desirable effect that large batch sizes increase capacity, why not choose the largest possible batch size to maximize capacity? While large batch sizes are desirable from a capacity perspective, they typically require a higher level of inventory, either within the process or at the finished goods level. Holding the flow rate constant, we can infer from Little's Law that such a higher inventory level also will lead to longer flow times. This is why batch-flow operations generally are not very fast in responding to customer orders (remember the last time you bought custom furniture?).

The interaction between batching and inventory is illustrated by the following two examples. First, consider an auto manufacturer producing a sedan and a station wagon on the same assembly line. For simplicity, assume both models have the same demand rate, 400 cars per day each. The metal stamping steps in the process preceding final assembly

FIGURE 7.4 The Impact of Batch Sizes on Inventory



are characterized by especially long setup times. Thus, to achieve a high level of capacity, the plant runs large production batches and produces sedans from the first of a month to the 15th and station wagons from the 16th to the end of the month.

However, it seems fairly unrealistic to assume that customers only demand sedans at the beginning of the month and station wagons at the end of the month. In other words, producing in large batches leads to a mismatch between the rate of supply and the rate of demand.

Thus, in addition to producing enough to cover demand in the first half of the month, to satisfy demand for sedans the company needs to produce 15 days of demand to inventory, which then fulfills demand while the line produces station wagons. This is illustrated by the left side of Figure 7.4. Observe that the average level of inventory is 3,000 cars for each of the two models. Now, ignoring setup times for a moment, consider the case in which the firm produces 400 station wagons and 400 sedans a day. In this setting, one would only need to carry 0.5 day of cycle inventory, a dramatic reduction in inventory. This is illustrated by the right side of Figure 7.4. Thus, smaller batches translate to lower inventory levels!

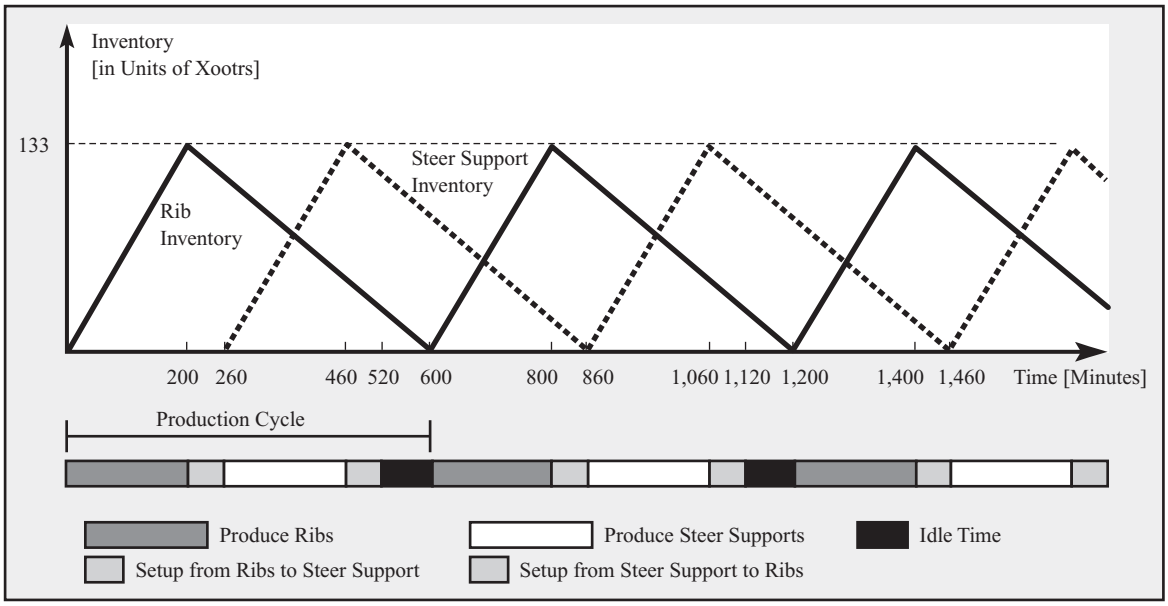
In the ideal case, which has been propagated by Toyota Production Systems under the word *heijunka* or *mixed-model* production, the company would alternate between producing one sedan and producing one station wagon, thereby producing in batch sizes of one. This way, a much better synchronization of the demand flow and the production flow is achieved and cycle inventory is eliminated entirely.

Second, consider a furniture maker producing chairs in batch sizes of 100. Starting with the wood-cutting step and all the way through the finishing process, the batch of 100 chairs would stay together as one entity.

Now, take the position of one chair in the batch. What is the most dominant activity throughout the process? Waiting! The larger the batch size, the longer the time the flow unit waits for the other "members" of the same batch—a situation comparable with going to the barber with an entire class of children. Given Little's Law, this increase in wait time (and thereby flow time) leads to a proportional increase in inventory.

With these observations, we can turn our attention back to the milling machine at Nova Cruz. Similar to Figure 7.4, we can draw the inventory of components (ribs and steer supports) over the course of a production cycle. Remember that the assembly process following the milling machine requires a supply of one unit every three minutes. This one

FIGURE 7.5 The Impact of Setup Times on Capacity



unit consists, from the view of the milling machine, of two ribs and a steer support unit. If we want to ensure a sufficient supply to keep the assembly process operating, we have to produce a sufficient number of ribs such that during the time we do not produce ribs (e.g., setup time and production of steer support) we do not run out of ribs. Say the milling machine operates with a batch size of 200 units, $B = 200$. In that case, the inventory of ribs changes as follows:

- During the production of ribs, inventory accumulates. As we produce ribs for one scooter per minute, but only supply ribs to the assembly line at a rate of one scooter every three minutes, rib inventory accumulates at the rate of two scooters per three minutes, or 2/3 scooters per minute.
- Because we produce for 200 minutes, the inventory of ribs at the end of the production run is 200 minutes $\times$ 2/3 scooters per minute $=$ 133 scooters (i.e., 266 ribs).
- How long does the rib inventory for 133 scooters last? The inventory ensures supply to the assembly for 400 minutes (cycle time of assembly operations was three minutes). After these 400 minutes, we need to start producing ribs again. During these 400 minutes, we have to accommodate two setups (together 120 minutes) and 200 minutes for producing the steer supports.

The resulting production plan as well as the corresponding inventory levels are summarized by Figure 7.5. Notice that each production cycle takes 600 minutes, and this includes 80 minutes of idle time. Why do we insert additional idle time into the milling machine's production schedule? The answer is that without the idle time, the milling machine would produce too quickly given our batch size of 200 units. To explain, assembly takes 200 units $\times$ 3 minute/unit $=$ 600 minutes to produce a batch of 200 scooters. The milling machine only needs 520 minutes to produce that batch of 600 scooters (120 minutes of setup and 400 minutes of production). Hence, if the milling machine produced one batch after another (without any idle time between them), it would produce 200 components sets every 520 minutes (or 200/520 $=$ 0.3846 components sets per minute), which is faster than assembly can use them (which is 1/3 component sets per minute). This analysis suggests that maybe we want to choose a different batch size, as we see in the next section.

7.3 Choosing a Batch Size in the Presence of Setup Times

When choosing an appropriate batch size for a process flow, it is important to balance the conflicting objectives: capacity and inventory. Large batches lead to large inventory; small batches lead to losses in capacity.

In balancing these two conflicting objectives, we benefit from the following two observations:

- Capacity at the bottleneck step is extremely valuable (as long as the process is capacity-constrained, i.e., there is more demand than capacity) as it constrains the flow rate of the entire process.
- Capacity at a nonbottleneck step is free, as it does not provide a constraint on the current flow rate.

This has direct implications for choosing an appropriate batch size at a process step with setups.

- If the setup occurs at the bottleneck step (and the process is capacity-constrained), it is desirable to increase the batch size, as this results in a larger process capacity and, therefore, a higher flow rate.
- If the setup occurs at a nonbottleneck step (or the process is demand-constrained), it is desirable to decrease the batch size, as this decreases inventory as well as flow time.

The scooter example summarized by Figure 7.6 illustrates these two observations and how they help us in choosing a good batch size. Remember that B denotes the batch size, S the setup time, and p the per unit processing time.

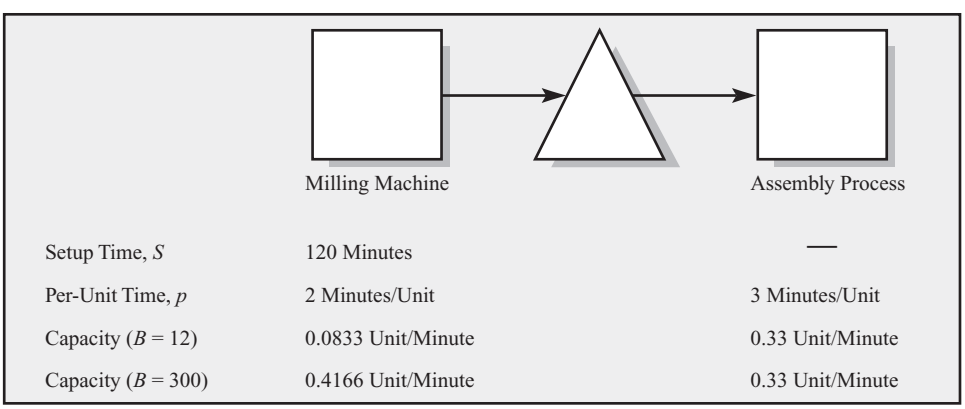
The process flow diagram in Figure 7.6 consists of only two activities: the milling machine and the assembly operations. We can combine the assembly operations into one activity, as we know that its slowest step (bottleneck of assembly) can create one Xootr every three minutes.

To determine the capacity of the milling machine for a batch size of 12, we apply the formula

$$\text{Capacity } (B) = \frac{\text{Batch size}}{\text{Setup time} + \text{Batch size} \times \text{Processing time}} = \frac{B}{S + B \times p} = \frac{12}{120 + 12 \times 2} = 0.0833 \text{ unit/minute}$$

The capacity of the assembly operation is easily computed based on its bottleneck capacity of $\frac{1}{3}$ unit per minute. Note that for $B = 12$, the milling machine is the bottleneck.

FIGURE 7.6
Data from the Scooter Case about Setup Times and Batching



Next consider, what happens to the same calculations if we increase the batch size from 12 to 300. While this does not affect the capacity of the assembly operations, the capacity of the milling machine now becomes

$$\text{Capacity } (B) = \frac{B}{S + B \times p} = \frac{300}{120 + 300 \times 2} = 0.4166 \text{ unit/minute}$$

Thus, we observe that the location of the bottleneck has shifted from the milling machine to the assembly operation, just by modifying the batch size. Now which of the two batch sizes is the "better" one, 12 or 300?

- The batch size of 300 is clearly too large. The milling machine incurs idle time as the overall process is constrained by the (substantially) smaller capacity of the assembly operations (note, based on Figure 7.5, we know that even for the smaller batch size of $B = 200$, there exists idle time at the milling machine). This large batch size is likely to create unnecessary inventory problems as described above.
- The batch size of 12 is likely to be more attractive in terms of inventory. Yet, the process capacity has been reduced to 0.0833 unit per minute, leaving the assembly operation starved for work.

As a batch size of 12 is too small and a batch size of 300 is too large, a good batch size is "somewhere in between." Specifically, we are interested in the smallest batch size that does not adversely affect process capacity.

To find this number, we equate the capacity of the step with setup (in this case, the milling machine) with the capacity of the step from the remaining process that has the smallest capacity (in this case, the assembly operations):

$$\frac{B}{120 + B \times 2} = \frac{1}{3}$$

and solve this equation for *B:*

$$\frac{B}{120 + B \times 2} = \frac{1}{3}$$

$$3 \times B = 120 + 2 \times B$$

$$B = 120$$

which gives us, in this case, $B = 120$. This algebraic approach is illustrated by Figure 7.7. If you feel uncomfortable with the calculus outlined above (i.e., solving the equation for the batch size *B*), or you want to program the method directly into Excel or another software package, you can use the following equation:

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}}$$

which is equivalent to the analysis performed above. To see this, simply substitute Setup time = 120 minutes, Flow rate = 0.333 unit per minute, and Processing time = 2 minutes per unit and obtain

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}} = \frac{0.333 \times 120}{1 - 0.333 \times 2} = 120$$

Figure 7.7 shows the capacity of the process step with setup, which increases with the batch size *B,* and for very high values of batch size *B* approaches $1/p$ (similar to the graph in Figure 7.3). As the capacity of the assembly operation does not depend on the batch size, it corresponds to a constant (flat line).

Exhibit 7.1

FINDING A GOOD BATCH SIZE IN THE PRESENCE OF SETUP TIMES

1. Compute Flow rate = Minimum {Available input, Demand, Process capacity}.
2. Define the production cycle, which includes the processing and setups of all flow units in a batch.
3. Compute the time in a production cycle that the resource is in setup; setup times are those times that are independent of the batch size.
4. Compute the time in a production cycle that the resource is processing; this includes all the processing times that are incurred per unit (i.e., are repeated for every member of the batch).
5. Compute the capacity of the resource with setup for a given batch size:

$$\text{Capacity } (B) = \frac{B}{\text{Setup time} + B \times \text{Processing time}}$$

6. We are looking for the batch size that leads to the lowest level of inventory without affecting flow rate; we find this by solving the equation

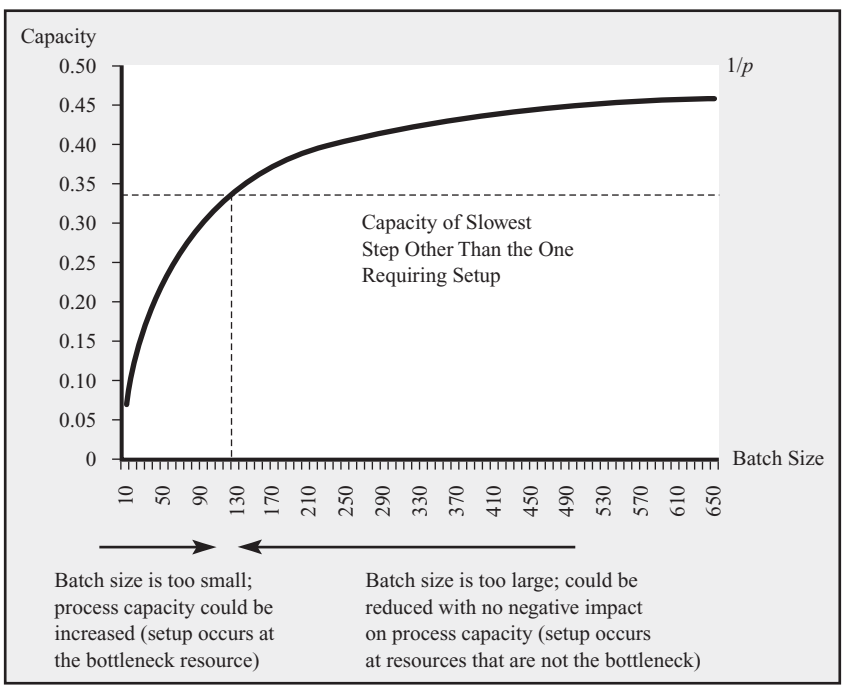
$$\text{Capacity } (B) = \text{Flow rate}$$

for the batch size *B*. This also can be done directly using the following formula:

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}}$$

The overall process capacity is—in the spirit of the bottleneck idea—the minimum of the two graphs. Thus, before the graphs intersect, the capacity is too low and flow rate is potentially given up. After the intersection point, the assembly operation is the bottleneck and any further increases in batch size yield no return. Exhibit 7.1 provides a summary of the computations leading to the recommended batch size in the presence of setup times.

FIGURE 7.7
Choosing a "Good" Batch Size



7.4 Setup Times and Product Variety

As we have seen in the case of the Xootr production process, setup times often occur due to the need to change over production from one product to another. This raises the following question: What is the impact of product variety on a process with setup times? To explore this question, let's consider a simple process that makes two kinds of soup: chicken noodle and tomato.

Demand for chicken soup is 100 gallons per hour, while demand for tomato soup is 75 gallons per hour. Switching from one type of soup to another requires 30 minutes to clean the production equipment so that one flavor does not disrupt the flavor of the next soup. Once production begins, the process can make 300 gallons per hour of either type of soup. Given these parameters, let's evaluate a production cycle that minimizes inventory while satisfying demand.

We first need to define our flow unit. In this case, it is natural to let our flow unit be one gallon of soup. Hence, a production cycle of soup contains a certain number of gallons, some chicken and some tomato. In this case, a "batch" is the set of gallons produced in a production cycle. While the plant manager is likely to refer to batches of tomato soup and batches of chicken soup individually, and unlikely to refer to the batch that combines both flavors, we cannot analyze the production process of tomato soup in isolation from the production process of chicken soup. (For example, if we dedicate more time to tomato production, then we will have less time for chicken noodle production.) Because we are ultimately interested in our capacity to make soup, we focus our analysis at the level of the production cycle and refer to the entire production within that cycle as a "batch."

Our desired flow rate is 175 gallons per hour (the sum of demand for chicken and tomato), the setup time is 1 hour (30 minutes per soup and two types of soup) and the processing time is 1/300 hour per gallon. The batch size that minimizes inventory while still meeting our demand is then

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}} = \frac{175 \times (2 \times 1/2)}{1 - 175 \times (1/300)} = 420 \text{ gallons}$$

We should produce in proportion to demand (otherwise at least one of the flavors will have too much production and at least one will have too little), so of the 420 gallons, $420 \times 100/(100 + 75) = 240$ gallons should be chicken soup and the remainder, $420 - 240 = 180$ gallons, should be tomato.

To understand the impact of variety on this process, suppose we were to add a third kind of soup to our product offering, onion soup. Furthermore, with onion soup added to the mix, demand for chicken remains 100 gallons per hour, and demand for tomato continues to be 75 gallons per hour, while onion now generates 30 gallons of demand on its own. In some sense, this is an ideal case for adding variety—the new variant adds incrementally to demand without stealing any demand from the existing varieties.

The desired flow rate is now $100 + 75 + 30 = 205$, the setup time is 1.5 hours (three setups per batch), and the inventory minimizing quantity for the production cycle is

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}} = \frac{205 \times (3 \times 1/2)}{1 - 205 \times (1/300)} = 971 \text{ gallons}$$

Again, we should produce in proportion to demand: $971 \times (100/205) = 474$ gallons of chicken, $971 \times (75/205) = 355$ gallons of tomato, and $971 \times (30/205) = 142$ gallons of onion.

So what happened when we added to variety? In short, we need more inventory. Our first hint of this is the size of the batch (the amount of soup across all flavors that

is produced in a production cycle)—420 gallons without onion, while 971 gallons with onion. We can explore this further by evaluating the maximum inventory of chicken soup in either case. (Average inventory of chicken soup is half of its peak inventory.) In our original case, during production, inventory of chicken soup increases at the rate of 300 − 100 = 200 gallons per hour. Production of 240 gallons of chicken soup requires 240/300 hours. So, peak inventory of chicken soup is 200 × (240/300) = 160 gallons. The analogous calculation with onion included yields a peak inventory of 200 × (474/300) = 316 gallons, a 98 percent increase in the amount of inventory needed!

Why did inventory of chicken soup increase when onion soup was added to the mix? Setup times are to blame. With more varieties in the production mix, the production process has to set up more often per production cycle. This reduces the capacity of the production cycle (no soup is made during a setup). To increase the capacity back to the desired flow rate (which is even higher now), we need to operate with larger batches (longer production cycles), and they lead to more inventory.

One may argue that the previous analysis is too optimistic—adding onion soup to the mix should steal some demand away from the other flavors. It turns out that our result is not sensitive to this assumption. To demonstrate, let's consider the opposite extreme—adding onion soup does not expand overall demand, it only steals demand from the other flavors. Specifically, the overall flow rate remains 175 gallons per hour, with or without onion soup. Furthermore, with onion soup, the demand rate for chicken, tomato, and onion are 80, 65, and 30 gallons per hour, respectively. The processing time is still 1/300 gallons per hour, and the setup time per batch is now 1.5 hours (three changeovers due to three types of soup). The batch size that minimizes our inventory while meeting our demand is

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Processing time}} = \frac{175 \times (3 \times 1/2)}{1 - 175 \times (1/300)} = 630 \text{ gallons}$$

Inventory does not increase as much in this case, but it still increases.

The conclusion from this investigation is that setup times and product variety do not mix very well. Consequently, there are two possible solutions to this challenge. The first is to offer only a limited amount of variety. That was Henry Ford's approach when he famously declared that "You can have any color Model-T you want, as long as it is black." While a convenient solution for a production manager, it is not necessarily the best strategy for satisfying demand in a competitive environment.

The other approach to the incompatibility of setups and variety is to work to eliminate setup times. This is the approach advocated by Shigeo Shingo, one of the most influential thought leader in manufacturing. When he witnessed changeover times of more than an hour in an automobile plant, he responded with the quote, "The flow must go on," meaning that every effort must be made to ensure a smooth flow of production. One way to ensure a smooth flow is to eliminate or reduce setup times. Shigeo Shingo developed a powerful technique for doing exactly that, which we will revisit later in the chapter.

7.5 Setup Time Reduction

Despite improvement potential from the use of "good" batch sizes and smaller transfer batches, setups remain a source of disruption of a smooth process flow. For this reason, rather than taking setups as "God-given" constraints and finding ways to accommodate them, we should find ways that directly address the root cause of the disruption.

This is the basic idea underlying the single minute exchange of die (SMED) method. The creators of the SMED method referred to any setup exceeding 10 minutes as an

unacceptable source of process flow disruption. The 10-minute rule is not necessarily meant to be taken literally: the method was developed in the automotive industry, where setup times used to take as much as four hours. The SMED method helps to define an aggressive, yet realistic setup time goal and to identify potential opportunities of setup time reduction.

The basic underlying idea of SMED is to carefully analyze all tasks that are part of the setup time and then divide those tasks into two groups, *internal* setup tasks and *external* setup tasks.

- Internal setup tasks are those tasks that can only be executed while the machine is stopped.
- External setup tasks are those tasks that can be done while the machine is still operating, meaning they can be done *before* the actual changeover occurs.

Experience shows that companies are biased toward using internal setups and that, even without making large investments, internal setups can be translated into external setups.

Similar to our discussion about choosing a good batch size, the biggest obstacles to overcome are ineffective cost accounting procedures. Consider, for example, the case of a simple heat treatment procedure in which flow units are moved on a tray and put into an oven. Loading and unloading of the tray is part of the setup time. The acquisition of an additional tray that can be loaded (or unloaded) while the other tray is still in process (before the setup) allows the company to convert internal setup tasks to external ones. Is this a worthwhile investment?

The answer is, as usual, it depends. SMED applied to nonbottleneck steps is not creating any process improvement at all. As discussed previously, nonbottleneck steps have excessive capacity and therefore setups are entirely free (except for the resulting increase in inventory). Thus, investing in any resource, technical or human, is not only wasteful, but it also takes scarce improvement capacity/funds away from more urgent projects. However, if the oven in the previous example were the bottleneck step, almost any investment in the acquisition of additional trays suddenly becomes a highly profitable investment.

The idea of internal and external setups as well as potential conversion from internal to external setups is best visible in car racing. Any pit stop is a significant disruption of the race car's flow toward the finish line. At any point and any moment in the race, an entire crew is prepared to take in the car, having prepared for any technical problem from tire changes to refueling. While the technical crew might appear idle and underutilized throughout most of the race, it is clear that any second they can reduce from the time the car is in the pit (internal setups) to a moment when the car is on the race track is a major gain (e.g., no race team would consider mounting tires on wheels during the race; they just put on entire wheels).

7.6 Balancing Setup Costs with Inventory Costs: The EOQ Model

Up to now, our focus has been on the role of setup times, as opposed to setup costs. Specifically, we have seen that setup time at the bottleneck leads to an overall reduction in process capacity. Assuming that the process is currently capacity-constrained, setup times thereby carry an opportunity cost reflecting the overall lower flow rate (sales).

Independent of such opportunity costs, setups frequently are associated with direct (out-of-pocket) costs. In these cases, we speak of setup costs (as opposed to setup times). Consider, for example, the following settings:

- The setup of a machine to process a certain part might require scrapping the first 10 parts that are produced after the setup. Thus, the material costs of these 10 parts constitute a setup cost.

• Assume that we are charged a per-time-unit usage fee for a particular resource (e.g., for the milling machine discussed above). Thus, every minute we use the resource, independent of whether we use it for setup or for real production, we have to pay for the resource. In this case, "time is money" and the setup time thereby translates directly into setup costs. However, as we will discuss below, one needs to be very careful when making the conversion from setup times to setup costs.

• When receiving shipments from a supplier, there frequently exists a fixed shipment cost as part of the procurement cost, which is independent of the purchased quantity. This is similar to the shipping charges that a consumer pays at a catalog or online retailer. Shipping costs are a form of setup costs.

All three settings reflect *economies of scale:* the more we order or produce as part of a batch, the more units there are in a batch over which we can spread out the setup costs.

If we can reduce per-unit costs by increasing the batch size, what keeps us from using infinitely (or at least very large) batches? Similar to the case of setup times, we again need to balance our desire for large batches (fewer setups) with the cost of carrying a large amount of inventory.

In the following analysis, we need to distinguish between two cases:

- If the quantity we order is produced or delivered by an outside supplier, all units of a batch are likely to arrive at the same time.
- In other settings, the units of a batch might not all arrive at the same time. This is especially the case when we produce the batch internally.

Figure 7.8 illustrates the inventory levels for the two cases described above. The lower part of Figure 7.8 shows the case of the outside supplier and all units of a batch arriving at the same moment in time. The moment a shipment is received, the inventory level jumps up by the size of the shipment. It then falls up to the time of the next shipment.

The upper part of Figure 7.8 shows the case of units created by a resource with (finite) capacity. Thus, while we are producing, the inventory level increases. Once we stop production, the inventory level falls. Let us consider the case of an outside supplier first (lower part of Figure 7.8). Specifically, consider the case of the Xootr handle caps that Nova Cruz sources from a supplier in Taiwan for $0.85 per unit. Note that the maximum inventory of handle caps occurs at the time we receive a shipment from Taiwan. The inventory is then depleted at the rate of the assembly operations, that is, at a flow rate, R, of 700 units (pairs of handle caps) per week, which is equal to one unit every three minutes.

For the following computations, we make a set of assumptions. We later show that these assumptions do not substantially alter the optimal decisions.

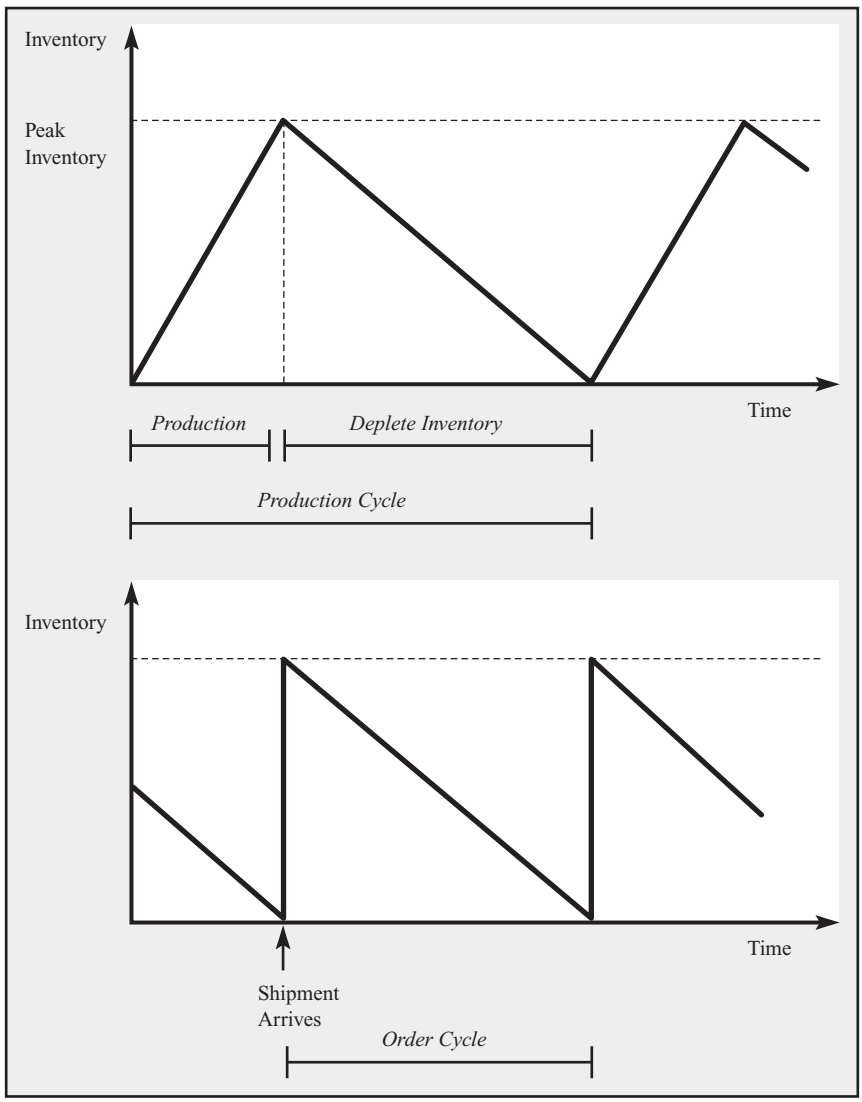
• We assume that production of Xootrs occurs at a constant rate of one unit every three minutes. We also assume our orders arrive on time from Taiwan. Under these two assumptions, we can deplete our inventory all the way to zero before receiving the next shipment.

• There is a fixed setup cost per order that is independent of the amount ordered. In the Xootr case, this largely consists of a $300 customs fee.

• The purchase price is independent of the number of units we order, that is, there are no quantity discounts. We talk about quantity discounts in the next section.

The objective of our calculations is to minimize the cost of inventory and ordering with the constraint that we must never run out of inventory (i.e., we can keep the assembly operation running).

FIGURE 7.8
Different Patterns of Inventory Levels



We have three costs to consider: purchase costs, delivery fees, and holding costs. We use 700 units of handle caps each week no matter how much or how frequently we order. Thus, we have no excuse for running out of inventory and there is nothing we can do about our purchase costs of

$$\$0.85/\text{unit} \times 700 \text{ units/week} = \$595 \text{ per week}$$

So when choosing our ordering policy (when and how much to order), we focus on minimizing the sum of the other two costs, delivery fees and inventory costs.

The cost of inventory depends on how much it costs us to hold one unit in inventory for a given period of time, say one week. We can obtain the number by looking at the annual inventory costs and dividing that amount by 52. The annual inventory costs need to account for financing the inventory (cost of capital, especially high for a start-up like Nova Cruz), costs of storage, and costs of obsolescence. Nova Cruz uses an annual inventory cost of 40 percent. Thus, it costs Nova Cruz 0.7692 percent to hold a piece of inventory for one week. Given that a handle cap costs $0.85 per unit, this translates to an

inventory cost of $h = 0.007692 \times \$0.85/\text{unit} = \$0.006538$ per unit per week. Note that the annual holding cost needs to include the cost of capital as well as any other cost of inventory (e.g., storage, theft, etc).

How many handle caps will there be, on average, in Nova Cruz's inventory? As we can see in Figure 7.8, the average inventory level is simply

$$\text{Average inventory} = \frac{\text{Order quantity}}{2}$$

If you are not convinced, refer in Figure 7.8 to the "triangle" formed by one order cycle. The average inventory during the cycle is half of the height of the triangle, which is half the order quantity, $Q/2$. Thus, for a given inventory cost, h, we can compute the inventory cost per unit of time (e.g., inventory costs per week):

$$\text{Inventory costs [per unit of time]} = \frac{1}{2}\,\text{Order quantity} \times h = \frac{1}{2}\,Q \times h$$

Before we turn to the question of how many handle caps to order at once, let's first ask ourselves how frequently we have to place an order. Say at time 0 we have I units in inventory and say we plan our next order to be Q units. The I units of inventory will satisfy demand until time I/R (in other words, we have I/R weeks of supply in inventory). At this time, our inventory will be zero if we don't order before then. We would then again receive an order of Q units (if there is a lead time in receiving this order, we simply would have to place this order earlier).

Do we gain anything by receiving the Q handle caps earlier than at the time when we have zero units in inventory? Not in this model: demand is satisfied whether we order earlier or not and the delivery fee is the same too. But we do lose something by ordering earlier: we incur holding costs per unit of time the Q units are held.

Given that we cannot save costs by choosing the order time intelligently, we must now work on the question of how much to order (the order quantity). Let's again assume that we order Q units with every order and let's consider just one order cycle. The order cycle begins when we order Q units and ends when the last unit is sold, Q/R time units later. For example, with $Q = 1{,}000$, an order cycle lasts 1,000 units/700 units per week = 1.43 weeks. We incur one ordering fee (setup costs), K, in that order cycle, so our setup costs per week are

$$\begin{aligned}\text{Setup costs [per unit of time]} &= \frac{\text{Setup cost}}{\text{Length of order cycle}} \\ &= \frac{K}{Q/R} = \frac{K \times R}{Q}\end{aligned}$$

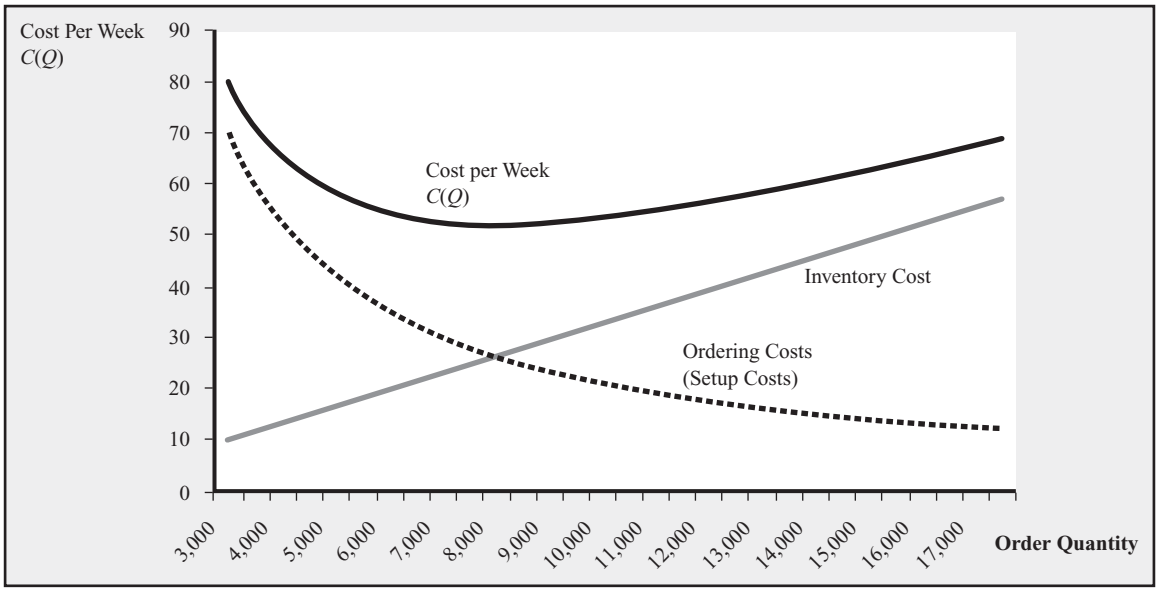
Let $C(Q)$ be the sum of our average delivery cost per unit time and our average holding cost per unit time (per week):

$$\begin{aligned}\text{Per unit of time cost } C(Q) &= \text{Setup costs} + \text{Inventory costs} \\ &= \frac{K \times R}{Q} + \frac{1}{2} \times h \times Q\end{aligned}$$

Note that purchase costs are not included in $C(Q)$ for the reasons discussed earlier. From the above we see that the delivery fee per unit time decreases as Q increases: we amortize the delivery fee over more units. But as Q increases, we increase our holding costs.

Figure 7.9 graphs the weekly costs of delivery, the average weekly holding cost, and the total weekly cost, $C(Q)$. As we can see, there is a single order quantity Q that minimizes the total cost $C(Q)$. We call this quantity Q^*, the economic order quantity, or EOQ for short. Hence the name of the model.

FIGURE 7.9 **Inventory and Ordering Costs for Different Order Sizes**



From Figure 7.9 it appears that Q^* is the quantity at which the weekly delivery fee equals the weekly holding cost. In fact, that is true, as can be shown algebraically. Further, using calculus it is possible to show that

$$\text{Economic order quantity} = \sqrt{\frac{2 \times \text{Setup cost} \times \text{Flow rate}}{\text{Holding cost}}}$$

$$Q^* = \sqrt{\frac{2 \times K \times R}{h}}$$

As our intuition suggests, as the setup costs K increase, we should make larger orders, but as holding costs h increase, we should make smaller orders.

We can use the above formula to establish the economic order quantity for handle caps:

$$Q^* = \sqrt{\frac{2 \times \text{Setup cost} \times \text{Flow rate}}{\text{Holding cost}}}$$

$$= \sqrt{\frac{2 \times 300 \times 700}{0.006538}} = 8,014.69$$

The steps required to find the economic order quantity are summarized by Exhibit 7.2.

7.7 Observations Related to the Economic Order Quantity

If we always order the economic order quantity, our cost per unit of time, $C(Q^*)$, can be computed as

$$C(Q^*) = \frac{K \times R}{Q^*} + \frac{1}{2} \times h \times Q^* = \sqrt{2 \times K \times R \times h}$$

Exhibit 7.2

FINDING THE ECONOMIC ORDER QUANTITY

1. Verify the basic assumptions of the EOQ model:
   - Replenishment occurs instantaneously.
   - Demand is constant and not stochastic.
   - There is a fixed setup cost K independent of the order quantity.
2. Collect information on
   - Setup cost, K (only include out-of-pocket cost, not opportunity cost).
   - Flow rate, R.
   - Holding cost, h (not necessarily the yearly holding cost; needs to have the same time unit as the flow rate).
3. For a given order quantity Q, compute

$$\text{Inventory costs [per unit of time]} = \frac{1}{2} Q \times h$$

$$\text{Setup costs [per unit of time]} = \frac{K \times R}{Q}$$

4. The economic order quantity minimizes the sum of the inventory and the setup costs and is

$$Q^* = \sqrt{\frac{2 \times K \times R}{h}}$$

   The resulting costs are

$$C(Q^*) = \sqrt{2 \times K \times R \times h}$$

While we have done this analysis to minimize our average cost per unit of time, it should be clear that Q^* would minimize our average cost per unit (given that the rate of purchasing handle caps is fixed). The cost per unit can be computed as

$$\text{Cost per unit} = \frac{C(Q^*)}{R} = \sqrt{\frac{2 \times K \times h}{R}}$$

As we would expect, the per-unit cost is increasing with the ordering fee K as well as with our inventory costs. Interestingly, the per-unit cost is decreasing with the flow rate R. Thus, if we doubled our flow rate, our ordering costs increase by less than a factor of 2. In other words, there are economies of scale in the ordering process: the per-unit ordering cost is decreasing with the flow rate R. Put yet another way, an operation with setup and inventory holding costs becomes more efficient as the demand rate increases.

While we have focused our analysis on the time period when Nova Cruz experienced a demand of 700 units per week, the demand pattern changed drastically over the product life cycle of the Xootr. As discussed in Chapter 4, Nova Cruz experienced a substantial demand growth from 200 units per week to over 1,000 units per week. Table 7.2 shows how increases in demand rate impact the order quantity as well as the per-unit cost of the handle caps. We observe that, due to scale economies, ordering and inventory costs are decreasing with the flow rate R.

TABLE 7.2
Scale Economies in the EOQ Formula

Flow Rate, R	Economic Order Quantity, Q^*	Per-Unit Ordering and Inventory Cost, $C(Q^*)/R$	Ordering and Inventory Costs as a Percentage of Total Procurement Costs
200	4,284	0.14 [$/unit]	14.1%
400	6,058	0.10	10.4%
600	7,420	0.08	8.7%
800	8,568	0.07	7.6%
1,000	9,579	0.06	6.8%

A nice property of the economic order quantity is that the cost function, $C(Q)$, is relatively flat around its minimum Q^* (see graph in Figure 7.9). This suggests that if we were to order Q units instead of Q^*, the resulting cost penalty would not be substantial as long as Q is reasonably close to Q^*. Suppose we order only half of the optimal order quantity, that is, we order $Q^*/2$. In that case, we have

$$C(Q^*/2) = \frac{K \times R}{Q^*/2} + \frac{1}{2} \times h \times Q^*/2 = \frac{5}{4} \times \sqrt{2 \times K \times R \times h} = \frac{5}{4} \times C(Q^*)$$

Thus, if we order only half as much as optimal (i.e., we order twice as frequently as optimal), then our costs increase only by 25 percent. The same holds if we order double the economic order quantity (i.e., we order half as frequently as optimal).

This property has several important implications:

• Consider the optimal order quantity $Q^* = 8{,}014$ established above. However, now also assume that our supplier is only willing to deliver in predefined quantities (e.g., in multiples of 5,000). The robustness established above suggests that an order of 10,000 will only lead to a slight cost increase (increased costs can be computed as $C(Q = 10{,}000) = \$53.69$, which is only 2.5 percent higher than the optimal costs).

• Sometimes, it can be difficult to obtain exact numbers for the various ingredients in the EOQ formula. Consider, for example, the ordering fee in the Nova Cruz case. While this fee of $300 was primarily driven by the $300 for customs, it also did include a shipping fee. The exact shipping fee in turn depends on the quantity shipped and we would need a more refined model to find the order quantity that accounts for this effect. Given the robustness of the EOQ model, however, we know that the model is "forgiving" with respect to small misspecifications of parameters.

A particularly useful application of the EOQ model relates to *quantity discounts.* When procuring inventory in a logistics or retailing setting, we frequently are given the opportunity to benefit from quantity discounts. For example:

- We might be offered a discount for ordering a full truckload of supply.
- We might receive a free unit for every five units we order (just as in consumer retailing settings of "buy one, get one free").
- We might receive a discount for all units ordered over 100 units.
- We might receive a discount for the entire order if the order volume exceeds 50 units (or say $2,000).

We can think of the extra procurement costs that we would incur from not taking advantage of the quantity discount—that is, that would result from ordering in smaller quantities—as a setup cost. Evaluating an order discount therefore boils down to a comparison between inventory costs and setup costs (savings in procurement costs), which we can do using the EOQ model.

If the order quantity we obtain from the EOQ model is sufficiently large to obtain the largest discount (the lowest per-unit procurement cost), then the discount has no impact on our order size. We go ahead and order the economic order quantity. The more interesting case occurs when the EOQ is less than the discount threshold. Then we must decide if we wish to order more than the economic order quantity to take advantage of the discount offered to us.

Let's consider one example to illustrate how to think about this issue. Suppose our supplier of handle caps gives us a discount of 5 percent off the entire order if the order exceeds 10,000 units. Recall that our economic order quantity was only 8,014. Thus, the question is "should we increase the order size to 10,000 units in order to get the 5 percent discount, yet incur higher inventory costs, or should we simply order 8,014 units?"

We surely will not order more than 10,000; any larger order does not generate additional purchase cost savings but does increase inventory costs. So we have two choices: either stick with the EOQ or increase our order to 10,000. If we order $Q^* = 8{,}014$ units, our total cost per unit time is

$$\begin{aligned} &700 \text{ units/week} \times \$0.85/\text{unit} + C(Q^*) \\ &= \$595/\text{week} + \$52.40/\text{week} \\ &= \$647.40/\text{week} \end{aligned}$$

Notice that we now include our purchase cost per unit time of 700 units/week × \$0.85/unit. The reason for this is that with the possibility of a quantity discount, our purchase cost now depends on the order quantity.

If we increase our order quantity to 10,000 units, our total cost per unit time would be

$$\begin{aligned} &700 \text{ units/week} \times \$0.85/\text{unit} \times 0.95 + C(10{,}000) \\ &= \$565.25/\text{week} + \$52.06/\text{week} \\ &= \$617.31/\text{week} \end{aligned}$$

where we have reduced the procurement cost by 5 percent (multiplied by 0.95) to reflect the quantity discount. (*Note:* The 5 percent discount also reduces the holding cost h in $C(.)$) Given that the cost per week is lower in the case of the increased order quantity, we want to take advantage of the quantity discount.

After analyzing the case of all flow units of one order (batch) arriving simultaneously, we now turn to the case of producing the corresponding units internally (upper part of Figure 7.8).

All computations we performed above can be easily transformed to this more general case (see, e.g., Nahmias 2005). Moreover, given the robustness of the economic order quantity, the EOQ model leads to reasonably good recommendations even if applied to production settings with setup costs. Hence, we will not discuss the analytical aspects of this. Instead, we want to step back for a moment and reflect on how the EOQ model relates to our discussion of setup times at the beginning of the chapter.

A common mistake is to rely too much on setup *costs* as opposed to setup *times.* For example, consider the case of Figure 7.6 and assume that the monthly capital cost for the milling machine is \$9,000, which corresponds to \$64 per hour (assuming four weeks of 35 hours each). Thus, when choosing the batch size, and focusing primarily on costs, Nova Cruz might shy away from frequent setups. Management might even consider using the economic order quantity established above and thereby quantify the impact of larger batches on inventory holding costs.

There are two major mistakes in this approach:

- This approach to choosing batch sizes ignores the fact that the investment in the machine is already sunk.
- Choosing the batch size based on cost ignores the effect setups have on process capacity. As long as setup costs are a reflection of the cost of capacity—as opposed to direct financial setup costs—they should be ignored when choosing the batch size. It is

the overall process flow that matters, not an artificial local performance measure! From a capacity perspective, setups at nonbottleneck resources are free. And if the setups do occur at the bottleneck, the corresponding setup costs not only reflect the capacity costs of the local resource, but of the entire process!

Thus, when choosing batch sizes, it is important to distinguish between setup costs and setup times. If the motivation behind batching results from setup times (or opportunity costs of capacity), we should focus on optimizing the process flow. Section 7.3 provides the appropriate way to find a good batch size. If we face "true" setup costs (in the sense of out-of-pocket costs) and we only look at a single resource (as opposed to an entire process flow), the EOQ model can be used to find the optimal order quantity.

Finally, if we encounter a combination of setup times and (out-of-pocket) setup costs, we should use both approaches and compare the recommended batch sizes. If the batch size from the EOQ is sufficiently large so that the resource with the setup is not the bottleneck, minimizing costs is appropriate. If the batch size from the EOQ, however, makes the resource with the setups the bottleneck, we need to consider increasing the batch size beyond the EOQ recommendation.

7.8 Other Flow Interruptions: Buffer or Suffer

In addition to illustrating the SMED method, the race car example also helps to illustrate how the concept of batching can be applied to *continuous process flows,* as opposed to discrete manufacturing environments. First of all, we observe that the calculation of the average speed of the race car is nothing but a direct application of the batching formula introduced at the beginning of this chapter:

$$\text{Average speed (number of miles between stops)} = \frac{\text{Number of miles between stops}}{\text{Duration of the stop} + \text{Time to cover one mile} \times \text{Number of miles between stops}}$$

In continuous flow processes, the quantity between two flow interruptions is frequently referred to as a production run.

Consider the production of orange juice, which is produced in a continuous flow process. At an abstract level, orange juice is produced in a three-step process: extraction, filtering, and bottling. Given that the filter at the second process step has to be changed regularly, the process needs to be stopped for 30 minutes following every four hours of production. While operating, the step can produce up to 100 barrels per hour.

To determine the capacity of the filtering step, we use

$$\text{Capacity } (B) = \frac{B}{S + B \times p}$$

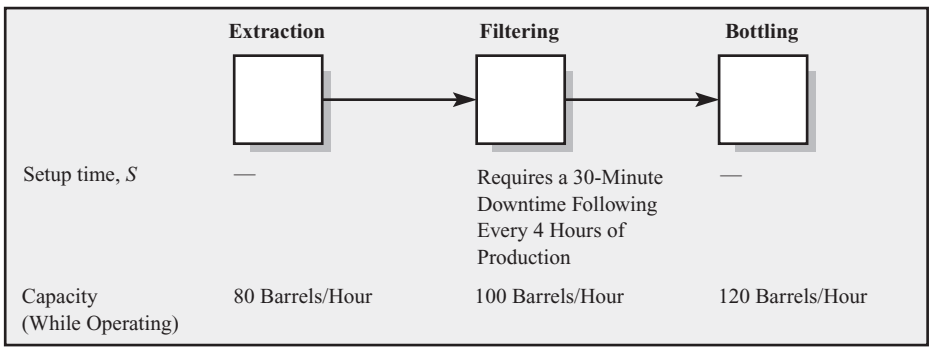
$$= \frac{\text{Amount processed between two stops}}{\text{Duration of stop} + \text{Time to produce one barrel} \times \text{Amount processed between two stops}}$$

$$= \frac{400 \text{ barrels}}{30 \text{ minutes} + 60/100 \text{ minutes per barrel} \times 400 \text{ barrels}}$$

$$= \frac{400 \text{ barrels}}{270 \text{ minutes}}$$

$$= 1.48 \ barrels/minute = 88.88 \ barrels/hour$$

FIGURE 7.10
Data for the Production of Orange Juice



While in the case of batch flow operations we have allowed for substantial buffer sizes between process steps, the process as described in Figure 7.10 is currently operating without buffers. This has substantial implications for the overall flow rate.

Analyzing each step in isolation would suggest that the extraction step is the bottleneck, which would give us a process capacity of 80 barrels per hour. However, in the absence of buffers, the extraction step needs to stop producing the moment the filtering step is shut down. Thus, while running, the process is constrained by the extraction step, producing an output of 80 barrels per hour, and while being shut down, the process step is constrained by the filtering step (at 0 barrel per hour).

Previously, we considered the setup step in isolation from the rest of the process. That is a valid analysis if the setup step indeed works in isolation from the rest of the process, that is, if there is sufficient inventory (buffers) between steps. That assumption is violated here: The filtering step cannot operate at 88 barrels per hour because it is constrained by the extraction step of 80 barrels per hour.

For this reason, when we use our equation

$$\text{Capacity} = \frac{\text{Amount processed between two stops}}{\text{Duration of stop} + \text{Time to produce one barrel} \times \text{Amount processed between two stops}}$$

it is important that we acknowledge that we are producing at a rate of 80 barrels per hour (i.e., $\frac{1}{80}$ hour per barrel) while we are at the filtering step. This leads to the following computation of process capacity:

$$\begin{aligned}\text{Capacity} &= \frac{320 \text{ barrels}}{0.5 \text{ hour} + \frac{1}{80} \text{ hour per barrel} \times 320 \text{ barrels}} \\ &= 320 \text{ barrels}/4.5 \text{ hours} \\ &= 71.11 \text{ barrels}/\text{hour}\end{aligned}$$

This prompts the following interesting observation: In the presence of flow interruptions, buffers can increase process capacity. Practitioners refer to this phenomenon as "buffer or suffer," indicating that flow interruptions can be smoothed out by introducing buffer inventories. In the case of Figure 7.10, the buffer would need to absorb the outflow of the extraction step during the downtime of the reduction step. Thus, adding a buffer between these two steps would indeed increase process capacity up to the level where, with 80 barrels per hour, the extraction step becomes the bottleneck.

7.9 Summary

Setups are interruptions of the supply process. These interruptions on the supply side lead to mismatches between supply and demand, visible in the form of inventory and—where this is not possible (see orange juice example)—lost throughput.

While in this chapter we have focused on inventory of components (handle caps), work-in-process (steer support parts), or finished goods (station wagons versus sedans, Figure 7.4), the supply–demand mismatch also can materialize in an inventory of waiting customer orders. For example, if the product we deliver is customized and built to the specifications of the customer, holding an inventory of finished goods is not possible. Similarly, if we are providing a substantial variety of products to the market, the risk of holding completed variants in finished goods inventory is large (this will be further discussed in Chapter 15). Independent of the form of inventory, a large inventory corresponds to long flow times (Little's Law). For this reason, batch processes are typically associated with very long customer lead times.

In this chapter, we discussed tools to choose a batch size. We distinguished between setup times and setup costs. To the extent that a process faces setup times, we need to extend our process analysis to capture the negative impact that setups have on capacity. We then want to look for a batch size that is large enough to not make the process step with the setup the bottleneck, while being small enough to avoid excessive inventory.

To the extent that a process faces (out-of-pocket) setup costs, we need to balance these costs against the cost of inventory. We discussed the EOQ model for the case of supply arriving in one single quantity (sourcing from a supplier), as well as the case of internal production. Figure 7.11 provides a summary of the major steps you should take when analyzing processes with flow interruptions, including setup times, setup costs, or machine downtimes. There are countless extensions to the EOQ model to capture, among other things, quantity discounts, perishability, learning effects, inflation, and quality problems.

Our ability to choose a "good" batch size provides another example of process improvement. Consider a process with significant setup times at one resource. As a manager of this process, we need to balance the conflicting objectives of

- Fast response to customers (short flow times, which correspond, because of Little's Law, to low inventory levels), which results from using small batch sizes.
- Cost benefits that result from using large batch sizes. The reason for this is that large batch sizes enable a high throughput, which in turn allows the firm to spread out its fixed costs over a maximum number of flow units.

FIGURE 7.11 Summary of Batching

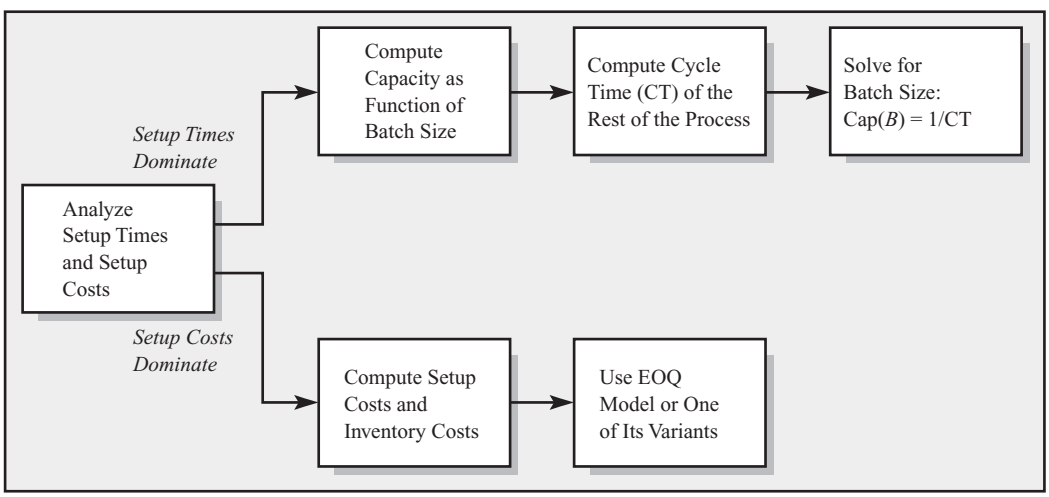
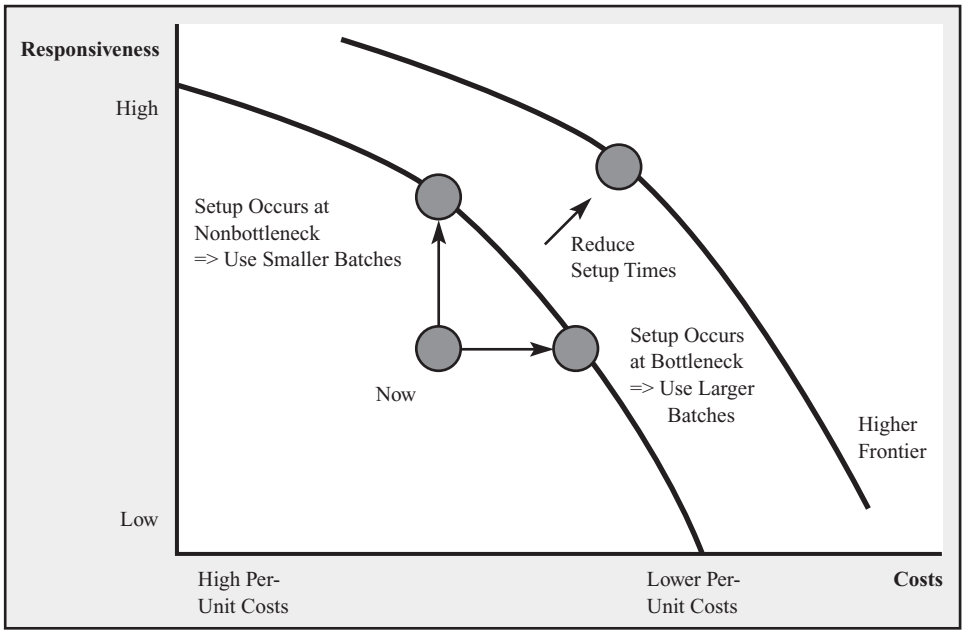


FIGURE 7.12
Choosing a Batch Size



This tension is illustrated by Figure 7.12. Similar to the case of line balancing, we observe that adjustments in the batch size are not trading in one performance measure against the other, but allow us to improve by reducing current inefficiencies in the process.

Despite our ability to choose batch sizes that mitigate the tension between inventory (responsiveness) and costs, there ultimately is only one way to handle setups: eliminate them wherever possible or at least shorten them. Setups do not add value and are therefore wasteful.

Methods such as SMED are powerful tools that can reduce setup times substantially. Similarly, the need for transfer batches can be reduced by locating the process resources according to the flow of the process.

7.10 Further Reading

Nahmias (2005) is a widely used textbook in operations management that discusses, among other things, many variants of the EOQ model.

7.11 Practice Problems

Q7.1* **(Window Boxes)** Metal window boxes are manufactured in two process steps: stamping and assembly. Each window box is made up of three pieces: a base (one part A) and two sides (two part Bs).

The parts are fabricated by a single stamping machine that requires a setup time of 120 minutes whenever switching between the two part types. Once the machine is set up, the processing time for each part A is one minute while the processing time for each part B is only 30 seconds.

Currently, the stamping machine rotates its production between one batch of 360 for part A and one batch of 720 for part B. Completed parts move from the stamping machine to the assembly only after the entire batch is complete.

At assembly, parts are assembled manually to form the finished product. One base (part A) and two sides (two part Bs), as well as a number of small purchased components, are required for each unit of final product. Each product requires 27 minutes of labor time

(* indicates that the solution is at the end of the book)

to assemble. There are currently 12 workers in assembly. There is sufficient demand to sell every box the system can make.

a. What is the capacity of the stamping machine?

b. What batch size would you recommend for the process?

Q7.2** **(PTests)** Precision Testing (PTests) does fluid testing for several local hospitals. Consider their urine testing process. Each sample requires 12 seconds to test, but after 300 samples, the equipment must be recalibrated. No samples can be tested during the recalibration process and that process takes 30 minutes.

a. What is PTest's maximum capacity to test urine samples (in samples per hour)?

b. Suppose 2.5 urine samples need to be tested per minute. What is the smallest batch size (in samples) that ensures that the process is not supply constrained? (Note; A batch is the number of tests between calibrations.)

c. PTest also needs to test blood samples. There are two kinds of tests that can be done—a "basic" test and a "complete" test. Basic tests require 15 seconds per sample, whereas "complete" tests require 1.5 minutes per sample. After 100 tests, the equipment needs to be cleaned and recalibrated, which takes 20 minutes. Suppose PTest runs the following cyclic schedule: 70 basic tests, 30 complete tests, recalibrate, and then repeat. With this schedule, how many *basic* tests can they complete per minute on average?

Q7.3 **(Gelato)** Bruno Fruscalzo decided to set up a small production facility in Sydney to sell to local restaurants that want to offer gelato on their dessert menu. To start simple, he would offer only three flavors of gelato: fragola (strawberry), chocolato (chocolate), and bacio (chocolate with hazelnut). After a short time he found his demand and setup times to be

	Fragola	**Chocolato**	**Bacio**
Demand (kg/hour)	**10**	**15**	**5**
Setup time (hours)	**3/4**	**1/2**	**1/6**

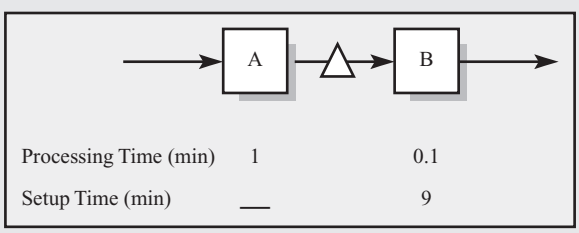
Bruno first produces a batch of fragola, then a batch of chocolato, then a batch of bacio and then he repeats that sequence. For example, after producing bacio and before producing fragola, he needs 45 minutes to set up the ice cream machine, but he needs only 10 minutes to switch from chocolato to bacio. When running, his ice cream machine produces at the rate of 50 kg per hour no matter which flavor it is producing (and, of course, it can produce only one flavor at a time).

a. Suppose Bruno wants to minimize the amount of each flavor produced at one time while still satisfying the demand for each of the flavors. (He can choose a different quantity for each flavor.) If we define a batch to be the quantity produced in a single run of each flavor, how many kilograms should he produce in each batch?

b. Given your answer in part (a), how many kilograms of fragola should he make with each batch?

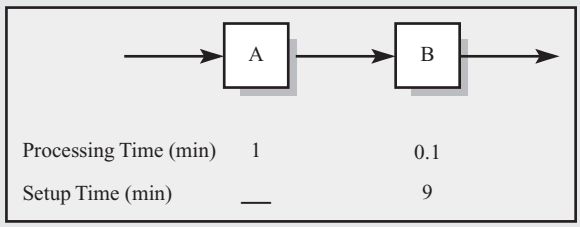
c. Given your answer in part (a), what is the maximum inventory of chocolato? (Assume production and demand occur at constant rates.)

Q7.4 **(Two-step)** Consider the following two step process:



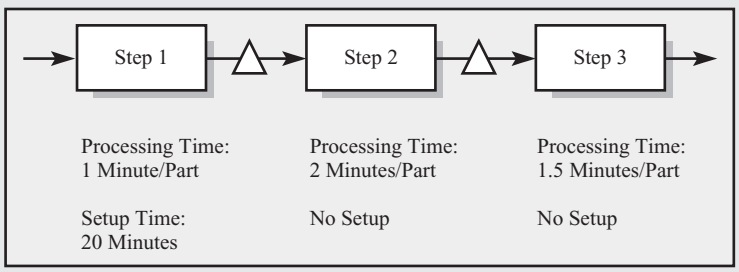
Step A has a processing time of 1 minute per unit, but no setup is required. Step B has an processing time of 0.1 minute per unit, but a setup time of 9 minutes is require per batch. A buffer with ample inventory is allowed between the two steps.

a. Suppose units are produced in batches of 5 (i.e., after each set of 5 units are produced, step B must incur a setup of 9 minutes). What is the capacity of the process (in units per minute)?
b. What is the batch size that maximizes the flow rate of this process with minimal inventory? Assume there is ample demand.
c. Now suppose the inventory buffer is removed between steps A and B:



Thus, when B is being set up, A cannot work because there is no place to put its completed units. Once B has finished its setup and is ready to work on units, A can resume its work because B is now ready to accept A's output. What batch size achieves a flow rate of 0.82 unit per minute?

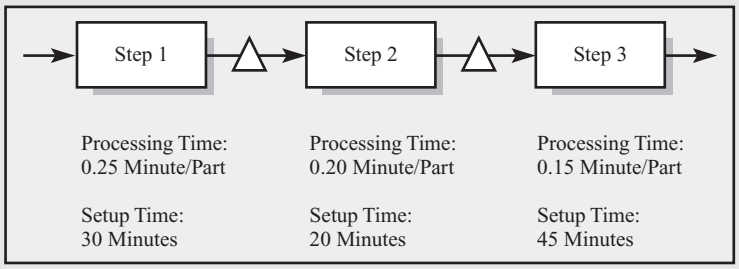
Q7.5 **(Simple Setup)** Consider the following batch flow process consisting of three process steps performed by three machines:



Work is processed in batches at each step. Before a batch is processed at step 1, the machine has to be set up. During a setup, the machine is unable to process any product.

a. Assume that the batch size is 50 parts. What is the capacity of the process?
b. For a batch size of 10 parts, which step is the bottleneck for the process?
c. Using the current production batch size of 50 parts, how long would it take to produce 20 parts starting with an empty system? Assume that the units in the batch have to stay together (no smaller transfer batches allowed) when transferred to step 2 and to step 3. A unit can leave the system the moment it is completed at step 3. Assume step 1 needs to be set up before the beginning of production.
d. Using the current production batch size of 50 parts, how long would it take to produce 20 parts starting with an empty system? Assume that the units in the batch do *not* have to stay together; specifically, units are transferred to the next step the moment they are completed at any step. Assume step 1 needs to be set up before the beginning of production.
e. What batch size would you choose, assuming that all units of a batch stay together for the entire process?

Q7.6 **(Setup Everywhere)** Consider the following batch-flow process consisting of three process steps performed by three machines:



Work is processed in batches at each step. Before a batch is processed at a step, the machine at that step must be set up. (During a setup, the machine is unable to process any product.) Assume that there is a dedicated setup operator for each machine (i.e., there is always someone available to perform a setup at each machine.)

a. What is the capacity of step 1 if the batch size is 35 parts?

b. For what batch sizes is step 1 (2, 3) the bottleneck?

Q7.7 **(JCL Inc.)** JCL Inc. is a major chip manufacturing firm that sells its products to computer manufacturers like Dell, HP, and others. In simplified terms, chip making at JCL Inc. involves three basic operations: depositing, patterning, and etching.

- **Depositing:** Using chemical vapor deposition (CVD) technology, an insulating material is deposited on the wafer surface, forming a thin layer of solid material on the chip.
- **Patterning:** Photolithography projects a microscopic circuit pattern on the wafer surface, which has a light-sensitive chemical like the emulsion on photographic film. It is repeated many times as each layer of the chip is built.
- **Etching:** Etching removes selected material from the chip surface to create the device structures.

The following table lists the required processing times and setup times at each of the steps. There is unlimited space for buffer inventory between these steps. Assume that the unit of production is a wafer, from which individual chips are cut at a later stage.

Note: A Setup can only begin once the batch has arrived at the machine.

Process Step	1 Depositing	2 Patterning	3 Etching
Setup time	45 min.	30 min.	20 min.
Processing time	0.15 min./unit	0.25 min./unit	0.20 min./unit

a. What is the process capacity in units per hour with a batch size of 100 wafers?

b. For the current batch size of 100 wafers, how long would it take to produce 50 wafers? Assume that the batch needs to stay together during deposition and patterning (i.e., the firm does not work with transfer batches that are less than the production batch). However, the 50 wafers can leave the process the moment all 50 wafers have passed through the etching stage. Recall that a setup can only be started upon the arrival of the batch at the machine.

c. For what batch size is step 3 (etching) the bottleneck?

d. Suppose JCL Inc. came up with a new technology that eliminated the setup time for step 1 (deposition), but increased the processing time to 0.45 min./unit. What would be the batch size you would choose so as to maximize the overall capacity of the process, assuming all units of a batch stay together for the entire process?

Q7.8 **(Kinga Doll Company)** Kinga Doll Company manufactures eight versions of its popular girl doll, Shari. The company operates on a 40-hour work week. The eight versions

differ in doll skin, hair, and eye color, enabling most children to have a doll with a similar appearance to them. It currently sells an average of 4,000 dolls (spread equally among its eight versions) per week to boutique toy retailers. In simplified terms, doll making at Kinga involves three basic operations: molding the body and hair, painting the face, and dressing the doll. Changing over between versions requires setup time at the molding and painting stations due to the different colors of plastic pellets, hair, and eye color paint required. The table below lists the setup times for a batch and the processing times for each unit at each step. Unlimited space for buffer inventory exists between these steps.

Assume that (i) setups need to be completed first, (ii) a setup can only start once the batch has arrived at the resource, and (iii) all flow units of a batch need to be processed at a resource before any of the units of the batch can be moved to the next resource.

Process Step	1 Molding	2 Painting	3 Dressing
Setup time	15 min.	30 min.	No setup
Processing time	0.25 min./unit	0.15 min./unit	0.30 min./unit

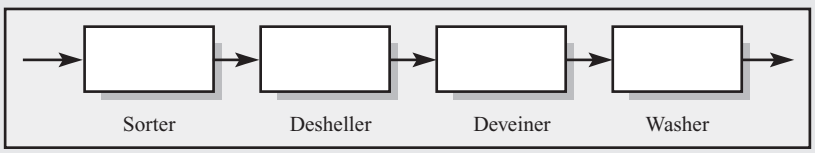
a. What is the process capacity in units per hour with a batch size of 500 dolls?

b. What is the time it takes for the first batch of 500 dolls to go through an empty process?

c. Which batch size would minimize inventory without decreasing the process capacity?

d. Which batch size would minimize inventory without decreasing the current flow rate?

Q7.9 **(Bubba Chump Shrimp)** The Bubba Chump Shrimp Company processes and packages shrimp for sale to wholesale seafood distributors. The shrimp are transported to the main plant by trucks that carry 1,000 pounds (lb) of shrimp. Once the continuous flow processing of the shrimp begins, *no* inventory is allowed in buffers due to spoilage and all of the shrimp must be processed within 12 hours to prevent spoilage. The processing begins at the sorter, where the trucks dump the shrimp onto a conveyor belt that feeds into the sorter, which can sort up to 500 lb per hour. The shrimp then proceed to the desheller, which can process shrimp at the rate of 400 lb per hour. However, after 3 hours and 45 minutes of processing, the desheller must be stopped for 15 minutes to clean out empty shrimp shells that have accumulated. The veins of the shrimp are then removed in the deveining area at a maximum rate of 360 lb per hour. The shrimp proceed to the washing area, where they are processed at 750 lb per hour. Finally, the shrimp are packaged and frozen.



Note: All unit weights given are in "final processed shrimp." You do *not* need to account for the weight of the waste in the deshelling area. The plant operates continuously for 12 hours per day beginning at 8:00 a.m. Finally, there is negligible time to fill the system in the morning.

a. What is the daily process capacity of the desheller (in isolation from the other processes)?

b. What is the daily process capacity of the deveiner (in isolation from the other processes)?

c. What is the daily process capacity of the processing plant (excluding the packaging and freezing)?

d. If five trucks arrive one morning at 8:00 a.m., what is the total number of pounds of shrimp that must be wasted?

Q7.10* **(Cat Food)** Cat Lovers Inc. (CLI) is the distributor of a very popular blend of cat food that sells for $1.25 per can. CLI experiences demand of 500 cans per week on average. They order the cans of cat food from the Nutritious & Delicious Co. (N&D). N&D sells cans to CLI at $0.50 per can and charges a flat fee of $7 per order for shipping and handling.

CLI uses the economic order quantity as their fixed order size. Assume that the opportunity cost of capital and all other inventory cost is 15 percent annually and that there are 50 weeks in a year.

a. How many cans of cat food should CLI order at a time?
b. What is CLI's total order cost for one year?
c. What is CLI's total holding cost for one year?
d. What is CLI's weekly inventory turns?

Q7.11* **(Beer Distributor)** A beer distributor finds that it sells on average 100 cases a week of regular 12-oz. Budweiser. For this problem assume that demand occurs at a constant rate over a 50-week year. The distributor currently purchases beer every two weeks at a cost of $8 per case. The inventory-related holding cost (capital, insurance, etc.) for the distributor equals 25 percent of the dollar value of inventory per year. Each order placed with the supplier costs the distributor $10. This cost includes labor, forms, postage, and so forth.

a. Assume the distributor can choose any order quantity it wishes. What order quantity minimizes the distributor's total inventory-related costs (holding and ordering)?

   For the next three parts, assume the distributor selects the order quantity specified in part (a).

b. What are the distributor's inventory turns per year?
c. What is the inventory-related cost per case of beer sold?
d. Assume the brewer is willing to give a 5 percent quantity discount if the distributor orders 600 cases or more at a time. If the distributor is interested in minimizing its total cost (i.e., purchase and inventory-related costs), should the distributor begin ordering 600 or more cases at a time?

Q7.12** **(Millennium Liquors)** Millennium Liquors is a wholesaler of sparkling wines. Their most popular product is the French Bete Noire. Weekly demand is for 45 cases. Assume demand occurs over 50 weeks per year. The wine is shipped directly from France. Millennium's annual cost of capital is 15 percent, which also includes all other inventory-related costs. Below are relevant data on the costs of shipping, placing orders, and refrigeration.

- Cost per case: $120
- Shipping cost (for any size shipment): $290
- Cost of labor to place and process an order: $10
- Fixed cost for refrigeration: $75/week

a. Calculate the weekly holding cost for one case of wine.
b. Use the EOQ model to find the number of cases per order and the average number of orders per year.
c. Currently orders are placed by calling France and then following up with a letter. Millennium and its supplier may switch to a simple ordering system using the Internet. The new system will require much less labor. What would be the impact of this system on the ordering pattern?

Q7.13 **(Powered by Koffee)** Powered by Koffee (PBK) is a new campus coffee store. PBK uses 50 bags of whole bean coffee every month, and you may assume that demand is perfectly steady throughout the year.

PBK has signed a year-long contract to purchase its coffee from a local supplier, Phish Roasters, for a price of $25 per bag and an $85 fixed cost for every delivery independent

(* indicates that the solution is at the end of the book)

of the order size. The holding cost due to storage is $1 per bag per month. PBK managers figure their cost of capital is approximately 2 percent per month.

a. What is the optimal order size, in bags?
b. Given your answer in (a), how many times a year does PBK place orders?
c. Given your answer in (a), how many months of supply of coffee does PBK have on average?
d. On average, how many dollars per month does PBK spend to hold coffee (including cost of capital)?

Suppose that a South American import/export company has offered PBK a deal for the next year. PBK can buy a years' worth of coffee directly from South America for $20 per bag and a fixed cost for delivery of $500. Assume the estimated cost for inspection and storage is $1 per bag per month and the cost of capital is approximately 2 percent per month.

e. Should PBK order from Phish Roasters or the South American import/export company? Quantitatively justify your answer.

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 8

Variability and Its Impact on Process Performance: Waiting Time Problems

For consumers, one of the most visible—and probably annoying—forms of supply–demand mismatches is waiting time. As consumers, we seem to spend a significant portion of our life waiting in line, be it in physical lines (supermarkets, check-in at airports) or in "virtual" lines (listening to music in a call center, waiting for a response e-mail).

It is important to distinguish between different types of waiting time:

- Waiting time predictably occurs when the expected demand rate exceeds the expected supply rate for some limited period of time. This happens especially in cases of constant capacity levels and demand that exhibits seasonality. This leads to implied utilization levels of over 100 percent for some time period. Queues forming at the gate of an airport after the flight is announced are an example of such queues.
- As we will see in the next section, in the presence of variability, queues also can arise if the implied utilization is below 100 percent. Such queues can thereby be fully attributed to the presence of variability, as there exists, on average, enough capacity to meet demand.

While the difference between these two types of waiting time probably does not matter much to the customer, it is of great importance from the perspective of operations management. The root cause for the first type of waiting time is a capacity problem; variability is only a secondary effect. Thus, when analyzing this type of a problem, we first should use the tools outlined in Chapters 3, 4, and 7 instead of focusing on variability.

The root cause of the second type of waiting time is variability. This makes waiting time unpredictable, both from the perspective of the customer as well as from the perspective of the operation. Sometimes, it is the customer (demand) waiting for service (supply) and, sometimes, it is the other way around. Demand just never seems to match supply in these settings.

Analyzing waiting times and linking these waiting times to variability require the introduction of new analytical tools, which we present in this chapter. We will discuss the tools for analyzing waiting times based on the example of An-ser Services, a call-center operation in

Wisconsin that specializes in providing answering services for financial services, insurance companies, and medical practices. Specifically, the objective of this chapter is to

- Predict waiting times and derive some performance metrics capturing the service quality provided to the customer.
- Recommend ways of reducing waiting time by choosing appropriate capacity levels, redesigning the service system, and outlining opportunities to reduce variability.

8.1 Motivating Example: A Somewhat Unrealistic Call Center

For illustrative purposes, consider a call center with just one employee from 7 A.M. to 8 A.M. Based on prior observations, the call-center management estimates that, on average, a call takes 4 minutes to complete (e.g., giving someone driving directions) and there are, on average, 12 calls arriving in a 60-minute period, that is, on average, one call every 5 minutes.

What will be the average waiting time for a customer before talking to a customer service representative? From a somewhat naïve perspective, there should be no waiting time at all. Since the call center has a capacity of serving 60/4 = 15 calls per hour and calls arrive at a rate of 12 calls per hour, supply of capacity clearly exceeds demand. If anything, there seems to be excess service capacity in the call center since its utilization, which we defined previously (Chapter 3) as the ratio between flow rate and capacity, can be computed as

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}} = \frac{12 \text{ calls per hour}}{15 \text{ calls per hour}} = 80\%$$

First, consider the arrivals and processing times as depicted in Figure 8.1. A call arrives exactly every 5 minutes and then takes exactly 4 minutes to be served. This is probably the weirdest call center that you have ever seen! No need to worry, we will return to "real operations" momentarily, but the following thought experiment will help you grasp how variability can lead to waiting time.

Despite its almost robotlike processing times and the apparently very disciplined customer service representative ("sorry, 4 minutes are over; thanks for your call"), this call center has one major advantage: no incoming call ever has to wait.

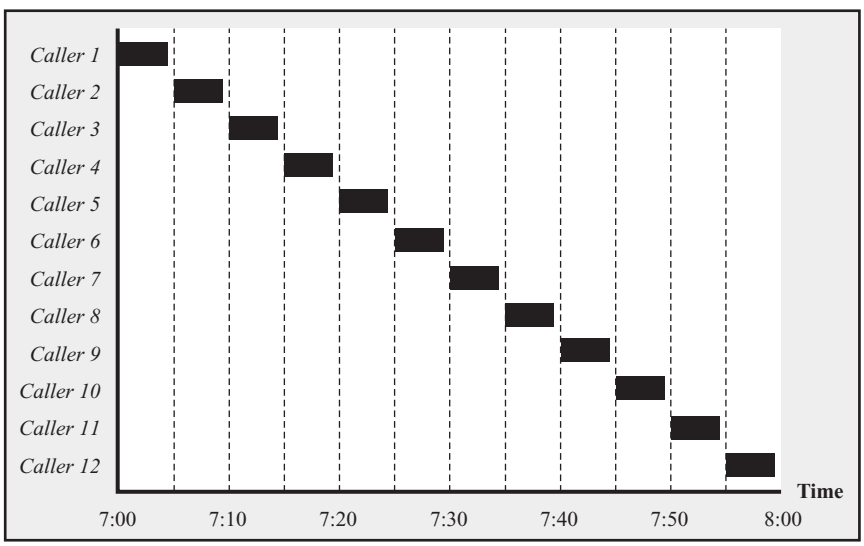
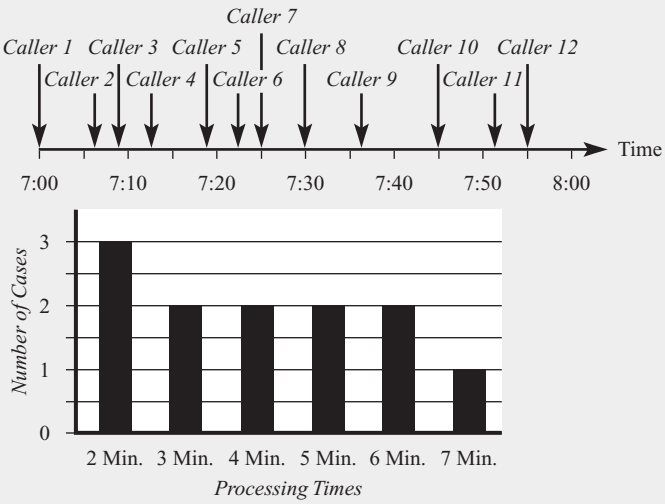


FIGURE 8.1
A Somewhat Odd Service Process

FIGURE 8.2
Data Gathered at a Call Center

Caller	Arrival Time	Processing Time
1	0	5
2	7	6
3	9	7
4	12	6
5	18	5
6	22	2
7	25	4
8	30	3
9	36	4
10	45	2
11	51	2
12	55	3



Assuming that calls arrive like kick scooters at an assembly line and are then treated by customer service representatives that act like robots reflects a common mistake managers make when calculating process performance. These calculations look at the process at an aggregate level and consider how much capacity is available over the entire hour (day, month, quarter), yet ignore how the requests for service are spaced out within the hour.

If we look at the call center on a minute-by-minute basis, a different picture emerges. Specifically, we observe that calls do not arrive like kick scooters appear at the end of the assembly line, but instead follow a much less systematic pattern, which is illustrated by Figure 8.2.

Moreover, a minute-by-minute analysis also reveals that the actual service durations also vary across calls. As Figure 8.2 shows, while the average processing time is 4 minutes, there exist large variations across calls, and the actual processing times range from 2 minutes to 7 minutes.

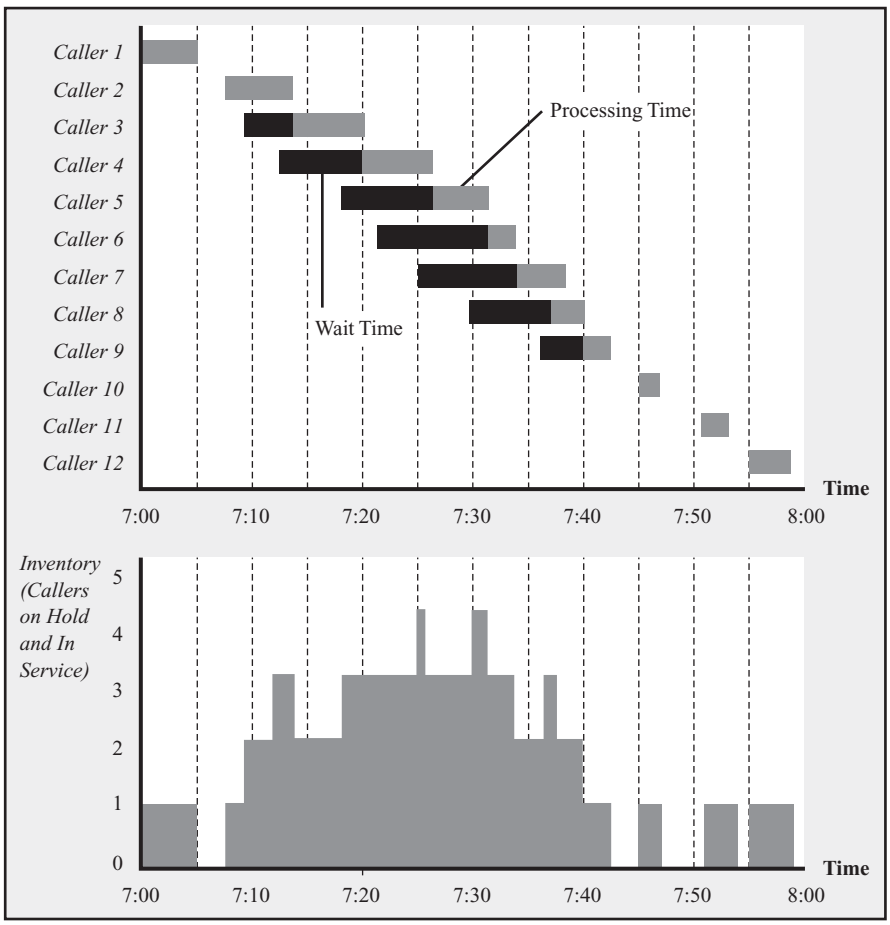
Now, consider how the hour from 7:00 A.M. to 8:00 A.M. unfolds. As can be seen in Figure 8.2, the first call comes in at 7:00 A.M. This call will be served without waiting time, and it takes the customer service representative 5 minutes to complete the call. The following 2 minutes are idle time from the perspective of the call center (7:05–7:07). At 7:07, the second call comes in, requiring a 6-minute processing time. Again, the second caller does not have to wait and will leave the system at 7:13. However, while the second caller is being served, at 7:09 the third caller arrives and now needs to wait until 7:13 before beginning the service.

Figure 8.3 shows the waiting time and processing time for each of the 12 customers calling between 7:00 A.M. and 8:00 A.M. Specifically, we observe that

- Most customers do have to wait a considerable amount of time (up to 10 minutes) before being served. This waiting occurs, although, on average, there is plenty of capacity in the call center.
- The call center is not able to provide a consistent service quality, as some customers are waiting, while others are not.
- Despite long waiting times and—because of Little's Law—long queues (see lower part of Figure 8.3), the customer service representative incurs idle time repeatedly over the time period from 7 A.M. to 8 A.M.

Why does variability not average out over time? The reason for this is as follows. In the call center example, the customer service representative can only serve a customer if there is capacity *and* demand at the same moment in time. Therefore, capacity can never "run

FIGURE 8.3
Detailed Analysis of Call Center



ahead" of demand. However, demand can "run ahead" of capacity, in which case the queue builds up. The idea that inventory can be used to decouple the supply process from demand, thereby restoring the flow rate to the level achievable in the absence of variability, is another version of the "buffer or suffer" principle that we already encountered in Chapter 7. Thus, if a service organization attempts to achieve the flow-rate levels feasible based on averages, long waiting times will result (unfortunately, in those cases, it is the customer who gets "buffered" and "suffers").

Taking the perspective of a manager attempting to match supply and demand, our objectives have not changed. We are still interested in calculating the three fundamental performance measures of an operation: inventory, flow rate, and flow time. Yet, as the above example illustrated, we realize that the process analysis tools we have discussed up to this point in the book need to be extended to appropriately deal with variability.

8.2 Variability: Where It Comes From and How It Can Be Measured

As a first step toward restoring our ability to understand a process's basic performance measures in the presence of variability, we take a more detailed look at the concept of variability itself. Specifically, we are interested in the sources of variability and how to measure variability.

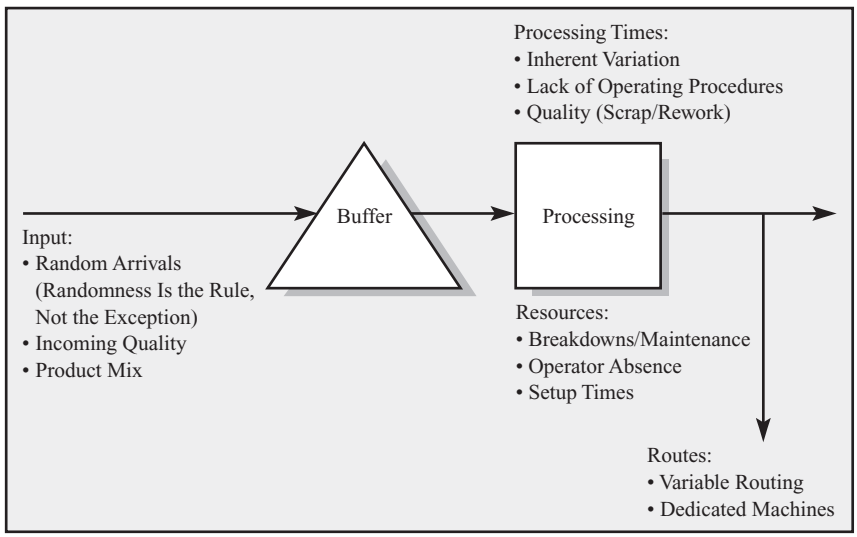


FIGURE 8.4
Variability and Where It Comes From

Why is there variability in a process to begin with? Drawing a simple (the most simple) process flow diagram suggests the following four sources of variability (these four sources are summarized in Figure 8.4):

• Variability from the inflow of flow units. The biggest source of variability in service organizations comes from the market itself. While some patterns of the customer-arrival process are predictable (e.g., in a hotel there are more guests checking out between 8 A.M. and 9 A.M. than between 2 P.M. and 3 P.M.), there always remains uncertainty about when the next customer will arrive.

• Variability in processing times. Whenever we are dealing with human operators at a resource, it is likely that there will be some variability in their behavior. Thus, if we would ask a worker at an assembly line to repeat a certain activity 100 times, we would probably find that some of these activities were carried out faster than others. Another source of variability in processing times that is specific to a service environment is that in most service operations, the customer him/herself is involved in many of the tasks constituting the processing time. At a hotel front desk, some guests might require extra time (e.g., the guest requires an explanation for items appearing on his or her bill), while others check out faster (e.g., simply use the credit card that they used for the reservation and only return their room key).

• Random availability of resources. If resources are subject to random breakdowns, for example, machine failures in manufacturing environments or operator absenteeism in service operations, variability is created.

• Random routing in case of multiple flow units in the process. If the path a flow unit takes through the process is itself random, the arrival process at each individual resource is subject to variability. Consider, for example, an emergency room in a hospital. Following the initial screening at the admissions step, incoming patients are routed to different resources. A nurse might handle easy cases, more complex cases might be handled by a general doctor, and severe cases are brought to specific units in the hospital (e.g., trauma center). Even if arrival times and processing times are deterministic, this random routing alone is sufficient to introduce variability.

In general, any form of variability is measured based on the standard deviation. In our case of the call center, we could measure the variability of call durations based on collecting

some data and then computing the corresponding standard deviation. The problem with this approach is that the standard deviation provides an *absolute* measure of variability. Does a standard deviation of 5 minutes indicate a high variability? A 5-minute standard deviation for call durations (processing times) in the context of a call center seems like a large number. In the context of a 2-hour surgery in a trauma center, a 5-minute standard deviation seems small.

For this reason, it is more appropriate to measure variability in *relative* terms. Specifically, we define the *coefficient of variation* of a random variable as

$$\text{Coefficient of variation} = \text{CV} = \frac{\text{Standard deviation}}{\text{Mean}}$$

As both the standard deviation and the mean have the same measurement units, the coefficient of variation is a unitless measure.

8.3 Analyzing an Arrival Process

Any process analysis we perform is only as good as the information we feed into our analysis. For this reason, Sections 8.3 and 8.4 focus on data collection and data analysis for the upcoming mathematical models. As a manager intending to apply some of the following tools, this data analysis is essential. However, as a student with only a couple of hours left to the final exam, you might be better off jumping straight to Section 8.5.

Of particular importance when dealing with variability problems is an accurate representation of the demand, which determines the timing of customer arrivals.

Assume we got up early and visited the call center of An-ser; say we arrived at their offices at 6:00 A.M. and we took detailed notes of what takes place over the coming hour. We would hardly have had the time to settle down when the first call comes in. One of the An-ser staff takes the call immediately. Twenty-three seconds later, the second call comes in; another 1:24 minutes later, the third call; and so on.

We define the time at which An-ser receives a call as the *arrival time.* Let AT_i denote the arrival time of the *i*th call. Moreover, we define the time between two consecutive arrivals as the *interarrival time,* IA. Thus, $IA_i = AT_{i+1} - AT_i$. Figure 8.5 illustrates these two definitions.

If we continue this data collection, we accumulate a fair number of arrival times. Such data are automatically recorded in call centers, so we could simply download a file that looks like Table 8.1.

Before we can move forward and introduce a mathematical model that predicts the effects of variability, we have to invest in some simple, yet important, data analysis. A major risk related to any mathematical model or computer simulation is that these tools always provide

FIGURE 8.5 **The Concept of Interarrival Times**

Call	Arrival Time, AT_i	Interarrival Time, $IA_i = AT_{i+1} - AT_i$
1	6:00:29	00:23
2	6:00:52	01:24
3	6:02:16	00:34
4	6:02:50	02:24
5	6:05:14	00:36
6	6:05:50	00:38
7	6:06:28	

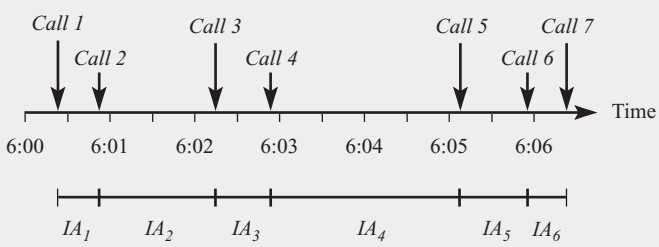


TABLE 8.1 **Call Arrivals at An-ser on April 2, from 6:00 A.M. to 10:00 A.M.**

6:00:29	6:52:39	7:17:57	7:33:51	7:56:16	8:17:33	8:28:11	8:39:25	8:55:56	9:21:58
6:00:52	6:53:06	7:18:10	7:34:05	7:56:24	8:17:42	8:28:12	8:39:47	8:56:17	9:22:02
6:02:16	6:53:07	7:18:17	7:34:19	7:56:24	8:17:50	8:28:13	8:39:51	8:57:42	9:22:02
6:02:50	6:53:24	7:18:38	7:34:51	7:57:39	8:17:52	8:28:17	8:40:02	8:58:45	9:22:30
6:05:14	6:53:25	7:18:54	7:35:10	7:57:51	8:17:54	8:28:43	8:40:09	8:58:49	9:23:13
6:05:50	6:54:18	7:19:04	7:35:13	7:57:55	8:18:03	8:28:59	8:40:23	8:58:49	9:23:29
6:06:28	6:54:24	7:19:40	7:35:21	7:58:26	8:18:12	8:29:06	8:40:34	8:59:32	9:23:45
6:07:37	6:54:36	7:19:41	7:35:44	7:58:41	8:18:21	8:29:34	8:40:35	8:59:38	9:24:10
6:08:05	6:55:06	7:20:10	7:35:59	7:59:12	8:18:23	8:29:38	8:40:46	8:59:45	9:24:30
6:10:16	6:55:19	7:20:11	7:36:37	7:59:20	8:18:34	8:29:40	8:40:51	9:00:14	9:24:42
6:12:13	6:55:31	7:20:26	7:36:45	7:59:22	8:18:46	8:29:45	8:40:58	9:00:52	9:25:07
6:12:48	6:57:25	7:20:27	7:37:07	7:59:22	8:18:53	8:29:46	8:41:12	9:00:53	9:25:15
6:14:04	6:57:38	7:20:38	7:37:14	7:59:36	8:18:54	8:29:47	8:41:26	9:01:09	9:26:03
6:14:16	6:57:44	7:20:52	7:38:01	7:59:50	8:18:58	8:29:47	8:41:32	9:01:31	9:26:04
6:14:28	6:58:16	7:20:59	7:38:03	7:59:54	8:19:20	8:29:54	8:41:49	9:01:55	9:26:23
6:17:51	6:58:34	7:21:11	7:38:05	8:01:22	8:19:25	8:30:00	8:42:23	9:02:25	9:26:34
6:18:19	6:59:41	7:21:14	7:38:18	8:01:42	8:19:28	8:30:01	8:42:51	9:02:30	9:27:02
6:19:11	7:00:50	7:21:46	7:39:00	8:01:56	8:20:09	8:30:08	8:42:53	9:02:38	9:27:04
6:20:48	7:00:54	7:21:56	7:39:17	8:02:08	8:20:23	8:30:23	8:43:24	9:02:51	9:27:27
6:23:33	7:01:08	7:21:58	7:39:35	8:02:26	8:20:27	8:30:23	8:43:28	9:03:29	9:28:25
6:24:25	7:01:31	7:23:03	7:40:06	8:02:29	8:20:44	8:30:31	8:43:47	9:03:33	9:28:37
6:25:08	7:01:39	7:23:16	7:40:23	8:02:39	8:20:54	8:31:02	8:44:23	9:03:38	9:29:09
6:25:19	7:01:56	7:23:19	7:41:34	8:02:47	8:21:12	8:31:11	8:44:49	9:03:51	9:29:15
6:25:27	7:04:52	7:23:48	7:42:20	8:02:52	8:21:12	8:31:19	8:45:05	9:04:11	9:29:52
6:25:38	7:04:54	7:24:01	7:42:33	8:03:06	8:21:25	8:31:20	8:45:10	9:04:33	9:30:47
6:25:48	7:05:37	7:24:09	7:42:51	8:03:58	8:21:28	8:31:22	8:45:28	9:04:42	9:30:58
6:26:05	7:05:39	7:24:45	7:42:57	8:04:07	8:21:43	8:31:23	8:45:31	9:04:44	9:30:59
6:26:59	7:05:42	7:24:56	7:43:23	8:04:27	8:21:44	8:31:27	8:45:32	9:04:44	9:31:03
6:27:37	7:06:37	7:25:01	7:43:34	8:05:53	8:21:53	8:31:45	8:45:39	9:05:22	9:31:55
6:27:46	7:06:46	7:25:03	7:43:43	8:05:54	8:22:19	8:32:05	8:46:24	9:06:01	9:33:08
6:29:32	7:07:11	7:25:18	7:43:44	8:06:43	8:22:44	8:32:13	8:46:27	9:06:12	9:33:45
6:29:52	7:07:24	7:25:39	7:43:57	8:06:47	8:23:00	8:32:19	8:46:40	9:06:14	9:34:07
6:30:26	7:07:46	7:25:40	7:43:57	8:07:07	8:23:02	8:32:59	8:46:41	9:06:41	9:35:15
6:30:32	7:09:17	7:25:46	7:45:07	8:07:43	8:23:12	8:33:02	8:47:00	9:06:44	9:35:40
6:30:41	7:09:34	7:25:48	7:45:32	8:08:28	8:23:30	8:33:27	8:47:04	9:06:48	9:36:17
6:30:53	7:09:38	7:26:30	7:46:22	8:08:31	8:24:04	8:33:30	8:47:06	9:06:55	9:36:37
6:30:56	7:09:53	7:26:38	7:46:38	8:09:05	8:24:17	8:33:40	8:47:15	9:06:59	9:37:23
6:31:04	7:09:59	7:26:49	7:46:48	8:09:15	8:24:19	8:33:47	8:47:27	9:08:03	9:37:37
6:31:45	7:10:29	7:27:30	7:47:00	8:09:48	8:24:26	8:34:19	8:47:40	9:08:33	9:37:38
6:33:49	7:10:37	7:27:36	7:47:15	8:09:57	8:24:39	8:34:20	8:47:46	9:09:32	9:37:42
6:34:03	7:10:54	7:27:50	7:47:53	8:10:39	8:24:48	8:35:01	8:47:53	9:10:32	9:39:03
6:34:15	7:11:07	7:27:50	7:48:01	8:11:16	8:25:03	8:35:07	8:48:27	9:10:46	9:39:10
6:36:07	7:11:30	7:27:56	7:48:14	8:11:30	8:25:04	8:35:25	8:48:48	9:10:53	9:41:37
6:36:12	7:12:02	7:28:01	7:48:14	8:11:38	8:25:07	8:35:29	8:49:14	9:11:32	9:42:58
6:37:21	7:12:08	7:28:17	7:48:50	8:11:49	8:25:16	8:36:13	8:49:19	9:11:37	9:43:27
6:37:23	7:12:18	7:28:25	7:49:00	8:12:00	8:25:22	8:36:14	8:49:20	9:11:50	9:43:37
6:37:57	7:12:18	7:28:26	7:49:04	8:12:07	8:25:31	8:36:23	8:49:40	9:12:02	9:44:09
6:38:20	7:12:26	7:28:47	7:49:48	8:12:17	8:25:32	8:36:23	8:50:19	9:13:19	9:44:21
6:40:06	7:13:16	7:28:54	7:49:50	8:12:40	8:25:32	8:36:29	8:50:38	9:14:00	9:44:32
6:40:11	7:13:21	7:29:09	7:49:59	8:12:41	8:25:45	8:36:35	8:52:11	9:14:04	9:44:37
6:40:59	7:13:22	7:29:27	7:50:13	8:12:42	8:25:48	8:36:37	8:52:29	9:14:07	9:44:44
6:42:17	7:14:04	7:30:02	7:50:27	8:12:47	8:25:49	8:37:05	8:52:40	9:15:15	9:45:10
6:43:01	7:14:07	7:30:07	7:51:07	8:13:40	8:26:01	8:37:11	8:52:41	9:15:26	9:46:15
6:43:05	7:14:49	7:30:13	7:51:31	8:13:41	8:26:04	8:37:12	8:52:43	9:15:27	9:46:44
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6:44:02	7:15:38	7:30:55	7:52:05	8:14:04	8:26:15	8:37:44	8:53:08	9:15:40	9:50:19
6:45:04	7:15:41	7:31:24	7:52:25	8:14:41	8:26:28	8:38:01	8:53:19	9:15:40	9:52:53
6:46:13	7:15:57	7:31:35	7:52:32	8:15:15	8:26:28	8:38:02	8:53:30	9:15:40	9:53:13
6:47:01	7:16:28	7:31:41	7:53:10	8:15:25	8:26:37	8:38:10	8:53:32	9:15:41	9:53:15
6:47:10	7:16:36	7:31:45	7:53:18	8:15:39	8:26:58	8:38:15	8:53:44	9:15:46	9:53:50
6:47:35	7:16:40	7:31:46	7:53:19	8:15:48	8:27:07	8:38:39	8:54:25	9:16:12	9:54:24
6:49:23	7:16:45	7:32:13	7:53:51	8:16:09	8:27:09	8:38:40	8:54:28	9:16:34	9:54:48
6:50:54	7:16:50	7:32:16	7:53:52	8:16:10	8:27:17	8:38:44	8:54:49	9:18:02	9:54:51
6:51:04	7:17:08	7:32:16	7:54:04	8:16:18	8:27:26	8:38:49	8:55:05	9:18:06	9:56:40
6:51:17	7:17:09	7:32:34	7:54:16	8:16:26	8:27:29	8:38:57	8:55:05	9:20:19	9:58:25
6:51:48	7:17:09	7:32:34	7:54:26	8:16:39	8:27:35	8:39:07	8:55:14	9:20:42	9:59:19
6:52:17	7:17:19	7:32:57	7:54:51	8:17:16	8:27:54	8:39:20	8:55:22	9:20:44	
6:52:17	7:17:22	7:33:13	7:55:13	8:17:24	8:27:57	8:39:20	8:55:25	9:20:54	
6:52:31	7:17:22	7:33:36	7:55:35	8:17:28	8:27:59	8:39:21	8:55:50	9:21:55	

us with a number (or a set of numbers), independent of the accuracy with which the inputs we enter into the equation reflect the real world.

Answering the following two questions before proceeding to any other computations improves the predictions of our models substantially.

- Is the arrival process *stationary;* that is, is the expected number of customers arriving in a certain time interval constant over the period we are interested in?
- Are the interarrival times *exponentially distributed,* and therefore form a so-called *Poisson* arrival process?

We now define the concepts of stationary arrivals and exponentially distributed interarrival times. We also describe how these two questions can be answered, both in general as well as in the specific setting of the call center described previously. We also discuss the importance of these two questions and their impact on the calculations in this and the next chapter.

Stationary Arrivals

Consider the call arrival pattern displayed in Table 8.1. How tempting it is to put these data into a spreadsheet, compute the mean and the standard deviation of the interarrival times over that time period, and end the analysis of the arrival pattern at this point, assuming that the mean and the standard deviation capture the entire behavior of the arrival process. Five minutes with Excel, and we could be done!

However, a simple graphical analysis (Figure 8.6) of the data reveals that there is more going on in the arrival process than two numbers can capture. As we can see graphically in Figure 8.6, the average number of customers calling within a certain time interval (e.g., 15 minutes) is not constant over the day.

To capture such changes in arrival processes, we introduce the following definitions:

- An arrival process is said to be *stationary* if, for any time interval (e.g., an hour), the expected number of arrivals in this time interval only depends on the length of the time interval, not on the starting time of the interval (i.e., we can move a time interval of a fixed length back and forth on a time line without changing the expected number of arrivals). In the context of Figure 8.6, we see that the arrival process is not stationary. For example, if we take a 3-hour interval, we see that there are many more customers arriving from 6 A.M. to 9 A.M. than there are from 1 A.M. to 4 A.M.
- An arrival process exhibits *seasonality* if it is not stationary.

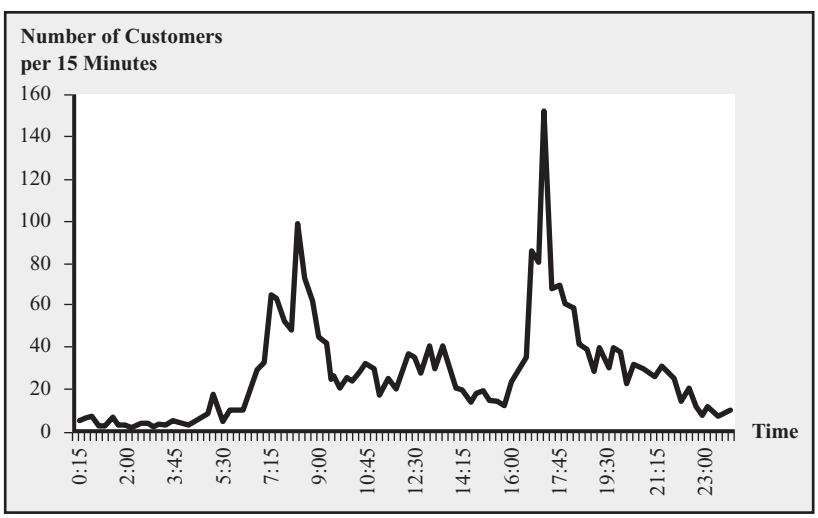
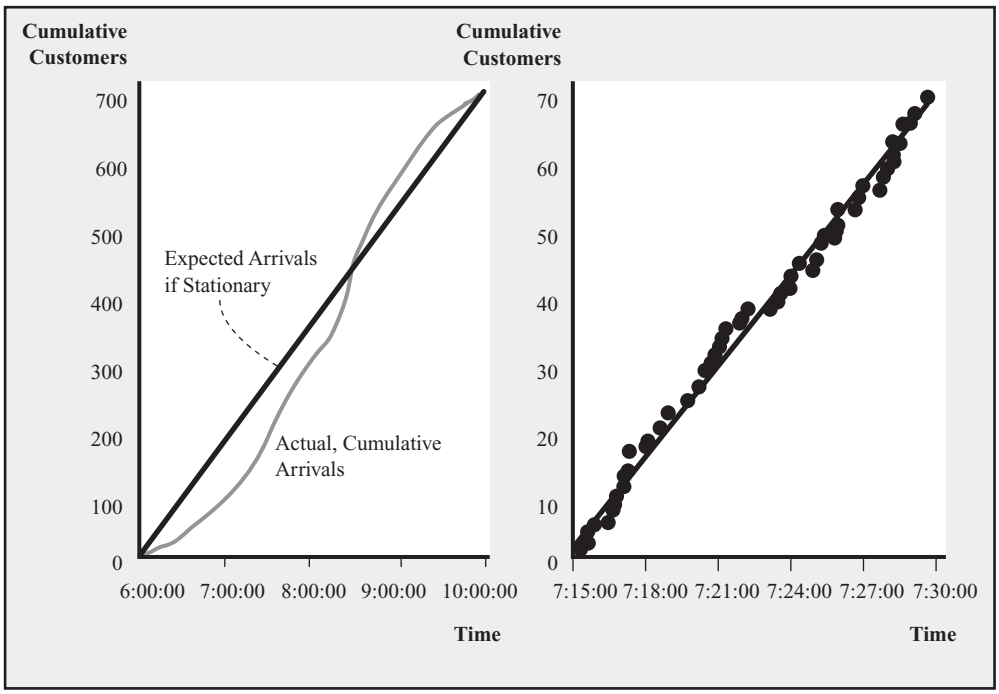


FIGURE 8.6
Seasonality over the Course of a Day

FIGURE 8.7 **Test for Stationary Arrivals**



When analyzing an arrival process, it is important that we distinguish between changes in demand (e.g., the number of calls in 15 minutes) that are a result of variability and changes in demand that are a result of seasonality. Both variability and seasonality are unpleasant from an operations perspecitive. However, the effect of seasonality alone can be perfectly predicted ex ante, while this is not possible for the case of variability (we might know the expected number of callers for a day, but the actual number is a realization of a random variable).

Based on the data at hand, we observe that the arrival process is not stationary over a period of several hours. In general, a simple analysis determines whether a process is stationary.

1. Sort all arrival times so that they are increasing in time (label them as $AT_1 \ldots AT_n$).
2. Plot a graph with (x AT_i; $y = i$) as illustrated by Figure 8.7.
3. Add a straight line from the lower left (first arrival) to the upper right (last arrival).

If the underlying arrival process is stationary, there will be no significant deviation between the graph you plotted and the straight line. In this case, however, in Figure 8.7 (left) we observe several deviations between the straight line and the arrival data. Specifically, we observe that for the first hour, fewer calls come in compared to the average arrival rate from 6 A.M. to 10 A.M. In contrast, around 8:30 A.M., the arrival rate becomes much higher than the average. Thus, our analysis indicates that the arrival process we face is not stationary.

When facing nonstationary arrival processes, the best way to proceed is to divide up the day (the week, the month) into smaller time intervals and have a separate arrival rate for each interval. If we then look at the arrival process within the smaller intervals—in our case, we use 15-minute intervals—we find that the seasonality within the interval is relatively low. In other words, within the interval, we come relatively close to a stationary arrival stream. The stationary behavior of the interarrivals within a 15-minute interval is illustrated by Figure 8.7 (right).

Figure 8.7 (left) is interesting to compare with Figure 8.7 (right): the arrival process behaves as stationary "at the micro-level" of a 15-minute interval, yet exhibits strong

seasonality over the course of the entire day, as we observed in Figure 8.6. Note that the peaks in Figure 8.6 correspond to those time slots where the line of "actual, cumulative arrivals" in Figure 8.7 grows faster than the straight line "predicted arrivals."

In most cases in practice, the context explains this type of seasonality. For example, in the case of An-ser, the spike in arrivals corresponds to people beginning their day, expecting that the company they want to call (e.g., a doctor's office) is already "up and running." However, since many of these firms are not handling calls before 9 A.M., the resulting call stream is channeled to the answering service.

Exponential Interarrival Times

Interarrival times commonly are distributed following an *exponential distribution.* If IA is a random interarrival time and the interarrival process follows an exponential distribution, we have

$$\text{Probability } \{\text{IA} \leq t\} = 1 - e^{-\frac{t}{a}}$$

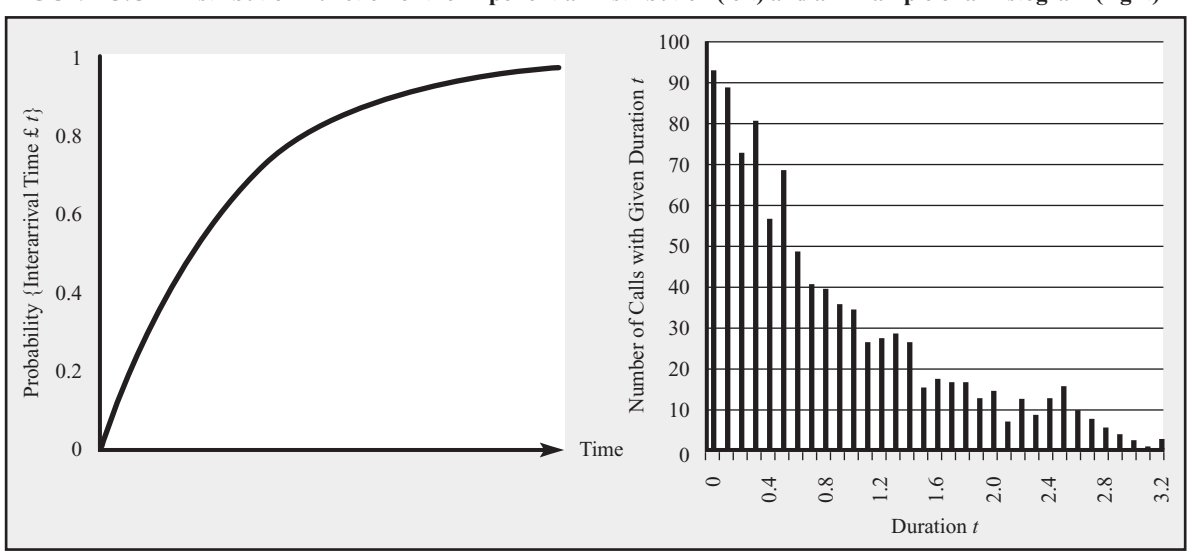
where a is the average interarrival time as defined above. Exponential functions are frequently used to model interarrival time in theory as well as practice, both because of their good fit with empirical data as well as their analytical convenience. If an arrival process has indeed exponential interarrival times, we refer to it as a *Poisson arrival process.*

It can be shown analytically that customers arriving independently from each other at the process (e.g., customers calling into a call center) form a demand pattern with exponential interarrival times. The shape of the cumulative distribution function for the exponential distribution is given in Figure 8.8. The average interarrival time is in minutes. An important property of the exponential distribution is that the standard deviation is also equal to the average, a.

Another important property of the exponential distribution is known as the *memoryless property.* The memoryless property simply states that the number of arrivals in the next time slot (e.g., 1 minute) is independent of when the last arrival has occurred.

To illustrate this property, consider the situation of an emergency room. Assume that, on average, a patient arrives every 10 minutes and no patients have arrived for the last

FIGURE 8.8 **Distribution Function of the Exponential Distribution (left) and an Example of a Histogram (right)**



20 minutes. Does the fact that no patients have arrived in the last 20 minutes increase or decrease the probability that a patient arrives in the next 10 minutes? For an arrival process with exponential interarrival times, the answer is *no.*

Intuitively, we feel that this is a reasonable assumption in many settings. Consider, again, an emergency room. Given that the population of potential patients for the ER is extremely large (including all healthy people outside the hospital), we can treat new patients as arriving independently from each other (the fact that Joan Wiley fell off her mountain bike has nothing to do with the fact that Joe Hoop broke his ankle when playing basketball).

Because it is very important to determine if our interarrival times are exponentially distributed, we now introduce the following four-step diagnostic procedure:

1. Compute the interarrival times $IA_1 \ldots IA_n$.
2. Sort the interarrival times in increasing order; let a_i denote the ith smallest interarrival time (a_1 is the smallest interarrival time; a_n is the largest).
3. Plot pairs ($x = a_i$, $y = i/n$). The resulting graph is called an empirical distribution function.
4. Compare the graph with an exponential distribution with "appropriately chosen parameter." To find the best value for the parameter, we set the parameter of the exponential distribution equal to the average interarrival time we obtain from our data. If a few observations from the sample are substantially remote from the resulting curve, we might adjust the parameter for the exponential distribution "manually" to improve fit.

Figure 8.9 illustrates the outcome of this process. If the underlying distribution is indeed exponential, the resulting graph will resemble the analytical distribution as in the case of Figure 8.9. Note that this procedure of assessing the goodness of fit works also for any other distribution function.

Nonexponential Interarrival Times

In some cases, we might find that the interarrival times are not exponentially distributed. For example, we might encounter a situation where arrivals are scheduled (e.g., every hour), which typically leads to a lower amount of variability in the arrival process.

FIGURE 8.9
Empirical versus Exponential Distribution for Interarrival Times

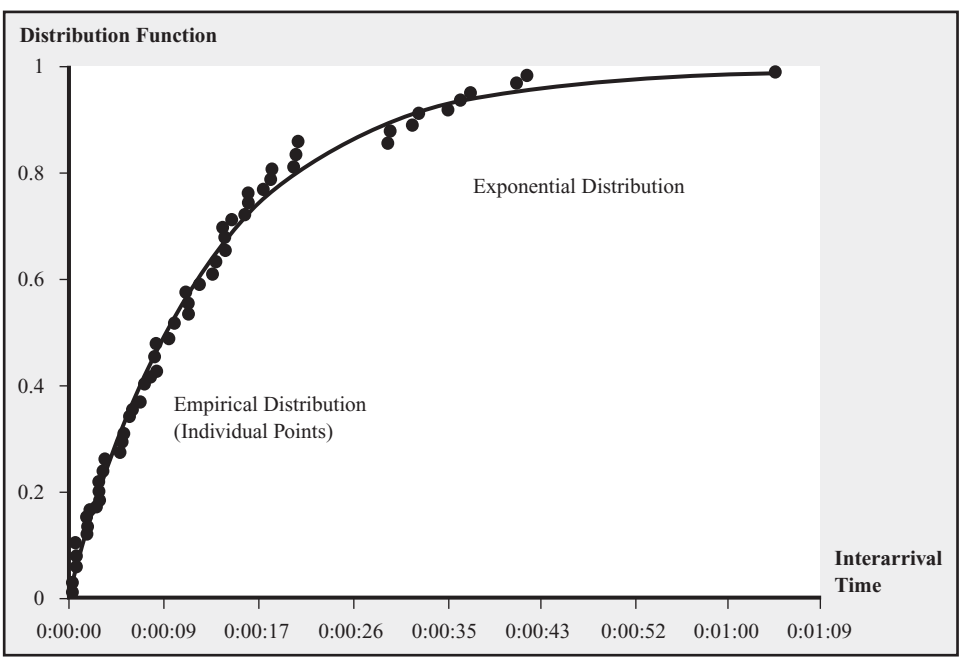
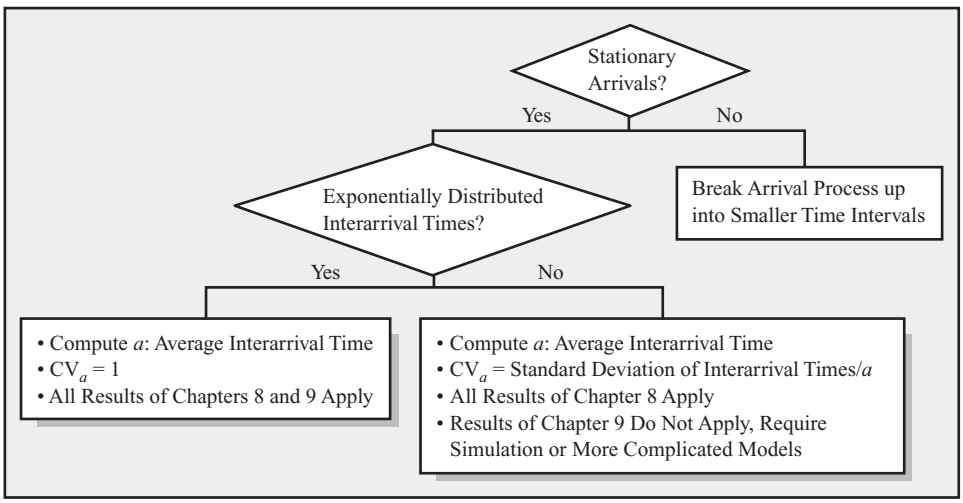


FIGURE 8.10
How to Analyze a Demand/Arrival Process



While in the case of the exponential distribution the mean interarrival time is equal to the standard deviation of interarrival times and, thus, one parameter is sufficient to characterize the entire arrival process, we need more parameters to describe the arrival process if interarrival times are not exponentially distributed.

Following our earlier definition of the coefficient of variation, we can measure the variability of an arrival (demand) process as

$$CV_a = \frac{\text{Standard deviation of interarrival time}}{\text{Average interarrival time}}$$

Given that for the exponential distribution the mean is equal to the standard deviation, its coefficient of variation is equal to 1.

Summary: Analyzing an Arrival Process

Figure 8.10 provides a summary of the steps required to analyze an arrival process. It also shows what to do if any of the assumptions required for the following models (Chapters 8 and 9) are violated.

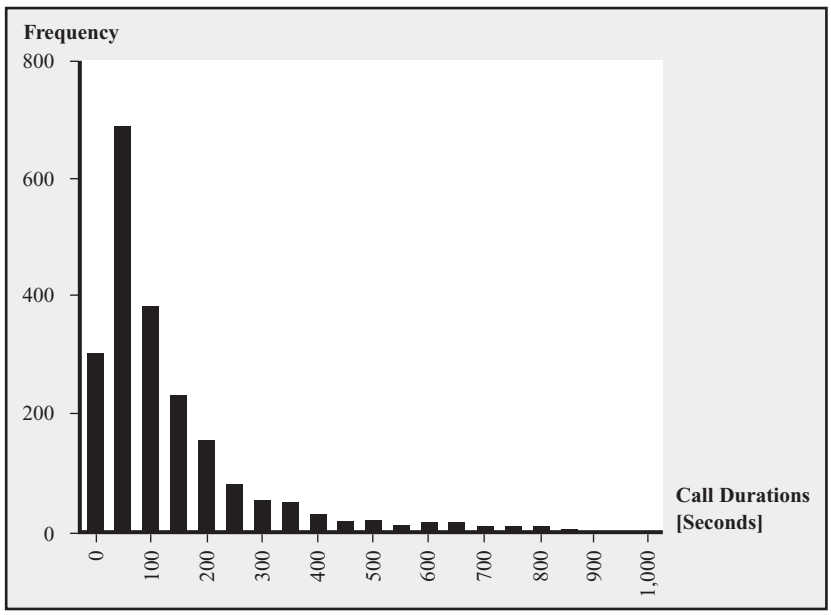
8.4 Processing Time Variability

Just as exact arrival time of an individual call is difficult to predict, so is the actual duration of the call. Thus, service processes also have a considerable amount of variability from the supply side. Figure 8.11 provides a summary of call durations for the case of the An-ser call center. From the perspective of the customer service representative, these call durations are the processing times. As mentioned previously, we will use the words *processing time, processing time,* and *processing time* interchangeably.

We observe that the variability in processing times is substantial. While some calls were completed in less than a minute, others took more than 10 minutes! Thus, in addition to the variability of demand, variability also is created within the process.

There have been reports of numerous different shapes of processing time distributions. For the purposes of this book, we focus entirely on their mean and standard deviation. In other words, when we collect data, we do not explicitly model the distribution of the processing times, but assume that the mean and standard deviation capture all the relevant information. This information is sufficient for all computations in Chapters 8 and 9.

FIGURE 8.11
Processing Times in Call Center

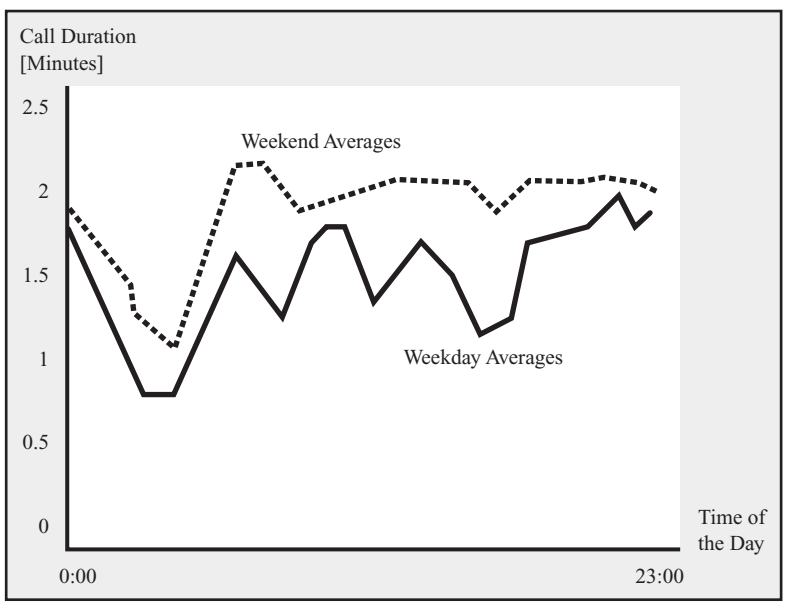


Based on the data summarized in Figure 8.11, we compute the mean processing time as 120 seconds and the corresponding standard deviation as 150 seconds. As we have done with the interarrival times, we can now define the coefficient of variation, which we obtain by

$$\text{CV}_p = \frac{\text{Standard deviation of processing time}}{\text{Average processing time}}$$

Here, the subscript p indicates that the CV measures the variability in the processing times.As with the arrival process, we need to be careful not to confuse variability with seasonality. Seasonality in processing times refers to known patterns of call durations as a function of the day of the week or the time of the day (as Figure 8.12 shows, calls take significantly longer on weekends than during the week). Call durations also differ depending on the time of the day.

FIGURE 8.12
Average Call Durations: Weekday versus Weekend



The models we introduce in Chapters 8 and 9 require a stationary service process (in the case of seasonality in the service process, just divide up the time line into smaller intervals, similar to what we did with the arrival process) but do not require any other properties (e.g., exponential distribution of processing time). Thus, the standard deviation and mean of the processing time are all we need to know.

8.5 Predicting the Average Waiting Time for the Case of One Resource

Based on our measures of variability, we now introduce a simple formula that restores our ability to predict the basic process performance measures: inventory, flow rate, and flow time.

In this chapter, we restrict ourselves to the most basic process diagram, consisting of one buffer with unlimited space and one single resource. This process layout corresponds to the call center example discussed above. Figure 8.13 shows the process flow diagram for this simple system.

Flow units arrive to the system following a demand pattern that exhibits variability. On average, a flow unit arrives every a time units. We labeled a as the average interarrival time. This average reflects the mean of interarrival times IA_1 to IA_n. After computing the standard deviation of the IA_1 to IA_n interarrival times, we can compute the coefficient of variation CV_a of the arrival process as discussed previously.

Assume that it takes on average p units of time to serve a flow unit. Similar to the arrival process, we can define p_1 to p_n as the empirically observed processing times and compute the coefficient of variation for the processing times, CV_p, accordingly. Given that there is only one single resource serving the arriving flow units, the capacity of the server can be written as $1/p$.

As discussed in the introduction to this chapter, we are considering cases in which the capacity exceeds the demand rate; thus, the resulting utilization is strictly less than 100 percent. If the utilization were above 100 percent, inventory would predictably build up and we would not need any sophisticated tools accounting for variability to predict that flow units will incur waiting times. However, the most important insight of this chapter is that flow units incur waiting time even if the server utilization is below 100 percent.

Given that capacity exceeds demand and assuming we never lose a customer (i.e., once a customer calls, he or she never hangs up), we are demand-constrained and, thus, the flow rate R is the demand rate. (Chapter 9 deals with the possibility of lost customers.) Specifically, since a customer arrives, on average, every a units of time, the flow rate $R = 1/a$. Recall that we can compute utilization as

$$\text{Utilization} = \frac{\text{Flow rate}}{\text{Capacity}} = \frac{1/a}{1/p} = p/a < 100\%$$

Note that, so far, we have not applied any concept that went beyond the deterministic process analysis we discussed in Chapters 3 to 7.

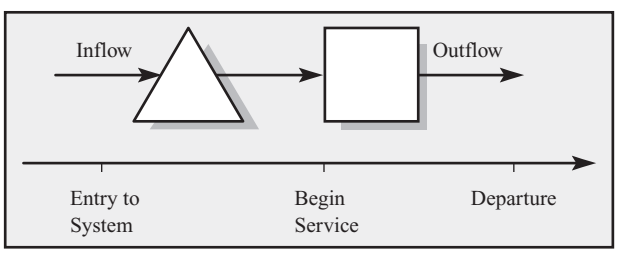
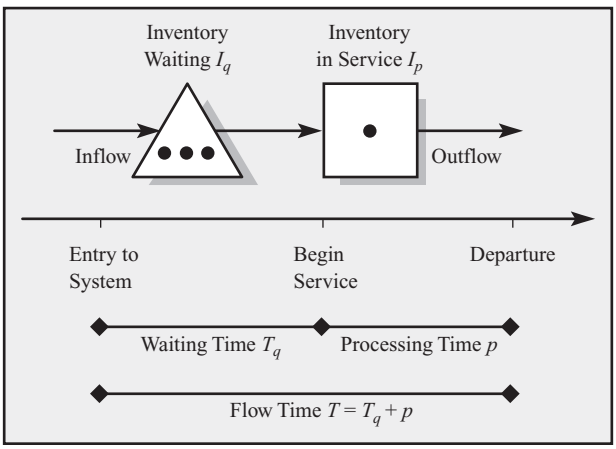


FIGURE 8.13
A Simple Process with One Queue and One Server

FIGURE 8.14
A Simple Process with One Queue and One Server



Now, take the perspective of a flow unit moving through the system (see Figure 8.14). A flow unit can spend time waiting in the queue (in a call center, this is the time when you listen to Music of the '70s). Let T_q denote the time the flow unit has to spend in the queue waiting for the service to begin. The subscript q denotes that this is only the time the flow unit waits in the queue. Thus, T_q does *not* include the actual processing time, which we defined as p. Based on the waiting time in the queue T_q and the average processing time p, we can compute the flow time (the time the flow unit will spend in the system) as

$$\text{Flow time} = \text{Time in queue} + \text{Processing time}$$
$$T = T_q + p$$

Instead of taking the perspective of the flow unit, we also can look at the system as a whole, wondering how many flow units will be in the queue and how many will be in service. Let I_q be defined as the inventory (number of flow units) that are in the queue and I_p be the number of flow units in process. Since the inventory in the queue I_q and the inventory in process I_p are the only places we can find inventory, we can compute the overall inventory in the system as $I = I_q + I_p$.

As long as there exists only one resource, I_p is a number between zero and one: sometimes there is a flow unit in service ($I_p = 1$); sometimes there is not ($I_p = 0$). The probability that at a random moment in time the server is actually busy, working on a flow unit, corresponds to the utilization. For example, if the utilization of the process is 30 percent, there exists a .3 probability that at a random moment in time the server is busy. Alternatively, we can say that over the 60 minutes in an hour, the server is busy for

$$.3 \times 60\ [\text{minutes/hour}] = 18 \text{ minutes}$$

While the inventory in service I_p and the processing time p are relatively easy to compute, this is unfortunately not the case for the inventory in the queue I_q or the waiting time in the queue T_q.

Based on the processing time p, the utilization, and the variability as measured by the coefficients of variation for the interarrival time CV_a and the processing time CV_p, we can compute the average waiting time in the queue using the following formula:

$$\text{Time in queue} = \text{Processing time} \times \left(\frac{\text{Utilization}}{1 - \text{Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

The formula does not require that the processing times or the interarrival times follow a specific distribution. Yet, for the case of nonexponential interarrival times, the formula

only approximates the expected time in the queue, as opposed to being 100 percent exact. The formula should be used only for the case of a stationary process (see Section 8.3 for the definition of a stationary process as well as for what to do if the process is not stationary).

The above equation states that the waiting time in the queue is the product of three factors:

- The waiting time is expressed as multiples of the processing time. However, it is important to keep in mind that the processing time also directly influences the utilization (as Utilization = Processing time/Interarrival time). Thus, one should not think of the waiting time as increasing linearly with the processing time.
- The second factor captures the utilization effect. Note that the utilization has to be less than 100 percent. If the utilization is equal to or greater than 100 percent, the queue continues to grow. This is not driven by variability, but simply by not having the requested capacity. We observe that the utilization factor is nonlinear and becomes larger and larger as the utilization level is increased closer to 100 percent. For example, for Utilization = 0.8, the utilization factor is $0.8/(1 - 0.8) = 4$; for Utilization = 0.9, it is $0.9/(1 - 0.9) = 9$; and for Utilization = 0.95, it grows to $0.95/(1 - 0.95) = 19$.
- The third factor captures the amount of variability in the system, measured by the average of the squared coefficient of variation of interarrival times CV_a and processing times CV_p. Since CV_a and CV_p affect neither the average processing time p nor the utilization u, we observe that the waiting time grows with the variability in the system.

The best way to familiarize ourselves with this newly introduced formula is to apply it and "see it in action." Toward that end, consider the case of the An-ser call center at 2:00 A.M. in the morning. An-ser is a relatively small call center and they receive very few calls at this time of the day (see Section 8.3 for detailed arrival information), so at 2:00 A.M., there is only one person handling incoming calls.

From the data we collected in the call center, we can quickly compute that the average processing time at An-ser at this time of the day is around 90 seconds. Given that we found in the previous section that the processing time does depend on the time of the day, it is important that we use the processing time data representative for these early morning hours: Processing time $p = 90$ seconds.

Based on the empirical processing times we collected in Section 8.4, we now compute the standard deviation of the processing time to be 120 seconds. Hence, the coefficient of variation for the processing time is

$$CV_p = 120 \text{ seconds}/90 \text{ seconds} = 1.3333$$

From the arrival data we collected (see Figure 8.6), we know that at 2:00 A.M. there are 3 calls arriving in a 15-minute interval. Thus, the interarrival time is $a = 5$ minutes $= 300$ seconds. Given the processing time and the interarrival time, we can now compute the utilization as

$$\begin{aligned} \text{Utilization} &= \text{Processing time/Interarrival time } (= p/a) \\ &= 90 \text{ seconds}/300 \text{ seconds} = 0.3 \end{aligned}$$

Concerning the coefficient of variation of the interarrival time, we can take one of two approaches. First, we could take the observed interarrival times and compute the standard deviation empirically. Alternatively, we could view the arrival process during the time period as random. Given the good fit between the data we collected and the exponential distribution (see Figure 8.9), we assume that arrivals follow a Poisson process (interarrival times are exponentially distributed). This implies a coefficient of variation of

$$CV_a = 1$$

Substituting these values into the waiting time formula yields

$$\begin{aligned}\text{Time in queue} &= \text{Processing time} \times \left(\frac{\text{Utilization}}{1 - \text{Utilization}}\right) \times \left(\frac{\text{CV}_a^2 + \text{CV}_p^2}{2}\right)\\ &= 90 \times \frac{0.3}{1 - 0.3} \times \frac{1^2 + 1.3333^2}{2}\\ &= 53.57 \text{ seconds}\end{aligned}$$

Note that this result captures the average waiting time of a customer before getting served. To obtain the customer's total time spent for the call, including waiting time and processing time, we need to add the processing time p for the actual service. Thus, the flow time can be computed as

$$T = T_q + p = 53.57 \text{ seconds} + 90 \text{ seconds} = 143.57 \text{ seconds}$$

It is important to point out that the value 53.57 seconds provides the average waiting time. The actual waiting times experienced by individual customers vary. Some customers get lucky and receive service immediately; others have to wait much longer than 53.57 seconds. This is discussed further below.

Waiting times computed based on the methodology outlined above need to be seen as long-run averages. This has the following two practical implications:

- If the system would start empty (e.g., in a hospital lab, where there are no patients before the opening of the waiting room), the first couple of patients are less likely to experience significant waiting time. This effect is transient: Once a sufficient number of patients have arrived, the system reaches a "steady-state." Note that given the 24-hour operation of An-ser, this is not an issue in this specific case.
- If we observe the system for a given time interval, it is unlikely that the average waiting time we observe within this interval is exactly the average we computed. However, the longer we observe the system, the more likely the expected waiting time T_q will indeed coincide with the empirical average. This resembles a casino, which cannot predict how much money a specific guest will win (or typically lose) in an evening, yet can well predict the economics of the entire guest population over the course of a year.

Now that we have accounted for the waiting time T_q (or the flow time T), we are able to compute the resulting inventory. With $1/a$ being our flow rate, we can use Little's Law to compute the average inventory I as

$$\begin{aligned}I &= R \times T = \frac{1}{a} \times (T_q + p)\\ &= 1/300 \times (53.57 + 90) = 0.479\end{aligned}$$

Thus, there is, on average, about half a customer in the system (it is 2:00 A.M. after all . . .). This inventory includes the two subsets we defined as inventory in the queue (I_q) and inventory in process (I_p):

- I_q can be obtained by applying Little's Law, but this time, rather than applying Little's Law to the entire system (the waiting line and the server), we apply it only to the waiting line in isolation. If we think of the waiting line as a mini process in itself (the corresponding process flow diagram consists only of one triangle), we obtain a flow time of T_q. Hence,

$$I_q = 1/a \times T_q = 1/300 \times 53.57 = 0.179$$

• At any given moment in time, we also can look at the number of customers that are currently talking to the customer service representative. Since we assumed there would only be one representative at this time of the day, there will never be more than one caller at this stage. However, there are moments in time when no caller is served, as the utilization of the employee is well below 100 percent. The average number of callers in service can thus be computed as

$$\begin{aligned} I_p &= \text{Probability}\{0 \text{ callers talking to representative}\} \times 0 \\ &\quad + \text{Probability}\{1 \text{ caller talking to representative}\} \times 1 \\ I_p &= (1-u) \times 0 + u \times 1 = u \end{aligned}$$

In this case, we obtain $I_p = 0.3$.

8.6 Predicting the Average Waiting Time for the Case of Multiple Resources

After analyzing waiting time in the presence of variability for an extremely simple process, consisting of just one buffer and one resource, we now turn to more complicated operations. Specifically, we analyze a waiting time model of a process consisting of one waiting area (queue) and a process step performed by multiple, identical resources.

We continue our example of the call center. However, now we consider time slots at more busy times over the course of the day, when there are many more customer representatives on duty in the An-ser call center. The basic process layout is illustrated in Figure 8.15.

Let m be the number of parallel servers we have available. Given that we have m servers working in parallel, we now face a situation where the average processing time is likely to be much longer than the average interarrival time. Taken together, the m resources have a capacity of m/p, while the demand rate continues to be given by $1/a$. We can compute the utilization u of the service process as

$$\begin{aligned} \text{Utilization} &= \frac{\text{Flow rate}}{\text{Capacity}} = \frac{1/\text{Interarrival time}}{(\text{Number of resources}/\text{Processing time})} \\ &= \frac{1/a}{m/p} = \frac{p}{a \times m} \end{aligned}$$

Similar to the case with one single resource, we are only interested in the cases of utilization levels below 100 percent.

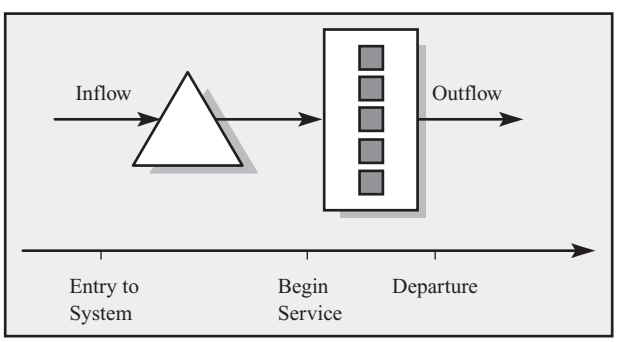


FIGURE 8.15
A Process with One Queue and Multiple, Parallel Servers ($m = 5$)

The flow unit will initially spend T_q units of time waiting for service. It then moves to the next available resource, where it spends p units of time for service. As before, the total flow time is the sum of waiting time and processing time:

$$\text{Flow time} = \text{Waiting time in queue} + \text{Processing time}$$
$$T = T_q + p$$

Based on the processing time p, the utilization u, the coefficients of variation for both service (CV_p) and arrival process (CV_a) as well as the number of resources in the system (m), we can compute the average waiting time T_q using the following formula:[1]

$$\text{Time in queue} = \left(\frac{\text{Processing time}}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}-1}}{1-\text{Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

As in the case of one single resource, the waiting time is expressed as the product of the processing time, a utilization factor, and a variability factor. We also observe that for the special case of $m = 1$, the above formula is exactly the same as the waiting time formula for a single resource. Note that all other performance measures, including the flow time (T), the inventory in the system (I), and the inventory in the queue (I_q), can be computed as discussed before.

While the above expression does not necessarily seem an inviting equation to use, it can be programmed without much effort into a spreadsheet. Furthermore, it provides the average waiting time for a system that otherwise could only be analyzed with much more sophisticated software packages.

Unlike the waiting time formula for the single resource case, which provides an exact quantification of waiting times as long as the interarrival times follow an exponential distribution, the waiting time formula for multiple resources is an approximation. The formula works well for most settings we encounter, specifically if the ratio of utilization u to the number of servers m is large (u/m is high).

Now that we have computed waiting time, we can again use Little's Law to compute the average number of flow units in the waiting area I_q, the average number of flow units in service I_p, and the average number of flow units in the entire system $I = I_p + I_q$. Figure 8.16 summarizes the key performance measures.

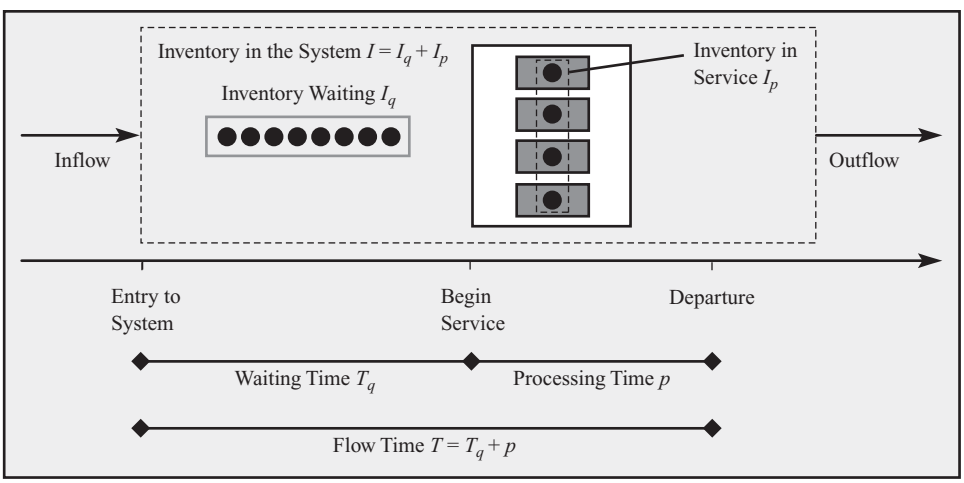


FIGURE 8.16
Summary of Key Performance Measures

[1] Hopp and Spearman (1996); the formula initially had been proposed by Sakasegawa (1977) and used successfully by Whitt (1983). For $m = 1$, the formula is exactly the same as in the previous section. The formula is an approximation for $m > 1$. An exact expression for this case does not exist.

Exhibit 8.1

SUMMARY OF WAITING TIME CALCULATIONS

1. Collect the following data:
   - Number of servers, m
   - Processing time, p
   - Interarrival time, a
   - Coefficient of variation for interarrival (CV_a) and processing time (CV_p)
2. Compute utilization: $u = \dfrac{p}{a \times m}$
3. Compute expected waiting time:

$$T_q = \left(\frac{\text{Processing time}}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}-1}}{1 - \text{Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

4. Based on T_q, we can compute the remaining performance measures as

$$\begin{aligned} \text{Flow time } T &= T_q + p \\ \text{Inventory in service } I_p &= m \times u \\ \text{Inventory in the queue } I_q &= T_q/a \\ \text{Inventory in the system } I &= I_p + I_q \end{aligned}$$

Note that in the presence of multiple resources serving flow units, there can be more than one flow unit in service simultaneously. If u is the utilization of the process, it is also the utilization of each of the m resources, as they process demand at the same rate. We can compute the expected number of flow units at any of the m resources *in isolation* as

$$u \times 1 + (1 - u) \times 0 = u$$

Adding up across the m resources then yields

$$\begin{aligned} \text{Inventory in process} &= \text{Number of resources} \times \text{Utilization} \\ I_p &= m \times u \end{aligned}$$

We illustrate the methodology using the case of An-ser services. Assuming we would work with a staff of 10 customer service representatives (CSRs) for the 8:00 A.M. to 8:15 A.M. time slot, we can compute the utilization as follows:

$$\text{Utilization } u = \frac{p}{a \times m} = \frac{90 \text{ [seconds/call]}}{11.39 \times 10 \text{ [seconds/call]}} = 0.79$$

where we obtained the interarrival time of 11.39 seconds between calls by dividing the length of the time interval (15 minutes = 900 seconds) by the number of calls received over the interval (79 calls). This now allows us to compute the average waiting time as

$$\begin{aligned} T_q &= \left(\frac{p}{m}\right) \times \left(\frac{u^{\sqrt{2(m+1)}-1}}{1 - u}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right) \\ &= \left(\frac{90}{10}\right) \times \left(\frac{0.79^{\sqrt{2(10+1)}-1}}{1 - 0.79}\right) \times \left(\frac{1 + 1.3333^2}{2}\right) = 24.98 \text{ seconds} \end{aligned}$$

The most important calculations related to waiting times caused by variability are summarized in Exhibit 8.1.

8.7 Service Levels in Waiting Time Problems

So far, we have focused our attention on the average waiting time in the process. However, a customer requesting service from our process is not interested in the average time he or she waits in queue or the average total time to complete his or her request (waiting time T_q and flow time T respectively), but in the wait times that he or she experiences personally.

Consider, for example, a caller who has just waited for 15 minutes listening to music while on hold. This caller is likely to be unsatisfied about the long wait time. Moreover, the response from the customer service representative of the type "we are sorry for your delay, but our average waiting time is only 4 minutes" is unlikely to reduce this dissatisfaction.

Thus, from a managerial perspective, we not only need to analyze the average wait time, but also the likelihood that the wait time exceeds a certain *target wait time* (*TWT*). More formally, we can define the *service level* for a given target wait time as the percentage of customers that will begin service in TWT or less units of waiting time:

$$\text{Service level} = \text{Probability}\{\text{Waiting time} \leq \text{TWT}\}$$

This service level provides us with a way to measure to what extent the service is able to respond to demand within a consistent waiting time. A service level of 95 percent for a target waiting time of TWT = 2 minutes means that 95 percent of the customers are served in less than 2 minutes of waiting time.

Figure 8.17 shows the empirical distribution function (see Section 8.3 on how to create this graph) for waiting times at the An-ser call center for a selected time slot. Based on the graph, we can distinguish between two groups of customers. About 65 percent of the customers did not have to wait at all and received immediate service. The remaining 35 percent of the customers experienced a waiting time that strongly resembles an exponential distribution.

We observe that the average waiting time for the entire calling population (not just the ones who had to wait) was, for this specific sample, about 10 seconds. For a target wait time TWT = 30 seconds, we find a service level of 90 percent; that is, 90 percent of the callers had to wait 30 seconds or less.

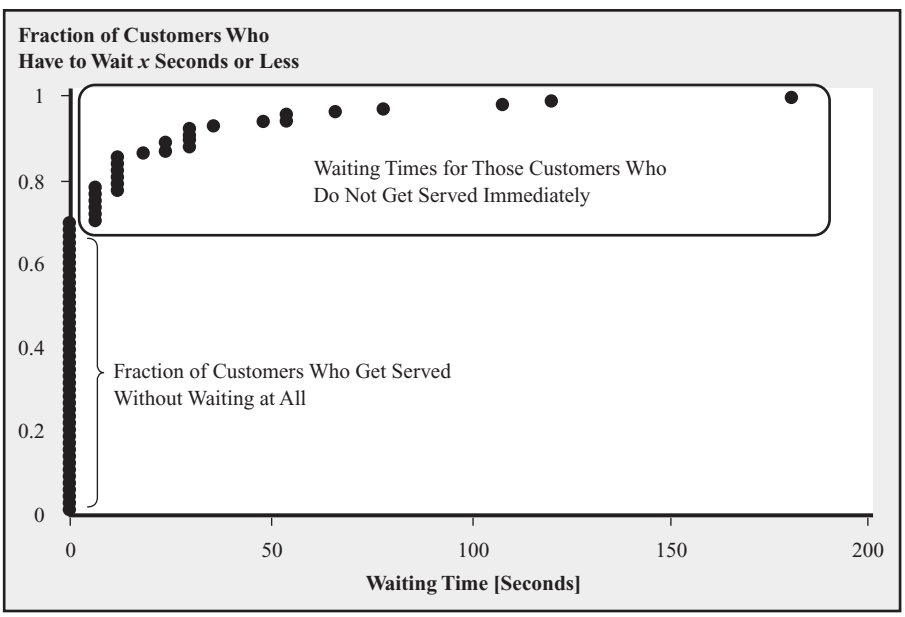


FIGURE 8.17
Empirical Distribution of Waiting Times at An-ser

Service levels as defined above are a common performance measure for service operations in practice. They are used internally by the firm in charge of delivering a certain service. They also are used frequently by firms that want to outsource a service, such as a call center, as a way to contract (and track) the responsiveness of their service provider.

There is no universal rule of what service level is right for a given service operation. For example, responding to large public pressure, the German railway system (Deutsche Bundesbahn) has recently introduced a policy that 80 percent of the calls to their customer complaint number should be handled within 20 seconds. Previously, only 30 percent of the calls were handled within 20 seconds. How fast you respond to calls depends on your market position and the importance of the incoming calls for your business. A service level that worked for the German railway system (30 percent within 20 seconds) is likely to be unacceptable in other, more competitive environments.

8.8 Economic Implications: Generating a Staffing Plan

So far, we have focused purely on analyzing the call center for a given number of customer service representatives (CSRs) on duty and predicted the resulting waiting times. This raises the managerial question of how many CSRs An-ser should have at work at any given moment in time over the day. The more CSRs we schedule, the shorter the waiting time, but the more we need to pay in terms of wages.

When making this trade-off, we need to balance the following two costs:

- Cost of waiting, reflecting increased line charges for 1-800 numbers and customer dissatisfaction (line charges are incurred for the actual talk time as well as for the time the customer is on hold).
- Cost of service, resulting from the number of CSRs available.

Additional costs that could be factored into the analysis are

- Costs related to customers calling into the call center but who are not able to gain access even to the waiting line, that is, they receive a busy signal (blocked customers; this will be discussed further in Chapter 9).
- Costs related to customers who hang up while waiting for service.

In the case of An-ser, the average salary of a CSR is $10 per hour. Note that CSRs are paid independent of being idle or busy. Variable costs for a 1-800 number are about $0.05 per minute. A summary of various costs involved in managing a call center—or service operations in general—is given by Figure 8.18.

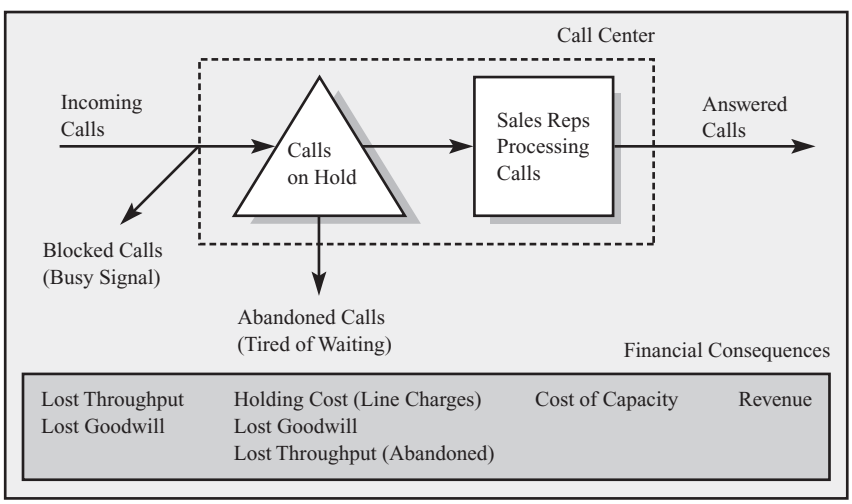


FIGURE 8.18
Economic Consequences of Waiting

TABLE 8.2
Determining the Number of CSRs to Support Target Wait Time

Number of CSRs, m	Utilization $u = p/(a \times m)$	Expected Wait Time T_q [seconds] Based on Waiting Time Formula
8	0.99	1221.23
9	0.88	72.43
10	0.79	24.98
11	0.72	11.11
12	0.66	5.50
13	0.61	2.89
14	0.56	1.58

When deciding how many CSRs to schedule for a given time slot, we first need to decide on how responsive we want to be to our customers. For the purpose of our analysis, we assume that the management of An-ser wants to achieve an average wait time of 10 seconds. Alternatively, we also could set a service level and then staff according to a TWT constraint, for example, 95 percent of customers to be served in 20 seconds or less.

Now, for a given arrival rate, we need to determine the number of CSRs that will correspond to an average wait time of 10 seconds. Again, consider the time interval from 8:00 A.M. to 8:15 A.M. Table 8.2 shows the utilization level as well as the expected wait time for different numbers of customer service representatives. Note that using fewer than 8 servers would lead to a utilization above one, which would mean that queues would build up independent of variability, which is surely not acceptable.

Table 8.2 indicates that adding CSRs leads to a reduction in waiting time. For example, while a staff of 8 CSRs would correspond to an average waiting time of about 20 minutes, the average waiting time falls below 10 seconds once a twelfth CSR has been added. Thus, working with 12 CSRs allows An-ser to meet its target of an average wait time of 10 seconds. In this case, the actual service would be even better and we expect the average wait time for this specific time slot to be 5.50 seconds.

Providing a good service level does come at the cost of increased labor. The more CSRs are scheduled to serve, the lower is their utilization. In Chapter 4 we defined the cost of direct labor as

$$\text{Cost of direct labor} = \frac{\text{Total wages per unit of time}}{\text{Flow rate per unit of time}}$$

where the total wages per unit of time are determined by the number of CSRs m times their wage rate (in our case, \$10 per hour or 16.66 cents per minute) and the flow rate is determined by the arrival rate. Therefore,

$$\text{Cost of direct labor} = \frac{m \times 16.66 \text{ cents/minute}}{1/a} = a \times m \times 16.66 \text{ cents/minute}$$

An alternative way of writing the cost of labor uses the definition of utilization ($u = p/(a \times m)$). Thus, in the above equation, we can substitute p/u for $a \times m$ and obtain

$$\text{Cost of direct labor} = \frac{p \times 16.66 \text{ cents/minute}}{u}$$

This way of writing the cost of direct labor has a very intuitive interpretation: The actual processing time p is inflated by a factor of 1/Utilization to appropriately account for idle time. For example, if utilization were 50 percent, we are charged a \$1 of idle time penalty

TABLE 8.3
Economic Implications of Various Staffing Levels

Number of Servers	Utilization	Cost of Labor per Call	Cost of Line Charges per Call	Total Cost per Call
8	0.988	0.2531	1.0927	1.3458
9	0.878	0.2848	0.1354	0.4201
10	0.790	0.3164	0.0958	0.4122
11	0.718	0.3480	0.0843	0.4323
12	0.658	0.3797	0.0796	0.4593
13	0.608	0.4113	0.0774	0.4887
14	0.564	0.4429	0.0763	0.5193
15	0.527	0.4746	0.0757	0.5503

for every \$1 we spend on labor productively. In our case, the utilization is 66 percent; thus, the cost of direct labor is

$$\text{Cost of direct labor} = \frac{1.5 \text{ minutes/call} \times 16.66 \text{ cents/minute}}{0.66} = 38 \text{ cents/call}$$

This computation allows us to extend Table 8.2 to include the cost implications of the various staffing scenarios (our calculations do not consider any cost of lost goodwill). Specifically, we are interested in the impact of staffing on the cost of direct labor per call as well as in the cost of line charges.

Not surprisingly, we can see in Table 8.3 that moving from a very high level of utilization of close to 99 percent (using 8 CSRs) to a more responsive service level, for example, as provided by 12 CSRs, leads to a significant increase in labor cost.

At the same time, though, line charges drop from over \$1 per call to almost \$0.075 per call. Note that \$0.075 per call is the minimum charge that can be achieved based on staffing changes, as it corresponds to the pure talk time.

Adding line charges and the cost of direct labor allows us to obtain total costs. In Table 8.3, we observe that total costs are minimized when we have 10 CSRs in service.

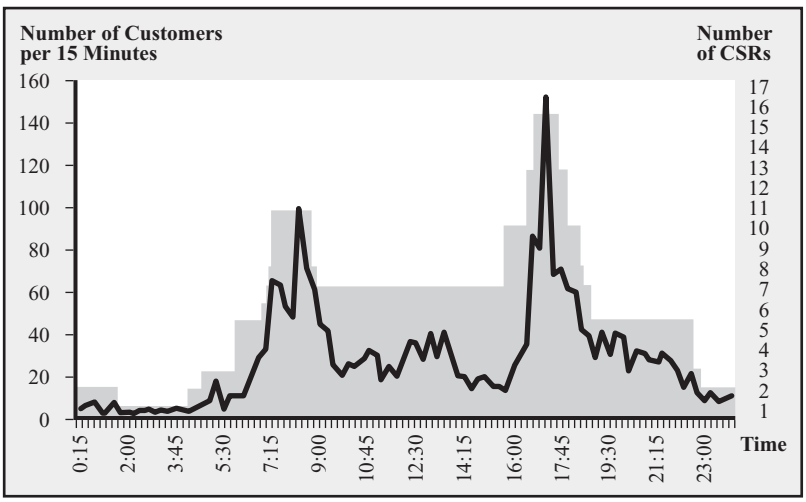
However, we need to be careful in labeling this point as the optimal staffing level, as the total cost number is a purely internal measure and does not take into account any information about the customer's cost of waiting. For this reason, when deciding on an appropriate staffing level, it is important to set acceptable service levels for waiting times as done in Table 8.2 and then staffing up to meet these service levels (opposed to minimizing internal costs).

If we repeat the analysis that we have conducted for the 8:00 A.M. to 8:15 A.M. time slot over the 24 hours of the day, we obtain a staffing plan. The staffing plan accounts for both the seasonality observed throughout the day as well as the variability and the resulting need for extra capacity. This is illustrated by Figure 8.19.

When we face a nonstationary arrival process as in this case, a common problem is to decide into how many intervals one should break up the time line to have close to a stationary arrival process within a time interval (in this case, 15 minutes). While we cannot go into the theory behind this topic, the basic intuition is this: It is important that the time intervals are large enough so that

- We have enough data to come up with reliable estimates for the arrival rate of the interval (e.g., if we had worked with 30-second intervals, our estimates for the number of calls arriving within a 30-second time interval would have been less reliable).
- Over the course of an interval, the queue needs sufficient time to reach a "steady state"; this is achieved if we have a relatively large number of arrivals and service completions within the duration of a time interval (more than 10).

FIGURE 8.19
Staffing and Incoming Calls over the Course of a Day



In practice, finding a staffing plan can be somewhat more complicated, as it needs to account for

- Breaks for the operators.
- Length of work period. It is typically not possible to request an operator to show up for work for only a one-hour time slot. Either one has to provide longer periods of time or one would have to temporarily route calls to other members of the organization (supervisor, back-office employees).

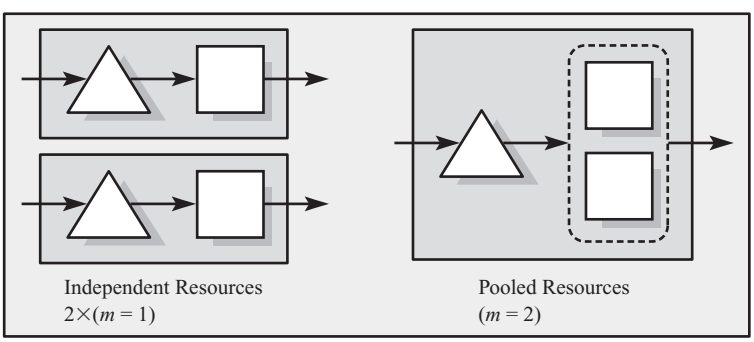
Despite these additional complications, the analysis outlined above captures the most important elements typical for making supply-related decisions in service environments.

8.9 Impact of Pooling: Economies of Scale

Consider a process that currently corresponds to two (m) demand arrival processes that are processed by two (m) identical servers. If demand cannot be processed immediately, the flow unit waits in front of the server where it initially arrived. An example of such a system is provided in Figure 8.20 (left).

Here is an interesting question: Does combining the two systems into a single system with one waiting area and two (m) identical servers lead to lower average waiting times? We refer to such a combination of multiple resources into one "mega-resource" as *pooling.*

FIGURE 8.20
The Concept of Pooling



Consider, for example, two small food services at an airport. For simplicity, assume that both of them have a customer arrival stream with an average interarrival time a of 4 minutes and a coefficient of variation equal to one. The processing time p is three minutes per customer and the coefficient of variation for the service process also is equal to one. Consequently, both food services face a utilization of $p/a = 0.75$.

Using our waiting time formula, we compute the average waiting time as

$$\begin{aligned} T_q &= \text{Processing time} \times \left(\frac{\text{Utilization}}{1 - \text{Utilization}}\right) \times \left(\frac{\text{CV}_a^2 + \text{CV}_p^2}{2}\right) \\ &= 3 \times \left(\frac{0.75}{1 - 0.75}\right) \times \left(\frac{1 + 1}{2}\right) \\ &= 3 \times (0.75/0.25) = 9 \text{ minutes} \end{aligned}$$

Now compare this with the case in which we combine the capacity of both food services to serve the demand of both services. The capacity of the pooled process has increased by a factor of two and now is $\frac{2}{3}$ unit per minute. However, the demand rate also has doubled: If there was one customer every four minutes arriving for service 1 and one customer every four minutes arriving for service 2, the pooled service experiences an arrival rate of one customer every $a = 2$ minutes (i.e., two customers every four minutes is the same as one customer every two minutes).

We can compute the utilization of the pooled process as

$$\begin{aligned} u &= \frac{p}{a \times m} \\ &= 3/(2 \times 2) = 0.75 \end{aligned}$$

Observe that the utilization has not changed compared to having two independent services. Combining two processes with a utilization of 75 percent leads to a pooled system with a 75 percent utilization. However, a different picture emerges when we look at the waiting time of the pooled system. Using the waiting time formula for multiple resources, we can write

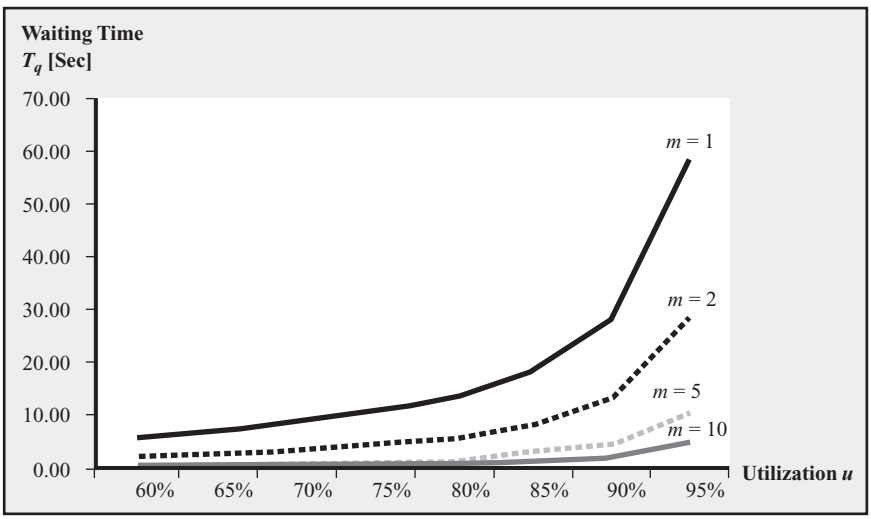
$$\begin{aligned} T_q &= \left(\frac{\text{Processing time}}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}-1}}{1 - \text{Utilization}}\right) \times \left(\frac{\text{CV}_a^2 + \text{CV}_p^2}{2}\right) \\ &= \left(\frac{3}{2}\right) \times \left(\frac{0.75^{\sqrt{2(2+1)}-1}}{1 - 0.75}\right) \times \left(\frac{1 + 1}{2}\right) = 3.95 \text{ minutes} \end{aligned}$$

In other words, the pooled process on the right of Figure 8.20 can serve the same number of customers using the same processing time (and thereby having the same utilization), but in only *half* the waiting time!

While short of being a formal proof, the intuition for this result is as follows. The pooled process uses the available capacity more effectively, as it prevents the case that one resource is idle while the other faces a backlog of work (waiting flow units). Thus, pooling identical resources balances the load for the servers, leading to shorter waiting times. This behavior is illustrated in Figure 8.21.

Figure 8.21 illustrates that for a given level of utilization, the waiting time decreases with the number of servers in the resource pool. This is especially important for higher levels of utilization. While for a system with one single server waiting times tend to "go

FIGURE 8.21
How Pooling Can Reduce Waiting Time



through the roof" once the utilization exceeds 85 percent, a process consisting of 10 identical servers can still provide reasonable service even at utilizations approaching 95 percent.

Given that a pooled system provides better service than individual processes, a service organization can benefit from pooling identical branches or work groups in one of two forms:

- The operation can use pooling to reduce customer waiting time without having to staff extra workers.
- The operation can reduce the number of workers while maintaining the same responsiveness.

These economic benefits of pooling can be illustrated nicely within the context of the An-ser case discussed above. In our analysis leading to Table 8.2, we assumed that there would be 79 calls arriving per 15-minute time interval and found that we would need 12 CSRs to serve customers with an average wait time of 10 seconds or less.

Assume we could pool An-ser's call center with a call center of comparable size; that is, we would move all CSRs to one location and merge both call centers' customer populations. Note that this would not necessarily require the two call centers to "move in" with each other; they could be physically separate as long as the calls are routed through one joint network.

Without any consolidation, merging the two call centers would lead to double the number of CSRs and double the demand, meaning 158 calls per 15-minute interval. What would be the average waiting time in the pooled call center? Or, alternatively, if we maintained an average waiting time of 10 seconds or less, how much could we reduce our staffing level? Table 8.4 provides the answers to these questions.

First, consider the row of 24 CSRs, corresponding to pooling the entire staff of the two call centers. Note specifically that the utilization of the pooled call center is not any different from what it was in Table 8.2. We have doubled the number of CSRs, but we also have doubled the number of calls (and thus cut the interarrival time by half). With 24 CSRs, we expect an average waiting time of 1.2 seconds (compared to almost 6 seconds before).

Alternatively, we could take the increased efficiency benefits resulting from pooling by reducing our labor cost. We also observe from Table 8.4 that a staff of 20 CSRs would be able to answer calls with an average wait time of 10 seconds. Thus, we could increase

TABLE 8.4
Pooling Two Call Centers

Number of CSRs	Utilization	Expected Wait Time [seconds]	Labor Cost per Call	Line Cost per Call	Total Cost
16	0.988	588.15	0.2532	0.5651	0.8183
17	0.929	72.24	0.2690	0.1352	0.4042
18	0.878	28.98	0.2848	0.0992	0.3840
19	0.832	14.63	0.3006	0.0872	0.3878
20	0.790	8.18	0.3165	0.0818	0.3983
21	0.752	4.84	0.3323	0.0790	0.4113
22	0.718	2.97	0.3481	0.0775	0.4256
23	0.687	1.87	0.3639	0.0766	0.4405
24	0.658	1.20	0.3797	0.0760	0.4558
25	0.632	0.79	0.3956	0.0757	0.4712
26	0.608	0.52	0.4114	0.0754	0.4868
27	0.585	0.35	0.4272	0.0753	0.5025
28	0.564	0.23	0.4430	0.0752	0.5182
29	0.545	0.16	0.4589	0.0751	0.5340
30	0.527	0.11	0.4747	0.0751	0.5498

utilization to almost 80 percent, which would lower our cost of direct labor from $0.3797 to $0.3165. Given an annual call volume of about 700,000 calls, such a saving would be of significant impact for the bottom line.

Despite the nice property of pooled systems outlined above, pooling should not be seen as a silver bullet. Specifically, pooling benefits are much lower than expected (and potentially negative) in the following situations:

- Pooling benefits are significantly lower when the systems that are pooled are not truly independent. Consider, for example, the idea of pooling waiting lines before cash registers in supermarkets, similar to what is done at airport check-ins. In this case, the individual queues are unlikely to be independent, as customers in the current, nonpooled layout will intelligently route themselves to the queue with the shortest waiting line. Pooling in this case will have little, if any, effect on waiting times.
- Similar to the concept of line balancing we introduced earlier in this book, pooling typically requires the service workforce to have a broader range of skills (potentially leading to higher wage rates). For example, an operator sufficiently skilled that she can take orders for hiking and running shoes, as well as provide answering services for a local hospital, will likely demand a higher wage rate than someone who is just trained to do one of these tasks.
- In many service environments, customers value being treated consistently by the same person. Pooling several lawyers in a law firm might be desirable from a waiting-time perspective but ignores the customer desire to deal with one point of contact in the law firm.
- Similarly, pooling can introduce additional setups. In the law-firm example, a lawyer unfamiliar with the situation of a certain client might need a longer time to provide some quick advice on the case and this extra setup time mitigates the operational benefits from pooling.
- Pooling can backfire if pooling combines different customer classes because this might actually increase the variability of the service process. Consider two clerks working in a retail bank, one of them currently in charge of simple transactions (e.g., processing time of 2 minutes per customer), while the other one is in charge of more complex cases (e.g., processing time of 10 minutes). Pooling these two clerks makes the service process more variable and might actually increase waiting time.

8.10 Priority Rules in Waiting Lines

Choosing an appropriate level of capacity helps to prevent waiting lines from building up in a process. However, in a process with variability, it is impossible to eliminate waiting lines entirely. Given, therefore, that at some point in time some customers will have to wait before receiving service, we need to decide on the order in which we permit them access to the server. This order is determined by a *priority rule,* sometimes also referred to as a queuing discipline.

Customers are assigned priorities by adding a (small) step at the point in the process where customers arrive. This process step is called the *triage step.* At triage, we collect information about some of the characteristics of the arriving customer, which we use as input for the priority rule. Below we discuss priority rules based on the following characteristics:

- The processing time or the expected processing time of the customer (processing-time-dependent priority rules).
- Processing-time-independent priority rules, including priority rules based on customer arrival time and priority rules based on customer importance or urgency.

Processing-Time-Dependent Priority Rules

If it is possible to observe the customer's processing time or his or her expected processing time prior to initiating the service process, this information should be incorporated when assigning a priority to the customer. The most commonly used service-time-dependent priority rule is the shortest processing time (SPT) rule.

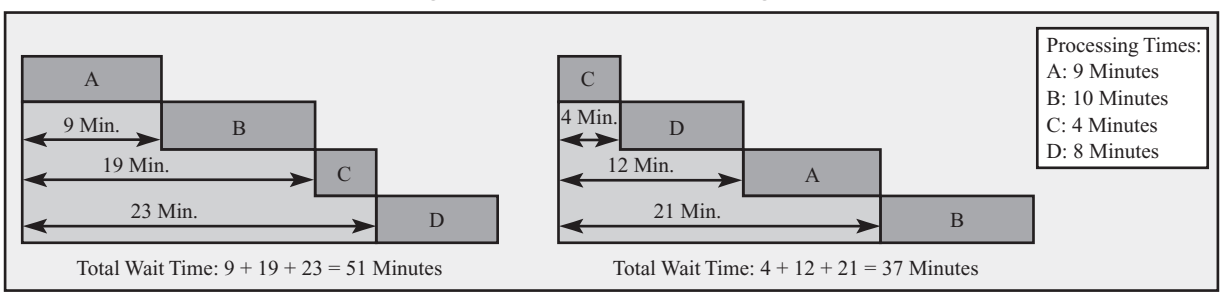
Under the SPT rule, the next available server is allocated to the customer with the shortest (expected) processing time of all customers currently in the waiting line. The SPT rule is extremely effective and performs well, with respect to the expected waiting time as well as to the variance of the waiting time. If the processing times are not dependent on the sequence with which customers are processed, the SPT rule can be shown to lead to the shortest average flow time. Its basic intuition is summarized by Figure 8.22.

Processing-Time-Independent Priority Rules

In many cases, it is difficult or impossible to assess the processing time or even the expected processing time prior to initiating the service process. Moreover, if customers are able to misrepresent their processing time, then they have an incentive to suggest that their processing time is less than it really is when the SPT rule is applied (e.g., "Can I just ask a quick question? . . ."). In contrast, the customer arrival time is easy to observe and difficult for the customer to manipulate.

For example, a call center receiving calls for airline reservations knows the sequence with which callers arrive but does not know which customer has already gathered all relevant information and is ready to order and which customer still requires explanation and discussion.

FIGURE 8.22 **The Shortest Processing time (SPT) Rule (used in the right case)**



The most commonly used priority rule based on arrival times is the first-come, first-served (FCFS) rule. With the FCFS rule, the next available server is allocated to the customer in the waiting line with the earliest arrival time.

In addition to using arrival time information, many situations in practice require that characteristics such as the urgency or the importance of the case are considered in the priority rule. Consider the following two examples:

- In an emergency room, a triage nurse assesses the urgency of each case and then assigns a priority to the patient. Severely injured patients are given priority, independent of their arrival times.
- Customers calling in for investor services are likely to experience different priorities, depending on the value of their invested assets. Customers with an investment of greater than $5 million are unlikely to wait, while customers investing only several thousand dollars might wait for 20 minutes or more.

Such urgency-based priority rules are also independent of the processing time. In general, when choosing a service-time-independent priority rule, the following property should be kept in mind: Whether we serve customers in the order of their arrival, in the reverse order of their arrival (last-come, first-served), or even in alphabetical order, the expected waiting time does not change. Thus, higher priority service (shorter waiting time) for one customer always requires lower priority (longer waiting time) for other customers.

From an implementation perspective, one last point is worth noting. Using priority rules other than FCFS might be perceived as unfair by the customers who arrived early and are already waiting the longest. Thus, while the average waiting time does not change, serving latecomers first increases the variance of the waiting time. Since variability in waiting time is not desirable from a service-quality perspective, the following property of the FCFS rule is worth remembering: Among service-time-independent priority rules, the FCFS rule minimizes the variance of waiting time and flow time.

8.11 Reducing Variability

In this chapter, we have provided some new methods to evaluate the key performance measures of flow rate, flow time, and inventory in the presence of variability. We also have seen that variability is the enemy of all operations (none of the performance measures improves as variability increases). Thus, in addition to just taking variability as given and adjusting our models to deal with variability, we should always think about ways to reduce variability.

Ways to Reduce Arrival Variability

One—somewhat obvious—way of achieving a match between supply and demand is by "massaging" demand such that it corresponds exactly to the supply process. This is basically the idea of *appointment systems* (also referred to as reservation systems in some industries).

Appointment systems have the potential to reduce the variability in the arrival process as they encourage customers to arrive at the rate of service. However, one should not overlook the problems associated with appointment systems, which include

- Appointment systems do not eliminate arrival variability. Customers do not perfectly arrive at the scheduled time (and some might not arrive at all, "no-shows"). Consequently, any good appointment system needs ways to handle these cases (e.g., extra charge or extra waiting time for customers arriving late). However, such actions are typically very difficult to implement, due to what is perceived to be "fair" and/or "acceptable," or because variability in processing times prevents service providers from always keeping on schedule (and if the doctor has the right to be late, why not the patient?).

- What portion of the available capacity should be reserved in advance. Unfortunately, the customers arriving at the last minute are frequently the most important ones: emergency operations in a hospital do not come through an appointment system and business travelers paying 5 to 10 times the fare of low-price tickets are not willing to book in advance (this topic is further explored in the revenue management chapter, Chapter 16).

The most important limitation, however, is that appointment systems might reduce the variability of the arrival process as seen by the operation, but they do not reduce the variability of the true underlying demand. Consider, for example, the appointment system of a dental office. While the system (hopefully) reduces the time the patient has to wait before seeing the dentist on the day of the appointment, this wait time is not the only performance measure that counts, as the patient might already have waited for three months between requesting to see the dentist and the day of the appointment. Thus, appointment systems potentially hide a much larger supply–demand mismatch and, consequently, any good implementation of an appointment system includes a continuous measurement of both of the following:

- The inventory of customers who have an appointment and are now waiting for the day they are scheduled to go to the dentist.
- The inventory of customers who wait for an appointment in the waiting room of the dentist.

In addition to the concept of appointment systems, we can attempt to influence the customer arrival process (though, for reasons similar to the ones discussed, not the true underlying demand pattern) by providing incentives for customers to avoid peak hours. Frequently observed methods to achieve this include

- Early-bird specials at restaurants or bars.
- Price discounts for hotels during off-peak days (or seasons).
- Price discounts in transportation (air travel, highway tolls) depending on the time of service.
- Pricing of air travel depending on the capacity that is already reserved.

It is important to point out that, strictly speaking, the first three items do not reduce variability; they level expected demand and thereby reduce seasonality (remember that the difference between the two is that seasonality is a pattern known already ex ante). The fourth item refers to the concept of revenue management, which is discussed in Chapter 16.

Ways to Reduce Processing Time Variability

In addition to reducing variability by changing the behavior of our customers, we also should consider how to reduce internal variability. However, when attempting to standardize activities (reducing the coefficient of variation of the processing times) or shorten processing times, we need to find a balance between operational efficiency (call durations) and the quality of service experienced by the customer (perceived courtesy).

Figure 8.23 compares five of An-ser's operators for a specific call service along these two dimensions. We observe that operators NN, BK, and BJ are achieving relatively short call durations while being perceived as friendly by the customers (based on recorded calls). Operator KB has shorter call durations, yet also scores lower on courtesy. Finally, operator NJ has the longest call durations and is rated medium concerning courtesy.

Based on Figure 8.23, we can make several interesting observations. First, observe that there seems to exist a frontier capturing the inherent trade-off between call duration and courtesy. Once call durations for this service go below 2.5 minutes, courtesy seems hard to maintain. Second, observe that operator NJ is away from this frontier, as he is neither

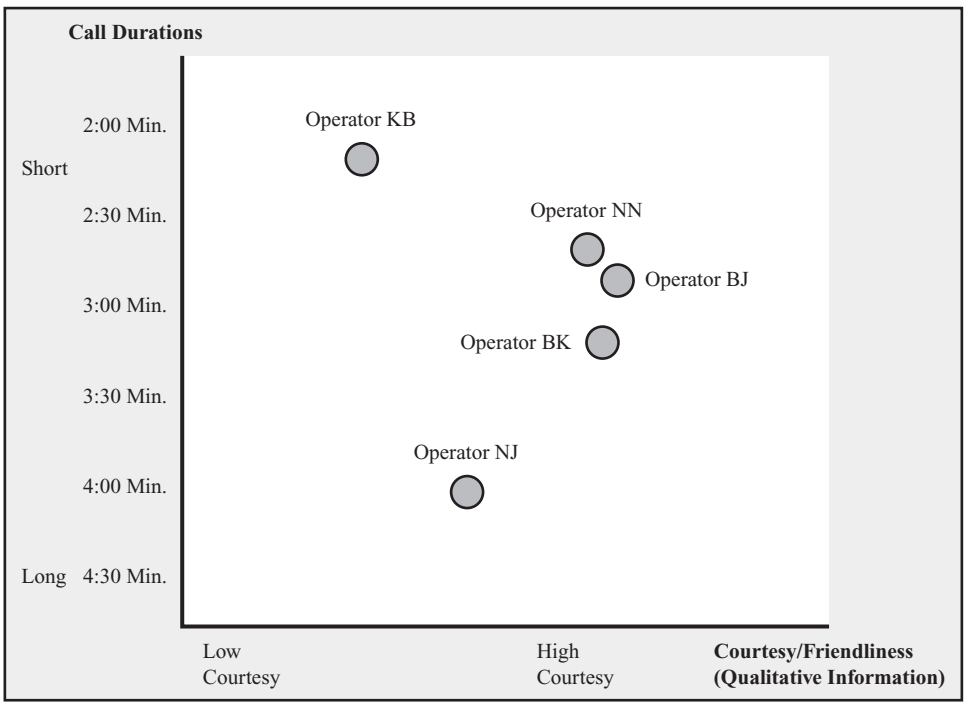


FIGURE 8.23
Operator Performance Concerning Call Duration and Courtesy

overly friendly nor fast. Remarkably, this operator also has the highest variability in call durations, which suggests that he is not properly following the operating procedures in place (this is not visible in the graph).

To reduce the inefficiencies of operators away from the frontier (such as NJ), call centers invest heavily in training and technology. For example, technology allows operators to receive real-time instruction of certain text blocks that they can use in their interaction with the customer (scripting). Similarly, some call centers have instituted training programs in which operators listen to audio recordings of other operators or have operators call other operators with specific service requests. Such steps reduce both the variability of processing times as well as their means and, therefore, represent substantial improvements in operational performance.

There are other improvement opportunities geared primarily toward reducing the variability of the processing times:

- Although in a service environment (or in a make-to-order production setting) the operator needs to acknowledge the idiosyncrasy of each customer, the operator still can follow a consistent process. For example, a travel agent in a call center might use predefined text blocks (scripts) for his or her interaction with the customer (welcome statement, first question, potential up-sell at the end of the conversation). This approach allowed operators NN, BK, and BJ in Figure 8.23 to be fast and friendly. Thus, being knowledgeable about the process (when to say what) is equally important as being knowledgeable about the product (what to say).

- Processing times in a service environment—unlike processing times in a manufacturing context—are not under the complete control of the resource. The customer him/herself plays a crucial part in the activity at the resource, which automatically introduces a certain amount of variability (e.g., having the customer provide his or her credit card number, having the customer bag the groceries, etc.) What is the consequence of this? At least from a variability perspective, the answer is clear: Reduce the involvement of the customer during the service

at a scarce resource wherever possible (note that if the customer involvement does not occur at a scarce resource, having the customer be involved and thereby do part of the work might be very desirable, e.g., in a self-service setting).

- Variability in processing times frequently reflects quality problems. In manufacturing environments, this could include reworking a unit that initially did not meet specifications. However, rework also occurs in service organizations (e.g., a patient who is released from the intensive care unit but later on readmitted to intensive care can be thought of as rework).

Many of these concepts are discussed further in Chapter 10.

8.12 Summary

In this chapter, we have analyzed the impact of variability on waiting times. As we expected from our more qualitative discussion of variability in the beginning of this chapter, variability causes waiting times, even if the underlying process operates at a utilization level of less than 100 percent. In this chapter, we have outlined a set of tools that allows us to quantify this waiting time, with respect to both the average waiting time (and flow time) as well as the service level experienced by the customer.

There exists an inherent tension between resource utilization (and thereby cost of labor) and responsiveness: Adding service capacity leads to shorter waiting times but higher costs of labor (see Figure 8.24). Waiting times grow steeply with utilization levels. Thus, any responsive process requires excess capacity. Given that capacity is costly, it is important that only as much capacity is installed as is needed to meet the service objective in place for the process. In this chapter, we have outlined a method that allows a service operation to find the point on the frontier that best supports their business objectives (service levels).

However, our results should be seen not only as a way to predict/quantify the waiting time problem. They also outline opportunities for improving the process. Improvement opportunities can be broken up into capacity-related opportunities and system-design-related opportunities, as summarized below.

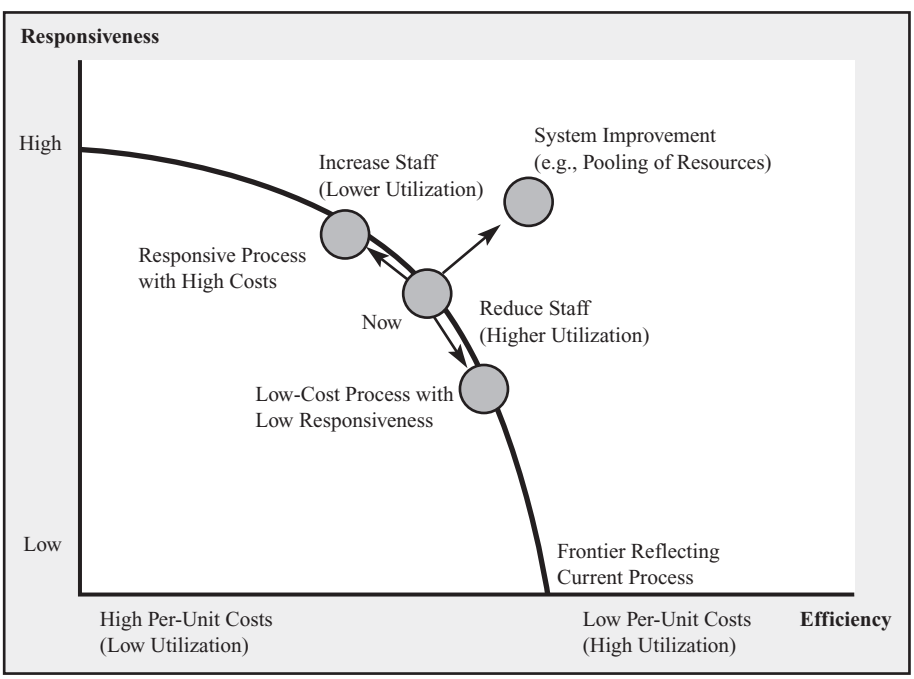


FIGURE 8.24
Balancing Efficiency with Responsiveness

Capacity-Related Improvements

Operations benefit from flexibility in capacity, as this allows management to adjust staffing levels to predicted demand. For example, the extent to which a hospital is able to have more doctors on duty at peak flu season is crucial in conducting the staffing calculations outlined in this chapter. A different form of flexibility is given by the operation's ability to increase capacity in the case of unpredicted demand. For example, the extent to which a bank can use supervisors and front-desk personnel to help with unexpected spikes in inbound calls can make a big difference in call center waiting times. This leads to the following two improvement opportunities:

- Demand (and sometimes supply) can exhibit seasonality over the course of the day. In such cases, the waiting time analysis should be done for individual time intervals over which the process behaves relatively stationary. System performance can be increased to the extent the organization is able to provide time-varying capacity levels that mirror the seasonality of demand (e.g., Figure 8.19).
- In the presence of variability, a responsive process cannot avoid excess capacity, and thereby will automatically face a significant amount of idle time. In many operations, this idle time can be used productively for tasks that are not (or at least are less) time critical. Such work is referred to as background work. For example, operators in a call center can engage in outbound calls during times of underutilization.

System-Design-Related Improvements

Whenever we face a trade-off between two conflicting performance measures, in this case between responsiveness and efficiency, finding the right balance between the measures is important. However, at least equally important is the attempt to improve the underlying process, shifting the frontier and allowing for higher responsiveness and lower cost (see Figure 8.24). In the context of services suffering from variability-induced waiting times, the following improvement opportunities should be considered:

- By combining similar resources into one joint resource pool (pooling resources), we are able to either reduce wait times for the same amount of capacity or reduce capacity for the same service level. Processes that face variability thereby exhibit very strong scale economies.
- Variability is not exogenous and we should remember to reduce variability wherever possible.
- By introducing a triage step before the actual service process that sequences incoming flow units according to a priority rule (service-time-dependent or service-time-independent), we can reduce the average wait time, assign priority to the most important flow units, or create a waiting system that is perceived as fair by customers waiting in line.

8.13 Further Reading

Gans, Koole, and Mandelbaum (2003) is a recent overview on call-center management from a queuing theory perspective. Further quantitative tools on queueing can be found in Hillier and Lieberman (2002).

Hall (1997) is a very comprehensive and real-world-focused book that provides numerous tools related to variability and its consequences in services and manufacturing.

8.14 Practice Problems

Q8.1* **(Online Retailer)** Customers send e-mails to a help desk of an online retailer every 2 minutes, on average, and the standard deviation of the interarrival time is also 2 minutes. The online retailer has three employees answering e-mails. It takes on average 4 minutes to write a response e-mail. The standard deviation of the processing times is 2 minutes.

(* indicates that the solution is at the end of the book)

a. Estimate the average customer wait before being served.

b. How many e-mails would there be, on average, that have been submitted to the online retailer but not yet answered?

Q8.2** **(My-law.com)** My-law.com is a recent start-up trying to cater to customers in search of legal services who are intimidated by the idea of talking to a lawyer or simply too lazy to enter a law office. Unlike traditional law firms, My-law.com allows for extensive interaction between lawyers and their customers via telephone and the Internet. This process is used in the upfront part of the customer interaction, largely consisting of answering some basic customer questions prior to entering a formal relationship.

In order to allow customers to interact with the firm's lawyers, customers are encouraged to send e-mails to my-lawyer@My-law.com. From there, the incoming e-mails are distributed to the lawyer who is currently "on call." Given the broad skills of the lawyers, each lawyer can respond to each incoming request.

E-mails arrive from 8 A.M. to 6 P.M. at a rate of 10 e-mails per hour (coefficient of variation for the arrivals is 1). At each moment in time, there is exactly one lawyer "on call," that is, sitting at his or her desk waiting for incoming e-mails. It takes the lawyer, on average, 5 minutes to write the response e-mail. The standard deviation of this is 4 minutes.

a. What is the average time a customer has to wait for the response to his/her e-mail, ignoring any transmission times? *Note:* This includes the time it takes the lawyer to start writing the e-mail *and* the actual writing time.

b. How many e-mails will a lawyer have received at the end of a 10-hour day?

c. When not responding to e-mails, the lawyer on call is encouraged to actively pursue cases that potentially could lead to large settlements. How much time on a 10-hour day can a My-law.com lawyer dedicate to this activity (assume the lawyer can instantly switch between e-mails and work on a settlement)?

To increase the responsiveness of the firm, the board of My-law.com proposes a new operating policy. Under the new policy, the response would be highly standardized, reducing the standard deviation for writing the response e-mail to 0.5 minute. The average writing time would remain unchanged.

d. How would the amount of time a lawyer can dedicate to the search for large settlement cases change with this new operating policy?

e. How would the average time a customer has to wait for the response to his/her e-mail change? *Note:* This includes the time until the lawyer starts writing the e-mail *and* the actual writing time.

Q8.3** **(Car Rental Company)** The airport branch of a car rental company maintains a fleet of 50 SUVs. The interarrival time between requests for an SUV is 2.4 hours, on average, with a standard deviation of 2.4 hours. There is no indication of a systematic arrival pattern over the course of a day. Assume that, if all SUVs are rented, customers are willing to wait until there is an SUV available. An SUV is rented, on average, for 3 days, with a standard deviation of 1 day.

a. What is the average number of SUVs parked in the company's lot?

b. Through a marketing survey, the company has discovered that if it reduces its daily rental price of $80 by $25, the average demand would increase to 12 rental requests per day and the average rental duration will become 4 days. Is this price decrease warranted? Provide an analysis!

c. What is the average time a customer has to wait to rent an SUV? Please use the initial parameters rather than the information in part (b).

d. How would the waiting time change if the company decides to limit all SUV rentals to *exactly* 4 days? Assume that if such a restriction is imposed, the average interarrival time will increase to 3 hours, with the standard deviation changing to 3 hours.

Q8.4 **(Tom Opim)** The following situation refers to Tom Opim, a first-year MBA student. In order to pay the rent, Tom decides to take a job in the computer department of a local

department store. His only responsibility is to answer telephone calls to the department, most of which are inquiries about store hours and product availability. As Tom is the only person answering calls, the manager of the store is concerned about queuing problems.

Currently, the computer department receives an average of one call every 3 minutes, with a standard deviation in this interarrival time of 3 minutes.

Tom requires an average of 2 minutes to handle a call. The standard deviation in this processing time is 1 minute.

The telephone company charges $5.00 per hour for the telephone lines whenever they are in use (either while a customer is in conversation with Tom or while waiting to be helped).

Assume that there are no limits on the number of customers that can be on hold and that customers do not hang up even if forced to wait a long time.

a. For one of his courses, Tom has to read a book (*The Pole,* by E. Silvermouse). He can read 1 page per minute. Tom's boss has agreed that Tom could use his idle time for studying, as long as he drops the book as soon as a call comes in. How many pages can Tom read during an 8-hour shift?
b. How long does a customer have to wait, on average, before talking to Tom?
c. What is the average total cost of telephone lines over an 8-hour shift? Note that the department store is billed whenever a line is in use, including when a line is used to put customers on hold.

Q8.5 **(Atlantic Video)** Atlantic Video, a small video rental store in Philadelphia, is open 24 hours a day, and—due to its proximity to a major business school—experiences customers arriving around the clock. A recent analysis done by the store manager indicates that there are 30 customers arriving every hour, with a standard deviation of interarrival times of 2 minutes. This arrival pattern is consistent and is independent of the time of day. The checkout is currently operated by one employee, who needs on average 1.7 minutes to check out a customer. The standard deviation of this check-out time is 3 minutes, primarily as a result of customers taking home different numbers of videos.

a. If you assume that every customer rents at least one video (i.e., has to go to the checkout), what is the average time a customer has to wait in line before getting served by the checkout employee, not including the actual checkout time (within 1 minute)?
b. If there are no customers requiring checkout, the employee is sorting returned videos, of which there are always plenty waiting to be sorted. How many videos can the employee sort over an 8-hour shift (assume no breaks) if it takes exactly 1.5 minutes to sort a single video?
c. What is the average number of customers who are at the checkout desk, either waiting or currently being served (within 1 customer)?
d. Now assume *for this question only* that 10 percent of the customers do not rent a video at all and therefore do not have to go through checkout. What is the average time a customer has to wait in line before getting served by the checkout employee, not including the actual checkout time (within 1 minute)? Assume that the coefficient of variation for the arrival process remains the same as before.
e. As a special service, the store offers free popcorn and sodas for customers waiting in line at the checkout desk. (*Note:* The person who is currently being served is too busy with paying to eat or drink.) The store owner estimates that every minute of customer waiting time costs the store 75 cents because of the consumed food. What is the optimal number of employees at checkout? Assume an hourly wage rate of $10 per hour.

Q8.6 **(RentAPhone)** RentAPhone is a new service company that provides European mobile phones to American visitors to Europe. The company currently has 80 phones available at Charles de Gaulle Airport in Paris. There are, on average, 25 customers per day requesting a phone. These requests arrive uniformly throughout the 24 hours the store is open. (*Note:* This means customers arrive at a faster rate than 1 customer per hour.) The corresponding coefficient of variation is 1.

Customers keep their phones on average 72 hours. The standard deviation of this time is 100 hours.

Given that RentAPhone currently does not have a competitor in France providing equally good service, customers are willing to wait for the telephones. Yet, during the waiting period, customers are provided a free calling card. Based on prior experience, RentAPhone found that the company incurred a cost of $1 per hour per waiting customer, independent of day or night.

a. What is the average number of telephones the company has in its store?
b. How long does a customer, on average, have to wait for the phone?
c. What are the total monthly (30 days) expenses for telephone cards?
d. Assume RentAPhone could buy additional phones at $1,000 per unit. Is it worth it to buy one additional phone? Why?
e. How would waiting time change if the company decides to limit all rentals to *exactly* 72 hours? Assume that if such a restriction is imposed, the number of customers requesting a phone would be reduced to 20 customers per day.

Q8.7 **(Webflux Inc.)** Webflux is an Internet-based DVD rental business specializing in hard-to-find, obscure films. Its operating model is as follows. When a customer finds a film on the Webflux Web site and decides to watch it, she puts it in the virtual shopping cart. If a DVD is available, it is shipped immediately (assume it can be shipped during weekends and holidays, too). If not available, the film remains in the customer's shopping cart until a rented DVD is returned to Webflux, at which point it is shipped to the customer if she is next in line to receive it. Webflux maintains an internal queue for each film and a returned DVD is shipped to the first customer in the queue (first-in, first-out).

Webflux has one copy of the 1990 film *Sundown, the Vampire in Retreat,* starring David Carradine and Bruce Campbell. The average time between requests for the DVD is 10 days, with a coefficient of variation of 1. On average, a customer keeps the DVD for 5 days before returning it. It also takes 1 day to ship the DVD to the customer and 1 day to ship it from the customer back to Webflux. The standard deviation of the time between shipping the DVD out from Webflux and receiving it back is 7 days (i.e., it takes on average 7 days to (a) ship it, (b) have it with the customer, and (c) ship it back); hence, the coefficient of variation of this time is 1.

a. What is the average time that a customer has to wait to receive *Sundown, the Vampire in Retreat* DVD after the request? Recall it takes 1 day for a shipped DVD to arrive at a customer address (i.e., in your answer, you have to include the 1-day shipping time).
b. On average, how many customers are in Webflux's internal queue for *Sundown?* Assume customers do not cancel their items in their shopping carts.

Thanks to David Carradine's renewed fame after the recent success of *Kill Bill Vol. I* and *II* which he starred in, the demand for *Sundown* has spiked. Now the average interarrival time for the DVD requests at Webflux is 3 days. Other numbers (coefficient of variation, time in a customer's possession, shipping time) remain unchanged. *For the following question only,* assume sales are lost for customers who encounter stockouts; that is those who cannot find a DVD on the Webflux Web site simply navigate away without putting it in the shopping cart.

c. To satisfy the increased demand, Webflux is considering acquiring a second copy of the *Sundown* DVD. If Webflux owns a total of two copies of *Sundown* DVDs (whether in Webflux's internal stock, in customer's possession, or in transit), what percentage of the customers are turned away because of a stockout? (*Note:* To answer this question, you will need material from Chapter 9.)

Q8.8 **(Security Walking Escorts)** A university offers a walking escort service to increase security around campus. The system consists of specially trained uniformed professional security officers that accompany students from one campus location to another. The service is operated 24 hours a day, seven days a week. Students request a walking escort by phone. Requests for escorts are received, on average, every 5 minutes with a coefficient of variation of 1. After receiving a request, the dispatcher contacts an available escort (via a

mobile phone), who immediately proceeds to pick up the student and walk her/him to her/his destination. If there are no escorts available (that is, they are all either walking a student to her/his destination or walking to pick up a student), the dispatcher puts the request in a queue until an escort becomes available. An escort takes, on average, 25 minutes for picking up a student and taking her/him to her/his desired location (the coefficient of variation of this time is also 1). Currently, the university has 8 security officers who work as walking escorts.

a. How many security officers are, on average, available to satisfy a new request?
b. How much time does it take—on average—from the moment a student calls for an escort to the moment the student arrives at her/his destination?

For the next two questions, consider the following scenario. During the period of final exams, the number of requests for escort services increases to 19.2 per hour (one request every 3.125 minutes). The coefficient of variation of the time between successive requests equals 1. However, if a student requesting an escort finds out from the dispatcher that her/his request would have to be put in the queue (i.e., all security officers are busy walking other students), the student cancels the request and proceeds to walk on her/his own.

c. How many students per hour who called to request an escort end up canceling their request and go walking on their own? (*Note:* To answer this question, you will need material from Chapter 9.)
d. University security regulations require that at least 80 percent of the students' calls to request walking escorts have to be satisfied. What is the minimum number of security officers that are needed in order to comply with this regulation?

Q8.9 **(Mango Electronics Inc.)** Mango Electronics Inc. is a *Fortune* 500 company that develops and markets innovative consumer electronics products. The development process proceeds as follows.

Mango researches new technologies to address unmet market needs. Patents are filed for products that have the requisite market potential. Patents are granted for a period of 20 years starting from the date of issue. After receiving a patent, the patented technologies are then developed into marketable products at five independent development centers. Each product is only developed at one center. Each center has all the requisite skills to bring any of the products to market (a center works on one product at a time). On average, Mango files a patent every 7 months (with standard deviation of 7 months). The average development process lasts 28 months (with standard deviation of 56 months).

a. What is the utilization of Mango's development facilities?
b. How long does it take an average technology to go from filing a patent to being launched in the market as a commercial product?
c. How many years of patent life are left for an average product launched by Mango Electronics?

Q8.10 **(UPS Shipping)** A UPS employee, Davis, packs and labels three types of packages: basic packages, business packages, and oversized packages. Business packages take priority over basic packages and oversized packages because those customers paid a premium to have guaranteed two-day delivery. During his nine-hour shift, he has, on average, one container of packages containing a variety of basic, business, and oversized packages to process every 3 hours. As soon as Davis processes a package, he passes it to the next employee, who loads it onto a truck. The times it takes him to process the three different types of packages and the average number of packages per container are shown in the table below.

	Basic	Business	Oversized
Average number of minutes to label and package each unit	5 minutes	4 minutes	6 minutes
Average number of units per container	10	10	5

Davis currently processes packages from each container as follows. First, he processes all business packages in the container. Then he randomly selects either basic packages or oversized packages for processing until the container is empty. However, his manager suggested to Davis that, for each container, he should process all the business packages first, second the basic packages, and last the oversized packages.

a. If Davis follows his supervisor's advice, what will happen to Davis's utilization?

b. What will happen to the average time that a package spends in the container?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 9

The Impact of Variability on Process Performance: Throughput Losses

After having analyzed waiting times caused by variability, we now turn to a second undesirable impact variability has on process performance: *throughput loss*. Throughput losses occur in the following cases, both of which differ from the case of flow units patiently waiting for service discussed in Chapter 8:

- There is a limited buffer size and demand arriving when this buffer is full is lost.
- Flow units are impatient and unwilling or unable to spend too much time waiting for service, which leads to flow units leaving the buffer before being served.

Analyzing processes with throughput losses is significantly more complicated compared to the case of patient customers discussed in Chapter 8. For this reason, we focus our analysis on the simplest case of throughput loss, which assumes that the buffer size is zero, that is, there is no buffer. We will introduce a set of analytical tools and discuss their application to time-critical emergency care provided by hospitals, especially trauma centers. In these settings, waiting times are not permissible and, when a trauma center is fully utilized, incoming ambulances are diverted to other hospitals.

There exist more general models of variability that allow for buffer sizes larger than zero, yet due to their complexity, we only discuss those models conceptually. Again, we start the chapter with a small motivating example.

9.1 Motivating Examples: Why Averages Do Not Work

Consider a street vendor who sells custom-made sandwiches from his truck parked along the sidewalk. Demand for these sandwiches is, on average, one sandwich in a five-minute time slot. However, the actual demand varies, and thus sometimes no customer places an order, while at other times the owner of the truck faces one or two orders. Customers are not willing to wait for sandwiches and leave to go to other street vendors if they cannot be served immediately.

TABLE 9.1
Street Vendor Example of Variability

Scenario	Demand	Capacity	Flow Rate
A	0	0	0
B	0	1	0
C	0	2	0
D	1	0	0
E	1	1	1
F	1	2	1
G	2	0	0
H	2	1	1
I	2	2	2
Average	1	1	$\frac{5}{9}$

The capacity leading to the supply of sandwiches over a five-minute time slot also varies and can take the values 0, 1, or 2 with equal probabilities (the variability of capacity might reflect different order sizes or operator absenteeism). The average capacity therefore is one, just as is the average demand.

From an aggregate planning perspective, demand and supply seem to match, and on average, the truck should be selling at a flow rate of one sandwich every five minutes:

$$\text{Flow rate} = \text{Minimum}\{\text{Demand}, \text{Capacity}\} = \text{Minimum}\{1, 1\} = 1$$

Now, consider an analysis that is conducted at the more detailed level. If we consider the potential outcomes of both the demand and the supply processes, we face nine possible scenarios, which are summarized in Table 9.1.

Consider each of the nine scenarios. But instead of averaging demand and capacity and then computing the resulting flow rate (as done above, leading to a predicted flow rate of one), we compute the flow rate for each of the nine scenarios and then take the average across scenarios. The last column in Table 9.1 provides the corresponding calculations.

Note that for the first three scenarios (Demand = 0), we are not selling a single sandwich. However, if we look at the last three scenarios (Demand = 2), we cannot make up for this loss, as we are constrained by capacity. Thus, even while demand is booming (Demand = 2), we are selling on average one sandwich every five minutes.

If we look at the average flow rate that is obtained this way, we observe that close to half of the sales we expected to make based on our aggregate analysis do not materialize! The explanation for this is as follows: In order to sell a sandwich, the street vendor needed demand (a customer) and supply (the capacity to make a sandwich) at the same moment in time. Flow rate could have been improved if the street vendor could have moved some supply to inventory and thereby stored it for periods of time in which demand exceeded supply, or, vice versa, if the street vendor could have moved some demand to a backlog of waiting customers and thereby stored demand for periods of time in which supply exceeded demand: another example of the "buffer or suffer" principle.

9.2 Ambulance Diversion

Now, let's move from analyzing a "cooked-up" food-truck to a problem of much larger importance, with respect to both its realism as well as its relevance. Over the last couple of years, reports have shown a substantial increase in visits to emergency departments. At the same time many hospitals, in response to increasing cost pressure, have downsized important resources that are part of the emergency care process. This has led to a decrease in the number of hours hospitals are "open" for emergency patients arriving by helicopter or ambulance.

Under U.S. federal law, all hospitals that participate in Medicare are required to screen—and, if an emergency condition is present, stabilize—any patient who comes to the emergency department, regardless of the individual's ability to pay.[1] Under certain circumstances where a hospital lacks staffing or facilities to accept additional emergency patients, the hospital may place itself on "diversion status" and direct en route ambulances to other hospitals.

In total, the General Accounting Office estimates that about two of every three hospitals went on diversion at least once during the fiscal year 2001. Moreover, the study estimates that about 2 in every 10 of these hospitals were on diversion for more than 10 percent of the time, and about 1 in every 10 was on diversion for more than 20 percent of the time—or about five hours per day.

We focus our analysis on trauma cases, that is, the most severe and also the most urgent type of emergency care. A triage system evaluates the patients while they are in the ambulance/helicopter and directs the arrival to the emergency department (less severe cases) or the trauma center (severe cases). Thus, the trauma center only receives patients who have had a severe trauma.

9.3 Throughput Loss for a Simple Process

Consider the following situation of a trauma center in a hospital in the Northeastern United States. Incoming patients are moved into one of three trauma bays. On average, patients spend two hours in the trauma bay. During that time, the patients are diagnosed and, if possible, stabilized. The most severe cases, which are difficult or impossible to stabilize, spend very little time in a trauma bay and are moved directly to the operating room.

Given the severe conditions of patients coming into the trauma center, any delay of care can have fatal consequences for the patient. Thus, having patients wait for service is not an option in this setting. If, as a result of either frequent arrivals or long service times, all three trauma bays are utilized, the trauma center has to move to the ambulance diversion status defined above.

We model the trauma center as a process flow diagram consisting of no buffer and multiple parallel resources (see Figure 9.1). Given that we have three trauma bays (and corresponding staff) available, there can be a maximum of three patients in the process. Once

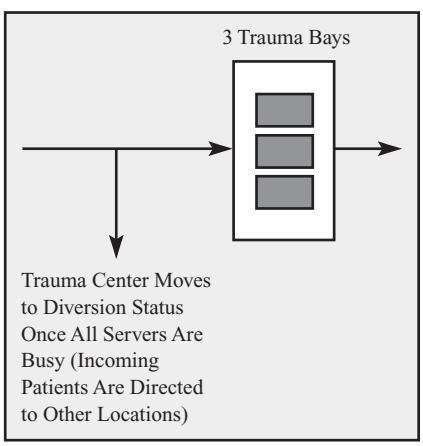


FIGURE 9.1
Process Flow Diagram for Trauma Center

[1] The following definitions and statistics are taken from the report "Hospital Emergency Departments," GAO-03-460, given by the General Accounting Office to the U.S. Senate.

all three bays are in use, the trauma center informs the regional emergency system that it has to go on diversion status; that is, any patients needing trauma services at that time are transported to other hospitals in the region.

The trauma center we analyze handles about 2,000 cases per year. For our analysis, we focus on the late evening hours, during which, on average, a new patient arrives every three hours. In addition to traffic rush hour, the late evening hours are among the busiest for the trauma center, as many of the incoming cases are results of vehicle accidents (alcohol-induced car accidents tend to happen in the evening) and victims of violence (especially in the summer months, many violent crimes occur in the evening hours).

Thus, we have a new patient every $a = 3$ hours and it takes, on average, $p = 2$ hours of time to get the patient out of the trauma center. In our analysis, we assume that the trauma bays are the resources and that there is sufficient staff to operate all three bays simultaneously, if the need arises.

Given that there are three trauma bays available, the capacity of the trauma center is

$$\text{Capacity} = \frac{\text{Number of resources}}{\text{Processing time}} = \frac{3}{2 \text{ hours/patient}}$$
$$= 1.5 \text{ patients per hour}$$

Since incoming patients arrive randomly, we use exponential interarrival times and consequently face a coefficient of variation of CV_a equal to one. The coefficient of variation of the service time in this case turns out to be above one (many medical settings are known to have extremely high variability). However, as we will see below, the following computations do not depend on the service time variability and apply to any service time distribution.

We are interested in analyzing the following performance measures:

- What percent of the time will the trauma center have to go on diversion status? Similarly, how many patients are diverted because all three trauma bays are utilized?
- What is the flow rate through the trauma center, that is, how many patients are treated every unit of time (e.g., every day)?

The most difficult, yet also most important step in our analysis is computing the probability with which the process contains m patients, P_m. This probability is of special importance, as once m patients are in the trauma center, the trauma center needs to divert any incoming requests until it has discharged a patient. The probability of having all m servers busy, P_m, depends on two variables:

- The *implied utilization.* Given that some patients are not admitted to the process (and thereby do not contribute to throughput), we no longer need to impose the condition that the capacity exceeds the demand rate ($1/a$). This assumption was necessary in the previous chapter, as otherwise the waiting line would have "exploded." In a system that automatically "shuts down" the process in case of high demand, this does not happen. Hence, u now includes the case of a utilization above 100 percent, which is why we speak of the implied utilization (Demand rate/Capacity) as opposed to utilization (Flow rate/Capacity).
- The number of resources (trauma bays) m.

We begin our analysis by computing the implied utilization:

$$u = \frac{\text{Demand rate}}{\text{Capacity}} = \frac{0.3333 \text{ patient per hour}}{1.5 \text{ patients per hour}} = 0.2222$$

Based on the implied utilization u and the number of resources m, we can use the following method to compute the probability that all m servers are busy, P_m. Define $r = u \times m = p/a$. Thus, $r = 0.67$.

TABLE 9.2
Finding the Probability $P_m(r)$ Using the Erlang Loss Table from Appendix B

Erlang Loss Table							
		m					
		1	2	3	4	5	6 . . .
	0.10	0.0909	0.0045	0.0002	0.0000	0.0000	0.0000
	0.20	0.1667	0.0164	0.0011	0.0001	0.0000	0.0000
	0.25	0.2000	0.0244	0.0020	0.0001	0.0000	0.0000
	0.30	0.2308	0.0335	0.0033	0.0003	0.0000	0.0000
r	0.33	0.2500	0.0400	0.0044	0.0004	0.0000	0.0000
	0.40	0.2857	0.0541	0.0072	0.0007	0.0001	0.0000
	0.50	0.3333	0.0769	0.0127	0.0016	0.0002	0.0000
	0.60	0.3750	0.1011	0.0198	0.0030	0.0004	0.0000
	0.67	0.4000	0.1176	0.0255	0.0042	0.0006	0.0001
	0.70	0.4118	0.1260	0.0286	0.0050	0.0007	0.0001
	0.75	0.4286	0.1385	0.0335	0.0062	0.0009	0.0001
	. . .						

We can then use the *Erlang loss formula* table (Appendix B) to look up the probability that all m resources are utilized and hence a newly arriving flow unit has to be rejected. First, we find the corresponding row heading in the table ($r = 0.67$) indicating the ratio of processing time to interarrival time (see Table 9.2). Second, we find the column heading ($m = 3$) indicating the number of resources. The intersection of that row with that column is

$$\text{Probability}\{\text{all } m \text{ servers busy}\} = P_m(r) = 0.0255 \quad \text{(Erlang loss formula)}$$

Thus, we find that our trauma center, on average, will be on diversion for 2.5 percent of the time, which corresponds to about 0.6 hour per day and about 18 hours per month.

A couple of remarks are in order to explain the impact of the processing time-to-interarrival-time ratio r and the number of resources m on the probability that all servers are utilized:

- The probability $P_m(r)$ and hence the analysis do not require the coefficient of variation for the service process. The analysis only applies to the (realistic) case of exponentially distributed interarrival times; therefore, we implicitly assume that the coefficient of variation for the arrival process is equal to one.
- The formula underlying the table in Appendix B is attributed to the work of Agner Krarup Erlang, a Danish engineer who invented many (if not most) of the models that we use in Chapters 8 and 9 for his employer, the Copenhagen Telephone Exchange. In this context, the arrivals were incoming calls for which there was either a telephone line available or not (in which case the calls were lost, which is why the formula is also known as the *Erlang loss formula*).
- At the beginning of Appendix B, we provide the formula that underlies the Erlang loss formula table. We can use the formula directly to compute the probability $P_m(r)$ for a given processing-time-to-interarrival-time ratio r and the number of resources m.

In addition to the probability that all resources are utilized, we also can compute the number of patients that will have to be diverted. Since demand for trauma care continues at a rate of $1/a$ independent of the diversion status of the trauma center, we obtain our flow rate as

$$\begin{aligned}\text{Flow rate} &= \text{Demand rate} \times \text{Probability that not all servers are busy} \\ &= 1/a \times (1 - P_m) = \tfrac{1}{3} \times 0.975 = 0.325 \text{ patient per hour}\end{aligned}$$

Similarly, we find that we divert ⅓ × 0.025 = 0.0083 patient per hour = 0.2 patient per day.

The case of the trauma center provides another example of how variability needs to be accommodated in a process by putting excess capacity in place. A utilization level of 22 percent in an environment of high fixed costs seems like the nightmare of any administrator. Yet, from the perspective of a person in charge of creating a responsive process, absolute utilization numbers should always be treated with care: The role of the trauma center is not to maximize utilization; it is to help people in need and ultimately save lives.

One main advantage of the formula outlined above is that we can quickly evaluate how changes in the process affect ambulance diversion. For example, we can compute the probability of diversion that would result from an increased utilization. Such a calculation would be important, both to predict diversion frequencies, as well as to predict flow rate (e.g., number of patients served per month).

Consider, for example, a utilization of 50 percent. Such a case could result from a substantial increase in arrival rate (e.g., consider the case that a major trauma center in the area closes because of the financial problems of its hospital).

Based on the increased implied utilization, $u = 0.5$, and the same number of trauma bays, $m = 3$, we compute $r = u \times m = 1.5$. We then use the Erlang loss formula table to look up the probability $P_m(r)$ that all m servers are utilized:

$$P_3(1.5) = 0.1343$$

Thus, this scenario of increased utilization would lead to ambulance diversion more than 13 percent of the time, corresponding to close to 100 hours of diversion every month.

Figure 9.2 shows the relationship between the level of implied utilization and the probability that the process cannot accept any further incoming arrivals. As we can see, similar to waiting time problems, there exist significant scale economies in loss systems: While a 50 percent utilization would lead to a diversion probability of 30 percent with one server ($m = 1$), it only leads to a 13 percent diversion probability with three servers and less than 2 percent for 10 servers.

Exhibit 9.1 summarizes the computations required for the Erlang loss formula.

FIGURE 9.2
Implied Utilization versus Probability of Having All Servers Utilized

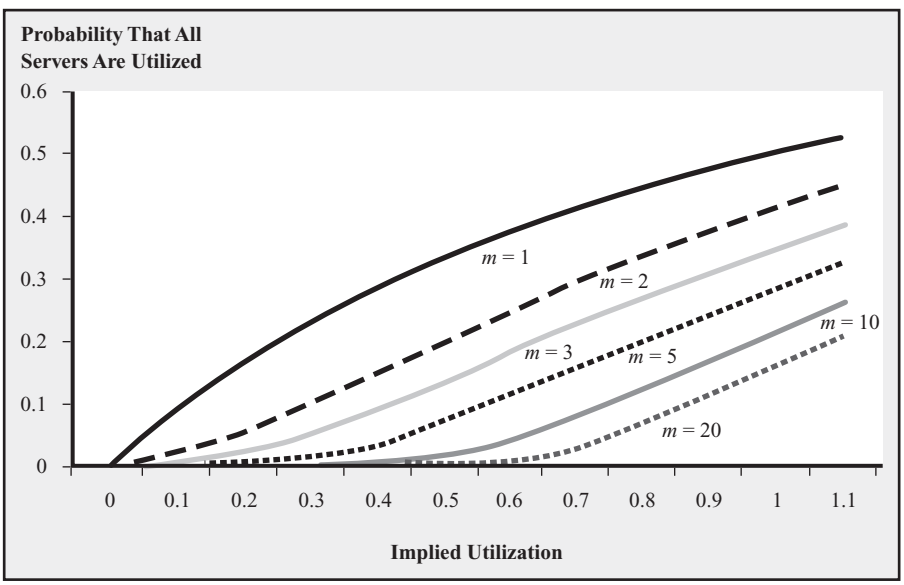


Exhibit 9.1

USING THE ERLANG LOSS FORMULA

1. Define $r = \frac{p}{a}$ where p is the processing time and a is the interarrival time
2. Use the Erlang loss formula table in Appendix B to look up the probability that all servers are busy:

$$\text{Probability } \{\text{all } m \text{ servers are busy}\} = P_m(r)$$

3. Compute flow rate based on

$$\text{Flow rate} = \text{Demand rate} \times \text{Probability that not all servers are busy}$$
$$R = 1/a \times (1 - P_m)$$

4. Compute lost customers as

$$\text{Customers lost} = \text{Demand rate} \times \text{Probability that all servers are busy}$$
$$= 1/a \times P_m$$

9.4 Customer Impatience and Throughput Loss

In Chapter 8 we analyzed a process in which flow units patiently waited in a queue until it was their turn to be served. In contrast, in the case of the trauma center, we have analyzed a process in which flow units never waited but, when all servers were busy, were turned immediately into lost flow units (were routed to other hospitals).

These two cases, a waiting problem on one side and a loss problem on the other side, are important, yet they also are extreme cases concerning the impact of variability on process performance. Many interesting applications that you might encounter are somewhere in between these two extremes. Without going into a detailed analysis, it is important that we at least discuss these intermediate cases at the conceptual level.

The first important intermediate case is a waiting problem in which there is a buffer that allows a limited number of flow units to wait for service. The limit of the buffer size might represent one of these situations:

- In a call center, there exist a maximum number of calls that can be on hold simultaneously; customers calling in when all these lines are in use receive a busy signal (i.e., they don't even get to listen to the 70s music!). Similarly, if one thinks of a queue in front of a drive-through restaurant, there exist a maximum number of cars that can fit in the queue; once this maximum is reached, cars can no longer line up.
- Given that, as a result of Little's Law, the number of customers in the queue can be translated into an expected wait time, a limit on the queue size might simply represent a maximum amount of time customers would be willing to wait. For example, customers looking at a queue in front of a movie theater might simply decide that the expected wait time is not justified by the movie they expect to see.

Although we will not discuss them in this book, there exist mathematical models to analyze this type of problem and for a given maximum size of the buffer, we can compute the usual performance measures, inventory, flow rate, and wait time (see, e.g., Hillier and Liebermann (2002)).

For the case of a single server, Figure 9.3 shows the relationship between the number of available buffers and the probability that all buffers are full; that is, the probability that the process can no longer accept incoming customers. As we can see, this probability

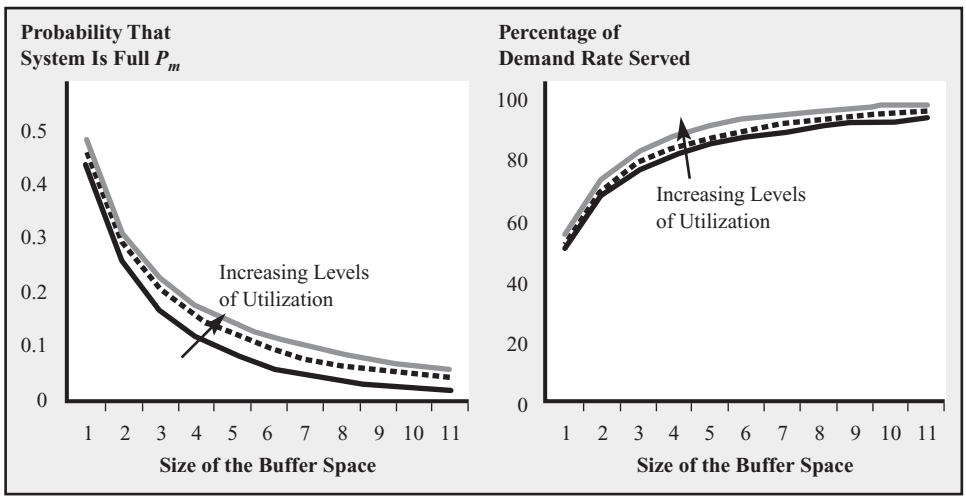


FIGURE 9.3
Impact of Buffer Size on the Probability P_m for Various Levels of Implied Utilization as well as on the Throughput of the Process in the Case of One Single Server

is quickly decreasing as we add more and more buffer space. Note that the graph shifts up as we increase the level of utilization, which corresponds to the intuition from earlier chapters.

Since we can compute the throughput of the system as

$$(1 - \text{Probability that all buffers are full}) \times \text{Demand rate}$$

we also can interpret Figure 9.3 as the throughput loss. The right part of Figure 9.3 shows the impact of buffer size on throughput. Even for a single server and a utilization of 90 percent, we need more than 10 buffers to come close to restoring the throughput we would expect in the absence of variability.

The second intermediate case between a waiting problem and a loss problem resembles the first case but is different in the sense that customers always enter the system (As opposed to not even joining the queue), but then leave the queue unserved as they become tired of waiting. The technical term for this is "customers *abandon* the queue" or the customers *balk.* This case is very common in call centers that have very long wait times. However, for call centers with high service levels for short target wait times, such as in the case of the An-ser call center discussed in Chapter 8, there are very few abandonment cases (this is why we could safely ignore customers abandoning the queue for our analysis in Chapter 8).

Figure 9.4 shows an example of call center data (collected by Gans, Koole, and Mandelbaum (2003)) in a setting with long waiting times. The horizontal axis shows how long customers had to wait before talking to an agent. The vertical axis represents the percentage of customers hanging up without being served. We observe that the longer customers have to wait, the larger the proportion of customers lost due to customer impatience.

There are three types of improvement opportunities for the two intermediate cases, limited buffer space and abandoning customers:

- Reduce wait times. Similar to our prior analysis, anything we can do to reduce wait times (intelligently choose capacity, reduce variability, etc.) helps reduce throughput losses resulting from customer impatience.
- Increase the maximum number of flow units that can be in the buffer. This can be achieved by either altering the actual buffer (adding more space, buying more telephone lines) or increasing the customers' willingness to tolerate waiting.
- Avoid customers leaving that have already waited. Having customers wait and then leave is even worse than having customers leave immediately, so it is important to

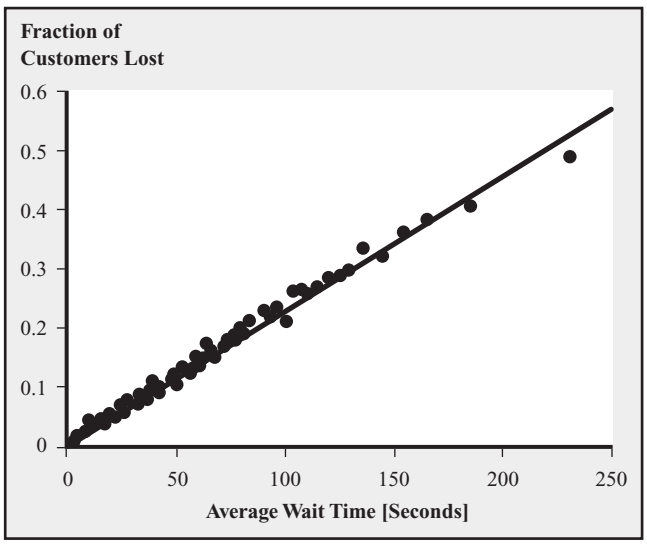


FIGURE 9.4
Impact of Waiting Time on Customer Loss

Source: Gans, Koole, and Mandelbaum, 2003.

avoid this case as much as possible. One way of achieving this is to reduce the perceived waiting duration by giving customers meaningful tasks to do (e.g., key in some information, help reduce the actual service time) or by creating an environment where waiting is not too painful (two generations of operations managers were told to install mirrors in front of elevators, so we are not going to repeat this suggestion). Obviously, mirrors at elevators and playing music in call centers alone do not solve the problem entirely; however, these are changes that are typically relatively inexpensive to implement. A more meaningful (and also low-cost) measure would be to communicate the expected waiting time upfront to the customer (e.g., as done in some call centers or in Disney's theme parks). This way, customers have expectations concerning the wait time and can make a decision whether or not to line up for this service (Disney case) or can even attempt to run other errands while waiting for service (call center case).

9.5 Several Resources with Variability in Sequence

After having analyzed variability and its impact on process performance for the case of very simple processes consisting of just one resource, we now extend our analysis to more complicated process flow diagrams.

Specifically, we analyze a sequence of resources as described in the process flow diagram in Figure 9.5. Such processes are very common, both in manufacturing and service environments:

- The kick-scooter assembly process that we analyzed in Chapter 4 consists (ignoring variability) of multiple resources in sequence.

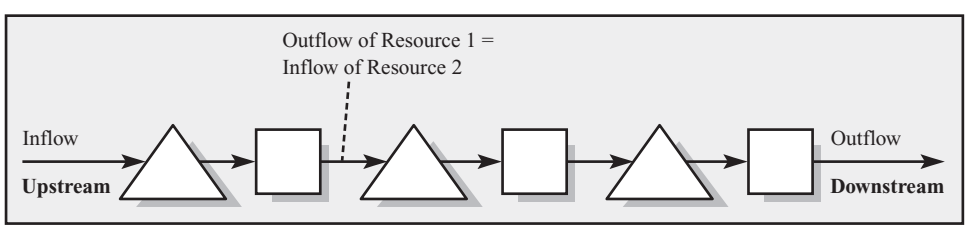


FIGURE 9.5
A Serial Queuing System with Three Resources

- As an example of a service process consisting of multiple resources in sequence, consider the immigration process at most U.S. airports. When arriving in the United States, travelers first have to make their way through the immigration authority and then line up at customs (see chapter 4).

A complicating factor in the analysis of such processes is that the subsequent resources do not operate independently from each other: The departure process of the first resource is the arrival process of the second resource, and so forth. Thus, the variability of the arrival process of the second resource depends on the variability of the arrival process of the first resource and on the variability of the service process of the first resource. What a mess!

Independent of our ability to handle the analytical challenges related to such processes, which also are referred to as tandem queues, we want to introduce some basic intuition of how such processes behave.

The Role of Buffers

Similar to what we have seen in the example of impatient customers and limited buffer space (Figure 9.3), buffers have the potential to improve the flow rate through a process. While, in the case of a single resource, buffers increase flow rate as they reduce the probability that incoming units are denied access to the system, the impact of buffers in *tandem queues* is somewhat more complicated. When looking at a tandem queue, we can identify two events that lead to reductions in flow rate (see Figure 9.6):

- A resource is *blocked* if it is unable to release the flow unit it has just completed as there is no buffer space available at the next resource downstream.
- A resource is *starved* if it is idle and the buffer feeding the resource is empty.

In the trauma center example discussed at the beginning of the chapter, blocking is the most important root cause of ambulance diversion. The actual time the trauma surgeon needs to care for a patient in the trauma bay is only, on average, one hour. However, on average, patients spend one additional hour in the trauma bay waiting for a bed in the intensive care unit (ICU) to become available. Since, during this time, the trauma bay cannot be used for newly arriving patients, a full ICU "backs up" and blocks the trauma center. The study of the General Accounting Office on emergency department crowding and ambulance diversion, mentioned above, pointed to the availability of ICU beds as the single largest source leading to ambulance diversion.

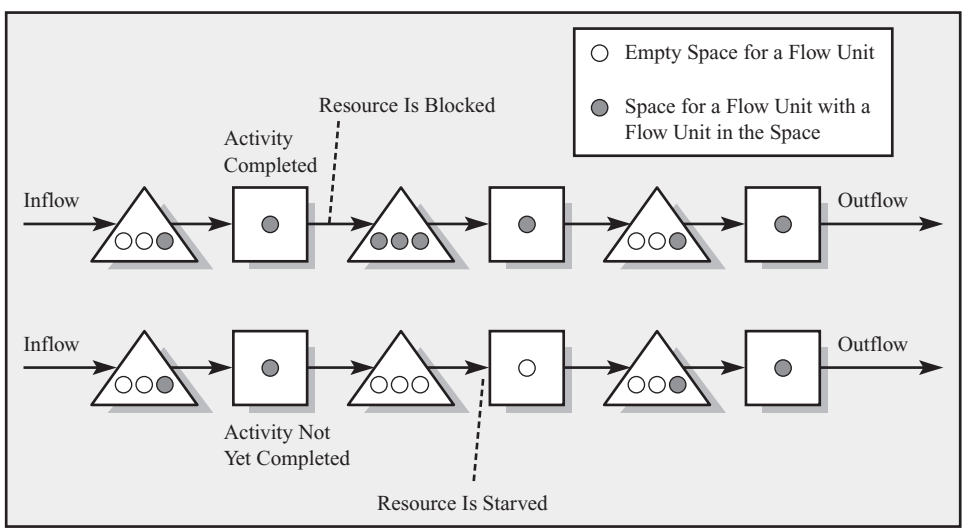


FIGURE 9.6
The Concepts of Blocking and Starving

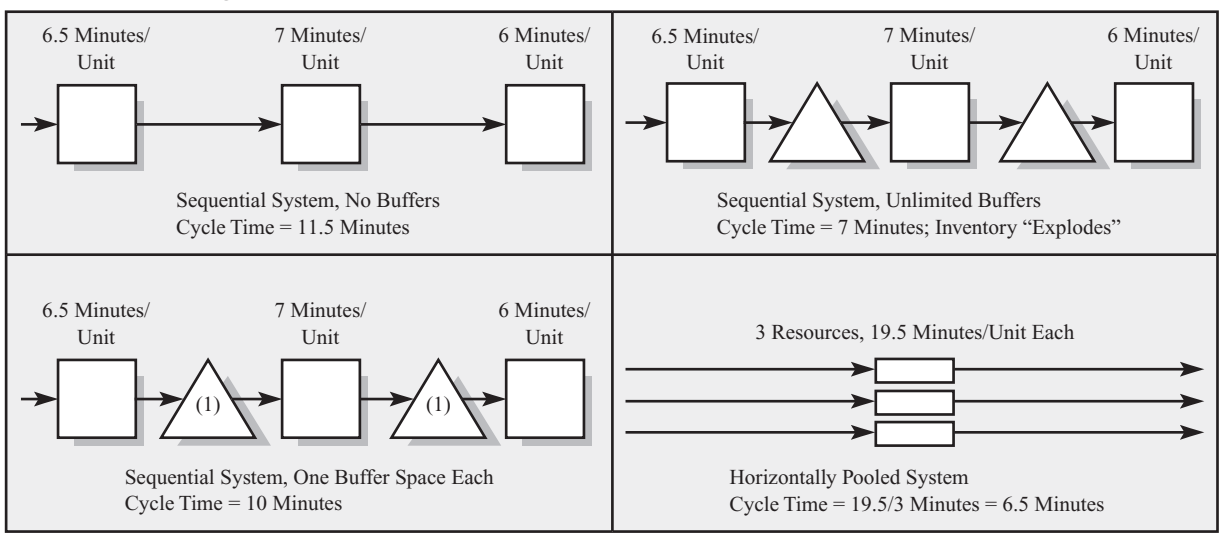
It is important to understand that the effects of blocking can snowball from one resource to additional resources upstream. This can be illustrated in the hospital setting outlined above. Consider a patient who is ready to be discharged from a general care unit at 11 A.M. However, as the patient wants to be picked up by a family member, the patient can only leave at 5 P.M. Consequently, the unit cannot make the bed available to newly arriving patients, including those who come from the ICU. This, in turn, can lead to a patient in the ICU who is ready to be discharged but now needs to wait in the ICU bed. And, yes, you guessed right, this in turn can lead to a patient in the trauma center, who could be moved to the ICU, but now has to stay in the trauma bay. Thus, in a process with limited buffer space, all resources are dependent on another. This is why we defined buffers that help management to relax these dependencies as *decoupling inventory* (Chapter 2).

Blocking and starving can be easily avoided by adding buffers. The buffers would have to contain a sufficient number of flow units so as to avoid starvation of the downstream resource. At the same time, the buffer should have enough space to prevent the resource upstream from ever being blocked. Several hospitals have recently experimented with introducing discharge rooms for patients who are ready to go home from a general care unit: Even a buffer at the end of the process (healthy patient) will reduce the probability that an incoming trauma patient has to be diverted because of a fully utilized trauma center.

In addition to the probability of not being able to admit newly arriving flow units, an important performance measure for our process continues to be the flow rate. Figure 9.7 uses simulation to compare four process layouts of three resources with variability. This situation corresponds to a worker-paced line, with one worker at every resource. The processing times are exponentially distributed with means of 6.5 minutes/unit, 7 minutes/unit, and 6 minutes/unit respectively.

Based on averages, we would expect the process to produce one unit of output every seven minutes. However, in the absence of any buffer space, the process only produces at a rate of one unit every 11.5 minutes (upper left). The process does not realize its full capacity, as the bottleneck is frequently blocked (station 2 has completed a flow unit but cannot forward it to station 3) or starved (station 2 wants to initiate production of the next flow unit but does not receive any input from upstream).

FIGURE 9.7 **Flow Rate Compared at Four Configurations of a Queuing System (Cycle times computed using simulation)**



If we introduce buffers to this process, the flow rate improves. Even just allowing for one unit in buffer before and after the bottleneck increases the output to one unit every 10 minutes (lower left). If we put no limits on buffers, the process is able to produce the expected flow rate of one unit every seven minutes (upper right). Yet, we also observe that the buffer between the first and the second steps will grow very rapidly.

Finally, the lower-right part of Figure 9.7 outlines an alternative way to restore the flow rate, different from the concept of "buffer or suffer" (in fact, the flow rate is even a little larger than in the case of the upper right). By combining the three activities into one activity, we eliminate starving and blocking entirely. This concept is called *horizontal pooling,* as it resembles the concept of pooling identical activities and their previously separate arrival streams that we discussed in Chapter 8. Observe further the similarities between horizontal pooling and the concept of a work cell discussed in Chapter 4.

Given the cost of inventory as well as its detrimental impact on quality discussed in Chapter 10, we need to be careful in choosing where and how much inventory (buffer space) we allow in the process. Since the bottleneck is the constraint limiting the flow rate through the process (assuming sufficient demand), we want to avoid the bottleneck being either starved or blocked. Consequently, buffers are especially helpful right before and right after the bottleneck.

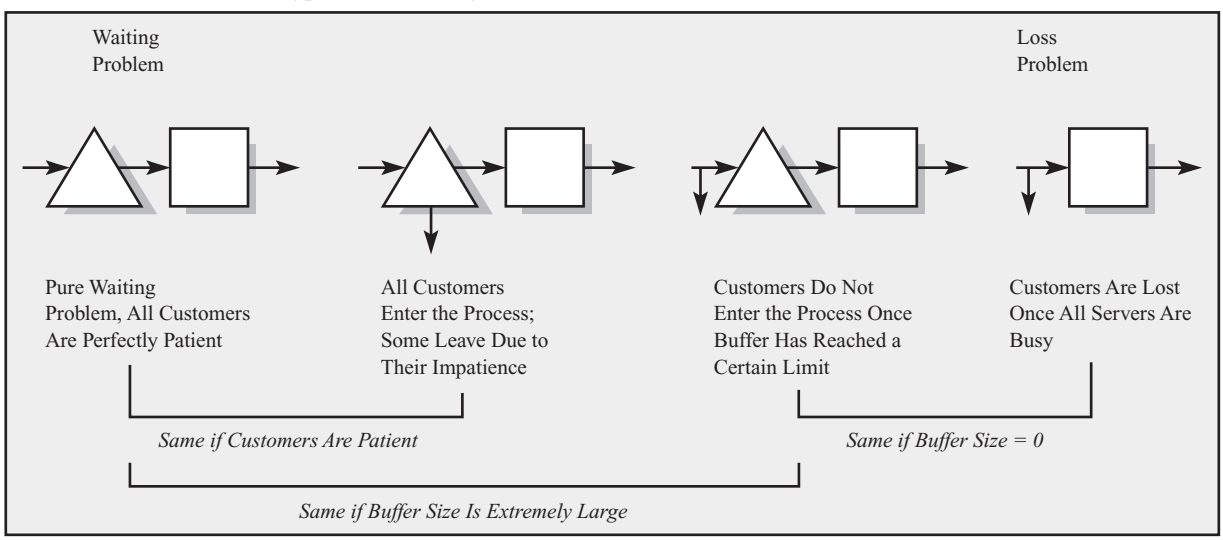
9.6 Summary

Variability not only impacts inventory and wait time but potentially also leads to losses in throughput. In this chapter, we have presented and analyzed the simplest case of such loss systems, consisting of multiple parallel resources with no buffer. The key computations for this case can be done based on the Erlang loss formula.

We then extended our discussion to the case in which customers potentially wait for service but are sufficiently impatient that a loss in throughput can still occur.

Figure 9.8 shows an overview of the various types of scenarios we discussed and, at least partially, analyzed. On the very left of the figure is the waiting problem of Chapter 8; on the very right is the no-buffer loss system (Erlang loss system) presented at the beginning of this chapter. In between are the intermediate cases of impatient customers. Observe that the four process types share a lot of similarities. For example, a wait system with limited, but large, buffer size is likely to behave very similarly to a pure waiting problem.

FIGURE 9.8 **Different Types of Variability Problems**



Similarly, as the buffer size approaches zero, the system behavior approaches the one of the pure loss system. Finally, we also looked at the case of several resources in series, forming a sequence of queues.

From a managerial perspective, the primary objective continues to be to reduce variability wherever possible. All concepts we discussed in Chapter 8 still apply, including the ideas to reduce the variability of service times through standardization and training.

However, since we cannot reduce variability entirely, it is important that we create processes that are robust enough so that they can accommodate as much of the remaining variability as possible. The following should be kept in mind to address throughput loss problems resulting from variability:

- *Use Buffers.* Nowhere else in this book is the concept of "buffer or suffer" so visible as in this chapter. To protect process resources, most importantly the bottleneck, from variability, we need to add buffers to avoid throughput losses of the magnitude in the example of Figure 9.7. In a sequence of resources, buffers are needed right before and right after the bottleneck to avoid the bottleneck either starving or becoming blocked.
- *Keep track of demand.* A major challenge in managing capacity-related decisions in a process with customer loss is to collect *real* demand information, which is required to compute the implied utilization level. Why is this difficult? The moment our process becomes sufficiently full that we cannot admit any new flow units (all trauma bays are utilized, all lines are busy in the call center), we lose demand, and, even worse, we do not even know how much demand we lose (i.e., we also lose the demand information). A common mistake that can be observed in practice is that managers use flow rate (sales) and utilization (Flow rate/Capacity) when determining if they need additional capacity. As we have discussed previously, utilization is by definition less than 100 percent. Consequently, the utilization measure always gives the impression that there is sufficient capacity in place. The metric that really matters is demand divided by capacity (implied utilization), as this reveals what sales could be if there were sufficient capacity.
- *Use background work.* Similar to what we discussed in Chapter 8 with respect to waiting time problems, we typically cannot afford to run a process at the low levels of utilization discussed in the trauma care setting. Instead, we can use less time-critical work to use potential idle time in a productive manner. However, a word of caution is in order. To qualify as background work, this work should not interfere with the time-critical work. Thus, it must be possible to interrupt or delay the processing of a unit of background work. Moreover, we have to ensure that background work does not compete for the same resource as time-critical work further downstream. For example, it has been reported that elective surgery (at first sight a great case of background work for a hospital) can lead to ambulance diversion, as it competes with trauma care patients for ICU capacity.

9.7 Further Reading

Gans, Koole, and Mandelbaum (2003), referenced in Chapter 8, is also a great reading with respect to customer loss patterns. Again, we refer the interested readers to Hillier and Lieberman (2002) and Hall (1997) for additional quantitative methods.

9.8 Practice Problems

Q9.1* **(Loss System)** Flow units arrive at a demand rate of 55 units per hour. It takes, on average, six minutes to serve a flow unit. Service is provided by seven servers.

a. What is the probability that all seven servers are utilized?
b. How many units are served every hour?
c. How many units are lost every hour?

(* indicates that the solution is at the end of the book)

Q9.2** **(Home Security)** A friend of yours approaches you with the business idea of a private home security service. This private home security service guarantees to either dispatch one of their own five guards immediately if one of their customers sends in an alarm or, in the case that all five guards are responding to other calls, direct the alarm to the local police. The company receives 12 calls per hour, evenly distributed over the course of the day.

The local police charges the home security company $500 for every call that the police responds to. It takes a guard, on average, 90 minutes to respond to an alarm.

a. What fraction of the time are incoming alarms directed to the police?

b. How much does the home security company have to pay the local police every month?

Q9.3** **(Video Store)** A small video store has nine copies of the DVD *Captain Underpants, The Movie* in its store. There are 15 customers every day who request this movie for their children. If the movie is not on the shelf, they leave and go to a competing store. Customers arrive evenly distributed over 24 hours.

The average rental duration is 36 hours.

a. What is the likelihood that a customer going to the video store will find the movie available?

b. Assume each rental is $5. How much revenue does the store make per day from the movie?

c. Assume each child that is not able to obtain the movie will receive a $1 bill. How much money would the store have to give out to children requesting *Captain Underpants* every day?

d. Assume the demand for the movie will stay the same for another six months. What would be the payback time (not considering interest rates) for purchasing an additional copy of the movie at $50? Consider the extra revenues related to question b and the potential cost savings (part (c)).

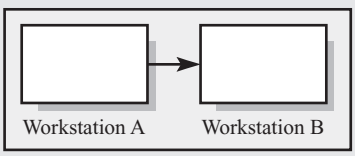
Q9.4 **(Gas Station)** Consider the situation of Mr. R. B. Cheney, who owns a large gas station on a highway in Vermont. In the afternoon hours, there are, on average, 1,000 cars per hour passing by the gas station, of which 2 percent would be willing to stop for refueling. However, since there are several other gas stations with similar prices on the highway, potential customers are not willing to wait and bypass Cheney's gas station.

The gas station has six spots that can be used for filling up vehicles and it takes a car, on average, five minutes to free up the spot again (includes filling up and any potential delay caused by the customer going inside the gas station).

a. What is the probability that all six spots are taken?

b. How many customers are served every hour?

Q9.5 **(Two Workstations)** Suppose a process contains two workstations that operate with no buffer between them.



Now consider the three possible scenarios below:

Scenario	Processing Time of Workstation A	Processing Time of Workstation B
Scenario 1	5 minutes	5 minutes
Scenario 2	5 minutes	4 minutes or 6 minutes equally likely
Scenario 3	5 minutes	3 minutes or 5 minutes equally likely

a. Which of the three scenarios will have, on average, the highest flow rate?

b. Which of the three scenarios will have, on average, the lowest flow time?

Q9.6 **(XTremely Fast Service Inc.)** XTremely Fast Service Inc. is a call center with several business units. One of its business units, Fabulous 4, currently staffs four operators who work eight hours per day Monday through Friday. They provide customer support for a mail-order catalog company. Assume customers call Fabulous 4 during business hours and that—on average—a call arrives every 3 minutes (standard deviation of the interarrival time is equal to 3 minutes). You do NOT have to consider any seasonality in this call arrival pattern. If all four staff members are busy, the customer is rerouted to another business unit instead of being put on hold. Suppose the processing time for each call is 5 minutes on average.

a. What is the probability that an incoming call is *not* processed by Fabulous 4?

b. Suppose that Fabulous 4 receives $1 for each customer that it processes. What is Fabulous 4's daily revenue?

c. Suppose Fabulous 4 pays $5 for every call that gets routed to another business unit. What is its daily transfer payment to the other business unit?

Q9.7 **(Gotham City Ambulance Services)** Gotham City Ambulance Services (GCAS) owns eight ambulances. On average, emergencies are reported to GCAS every 15 minutes (with a coefficient of variation of 1, no seasonality exists). If GCAS has available ambulances, it immediately dispatches one. If there are no ambulances available, the incident is served by the emergency services at a neighboring community. You can assume that in the neighboring community, there is always an ambulance available. On average, an ambulance and its crew are engaged for 1.5 hours (with a coefficient of variation of 1.5) on every call. GCAS operates 24 hours a day.

a. What fraction of the emergencies reported to GCAS are handled by the emergency services at the neighboring community?

b. How many emergencies are served by GCAS during an average 24-hour day?

c. GCAS updated the operating procedures for its staff. This led to a reduction in the coefficient of variation of the time spent on each trip by its staff from 1.5 to 1.25. How will this training program affect the number of emergencies attended to by the GCAS?

d. New regulations require that every emergency service respond to at least 95 percent of all incidents reported in its area of service. Does GCAS need to buy more ambulances to meet this requirement? If yes, how many ambulances will be required? (Assume that the mean time spent on each trip cannot be changed.)

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 10

Quality Management, Statistical Process Control, and Six-Sigma Capability

Many production and service processes suffer from quality problems. Airlines lose baggage, computer manufacturers ship laptops with defective disk drives, pharmacies distribute wrong medications to patients, and postal services lose or misdeliver articles by mail. In addition to these quality problems directly visible to consumers, many quality problems remain hidden from the perspective of the consumer, as they are detected and corrected within the boundaries of the process. For example, products arriving at the end of an assembly process might not pass final inspection, requiring that components be disassembled, reworked, and put together again. Although hidden to the consumer, such quality problems have a profound impact on the economics of business processes.

The main purpose of this chapter is to understand quality problems and to improve business processes with respect to quality. We will do this in five steps:

1. We first introduce the methodology of statistical process control, a powerful method that allows an organization to detect quality problems and to measure the effectiveness of process improvement efforts.
2. We introduce various ways to measure the capability of a process, including the concept of six sigma.
3. One way to achieve a high process capability is to build a process that is sufficiently robust so that deviations from the desired process behavior do not automatically lead to defects.
4. We then discuss how quality problems impact the process flow, thereby extending the process analysis discussion we started in Chapters 3 and 4. Specifically, we analyze how quality problems affect flow rate as well as the location of the bottleneck.
5. We conclude this chapter with a brief description of how to organize and implement quality improvement projects using structured problem-solving techniques.

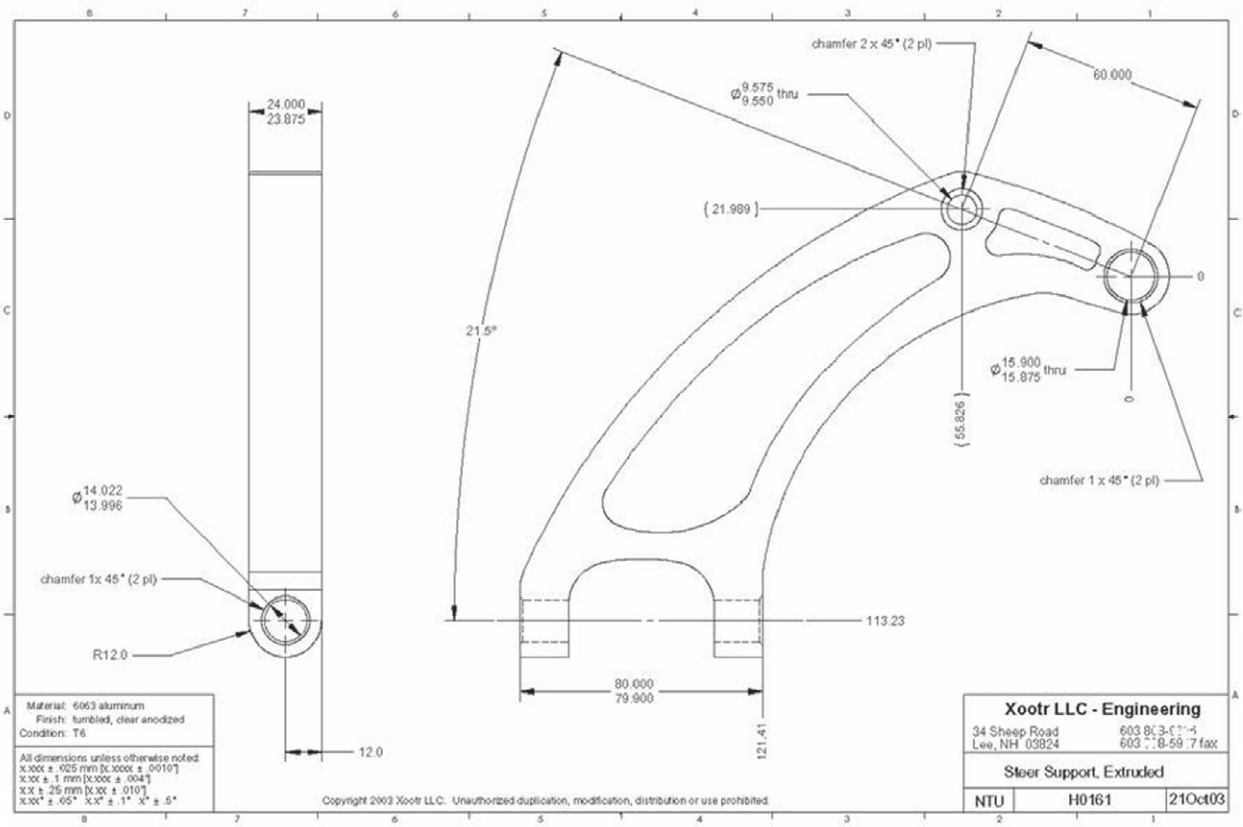
10.1 Controlling Variation: Practical Motivation

Variation is the root cause of all quality problems. To see this, imagine a process without any variation. In this case, the process would either always function as desired, in which case we would not need a chapter on quality, or it would never function as desired, in which case it would be unlikely that our organization would be in business to begin with. We might face variation with respect to durations, as we have discussed in Chapters 8 and 9, but also could encounter variation with respect to other measures, such as the courtesy of a customer service representative in a call center or the physical dimensions of a manufactured component. Thus, (once again) understanding variation, including its sources and its measurement, is essential to improve our operation.

As an example, consider the production of the steer support for the Xootr kick scooter discussed in Chapter 4.[1] The component is obtained via extrusion from aluminum and subsequent refinement at a computer-controlled machine tool (CNC machine). Figures 10.1 and 10.2 show the engineering drawing and the component in the assembly. Despite the fact that every steer support component is refined by the CNC machine, there still exists some variation with respect to the exact geometry of the output. This variation is the result of many causes, including differences in raw materials, the way the component is placed

[1] The authors thank Karl Ulrich of Xootr LLC for his invaluable input.

FIGURE 10.1 **Engineering Drawing of the Steer Support, a Critical Component of the Xootr Scooter**
The height of the steer support is specified by the dimensions (shown in the lower center portion of the drawing) as falling between 79.900 and 80.000 mm.



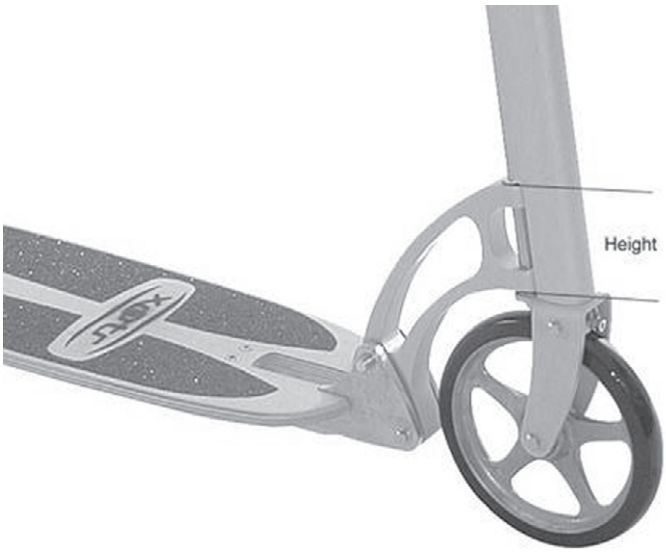


FIGURE 10.2
Steer Support within Xootr Scooter Assembly
The height of the steer support must closely match the opening in the lower handle.

in the machine, the temperature of the room at the time of the processing, an occasional mistake in programming the CNC machine, or some of the many other factors that we discuss further below.

According to the design of the product, the ideal steer support would measure 79.950 mm; the drawing specifies that the height must fall between 79.900 mm and 80.000 mm. If the height is less than 79.900 mm, the part may rattle excessively because it fits loosely. If the height is greater than 80.000 mm, then the part may not fit in the available gap in the handle assembly.

Given that variation of the steer support's height can cause quality problems, the engineers of the company (Xootr LLC) monitor the height very carefully. Every day, a sample of components is taken and measured accurately. Xootr engineers use *statistical process control (SPC)* to achieve the following:

- The company wants to achieve a consistent process that meets the specification as often as possible. SPC allows Xootr LLC to define performance measures that objectively describe the company's ability to produce according to their specifications.
- While a certain amount of variation seems natural, SPC allows Xootr LLC to quickly identify any "abnormally" large variation or changes in the underlying geometry.

10.2 The Two Types of Variation

Before we introduce the method of SPC, it is helpful to reflect a little more about the potential sources of variation. Following the work by W. A. Shewhart and W. E. Deming, we distinguish between two types of variation. *Common causes* of variation refer to constant variation reflecting pure randomness in the process. At the risk of being overly poetic for an operations management textbook, let us note that no two snowflakes are alike and no two flowers are exactly identical. In the same way, there is inherent variation in any business process and consequently no two steer support parts that Xootr can build will be exactly identical. Given that common-cause variation corresponds to "pure" randomness, a plot of the heights for a sample of steer support parts would have a shape similar to the normal distribution. Thus, for the case of common-cause variation, we cannot predict the exact

outcome for the randomness in every single flow unit, yet we can describe the underlying randomness in the form of a statistical distribution that applies to the larger population.

Assignable causes of variation are those effects that result in changes of the parameters of the underlying statistical distribution of the process. For example, a mistake in programming the CNC machine, an operator error, or wear and tear of the extrusion machine would be assignable causes. Such causes are not common for all steer support parts; they only affect a subset. For those parts affected by the assignable cause, the distribution of heights looks statistically different and might have a higher variance or a different mean. The objective of many process improvement projects is to "assign" changes in process behavior to such causes and then to prevent them from recurring in the future.

To understand the notion of common causes of variation and how they differ from assignable causes, consider the following illustrative example. Take a piece of paper and write three rows, each containing the capital letter R eight times. Use your "normal" writing hand for the first row. Then, switch hands and write the eight Rs in the second row with the hand that you typically do not write with. Finally, for the last row, use your "normal" writing hand for the first four Rs and then switch hands for the last four. The outcome is likely to resemble what is shown in Figure 10.3.

The first row of Rs looks relatively consistent. While not every letter is exactly the same, there exists some (common-cause) variation from one letter to the next. In the second row, we observe a much larger (common-cause) variation with respect to the shape of the eight Rs. However, just as in the first row, there exists no obvious pattern that would allow us to predict the shape of the next letter (e.g., it is not possible to predict the shape of the sixth letter based on the first five letters in the same row). The pattern of letters in the last row is different. Following the fourth R, the process changes substantially. This variation can be clearly assigned to the cause of switching hands.

The distinction between common causes of variation and assignable causes is not a universal truth; it depends on the degree of knowledge of the observer. For example, to a layman, the movement of the Dow Jones Industrial Index might appear totally random, while an experienced trader can easily point to specific causes (earnings announcements, information releases by the government or rating agencies) that explain certain patterns of the market. Thus, just as the layman might learn and understand patterns that currently appear random to her, a process observer will discover new assignable causes in variation that she previously fully attributed to common causes.

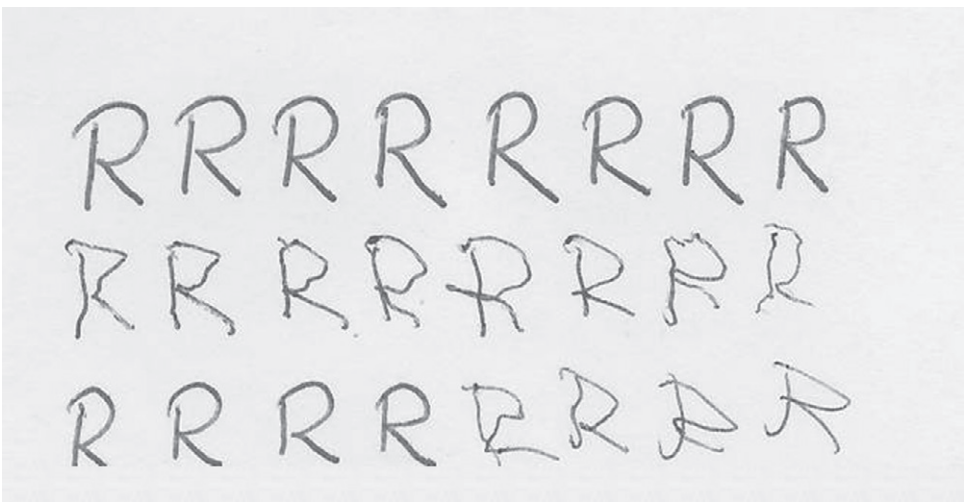


FIGURE 10.3
Examples for Variation Types

The objective of statistical process control is to

- Alert management to assignable causes (i.e., in the case of the third row, we want to set off an alarm as soon after the fifth letter as possible). However, we do not want to alert management to the small variation from one letter to the next in the first two rows.
- Measure the amount of variation in the process, creating an objective measure of consistency (i.e., we want some way to measure that the first row is "better" than the second row).
- Assign causes to variation that currently is perceived as pure randomness and subsequently control these causes, leading to reduced variation and a higher consistency in outcomes.

10.3 Constructing Control Charts

Control charts are graphical tools to statistically distinguish between assignable and common causes of variation. Control charts visualize variation, thereby enabling the user to judge whether the observed variation is due to common causes or assignable causes, such as the breakdown of a machine or an operator mistake.

Control charts are part of a larger set of tools known as statistical process control, a quality movement that goes back to the 1930s, and over the decades included the "quality gurus" W. A. Shewhart, W. E. Deming, and J. M. Juran. Control charts have recently become fashionable again as they are an integral part of the six-sigma movement, introduced by Motorola and publicized widely by General Electric. Although their origin lies in the manufacturing domain, control charts are applicable to service processes equally well. At the end of this section, we discuss an application of control charts in a call center setting.

In order to distinguish between assignable and common causes of variation concerning a specific process outcome, control charts track the process outcome over time. Such process outcomes could be the physical size of a component that is assembled into a scooter or the time it takes a customer service representative to answer a call.

Given that data collection in many environments is costly, control charts are frequently based on samples taken from the process, as opposed to assessing every individual flow unit. Common sample sizes for control charts range between 2 and 10. When constructing a control chart, a sample is drawn in each of several time periods for typically 20 to 50 time periods. In the Xootr case, we will create a control chart based on one month of data and five units sampled every day.

Control charts plot data over time in a graph similar to what is shown in Figure 10.4. The x-axis of the control chart captures the various time periods at which samples from the

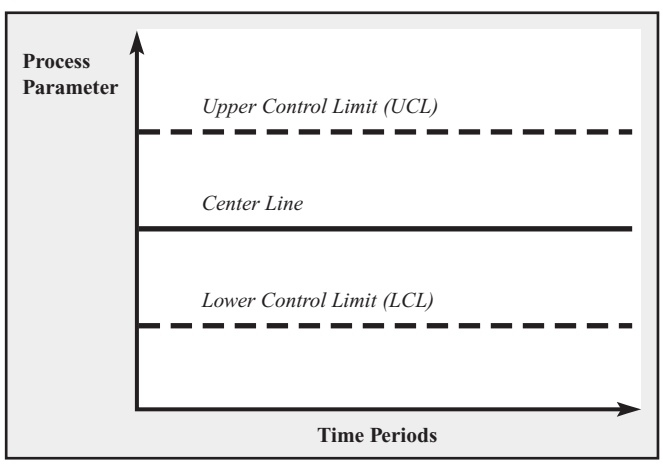


FIGURE 10.4
A Generic Control Chart

process are taken. For the two types of control charts that we discuss in this section, the y-axis plots one of the following two metrics:

- In the $\overline{X}$ chart (pronounced "X-bar chart"), the y-axis corresponds to the mean of each sample. $\overline{X}$ charts can be used to document trends over time and to identify unexpected drifts (e.g., resulting from the wear of a tool) or jumps (e.g., resulting from a new person operating a process step), corresponding to assignable causes of variation.

$$\overline{X} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

  where n is the sample size in each period.
- In the R (range) chart, the y-axis corresponds to the range of each sample. The range is the difference between the highest value in the sample and the lowest value in the sample. Thus,

$$R = \max\{x_1, x_2, \ldots x_n\} - \min\{x_1, x_2, \ldots x_n\}$$

Instead of using the range of the sample, an alternative measure of variability is the standard deviation. The main reason why control charts have historically focused on the range instead of the standard deviation lies in its simplicity with respect to computation and explanation to a broad set of people in an organization.

To familiarize ourselves with the control chart methodology introduced up to this point, consider the data, displayed in Table 10.1, the Xootr engineers collected related to the

TABLE 10.1
Measurements of Steer Support Dimension in Groups of Five Observations

Period	X_1	X_2	X_3	X_4	X_5	Mean	Range
1	79.941	79.961	79.987	79.940	79.956	79.957	0.047
2	79.953	79.942	79.962	79.956	79.944	79.951	0.020
3	79.926	79.986	79.958	79.964	79.950	79.957	0.059
4	79.960	79.970	79.945	79.967	79.967	79.962	0.025
5	79.947	79.933	79.932	79.963	79.954	79.946	0.031
6	79.950	79.955	79.967	79.928	79.963	79.953	0.039
7	79.971	79.960	79.941	79.962	79.918	79.950	0.053
8	79.970	79.952	79.946	79.928	79.970	79.953	0.043
9	79.960	79.957	79.944	79.945	79.948	79.951	0.016
10	79.936	79.945	79.961	79.958	79.947	79.949	0.025
11	79.911	79.954	79.968	79.947	79.918	79.940	0.057
12	79.950	79.955	79.992	79.964	79.940	79.960	0.051
13	79.952	79.945	79.955	79.945	79.952	79.950	0.010
14	79.973	79.986	79.942	79.978	79.979	79.972	0.044
15	79.931	79.962	79.935	79.953	79.937	79.944	0.031
16	79.966	79.943	79.919	79.958	79.923	79.942	0.047
17	79.960	79.941	80.003	79.951	79.956	79.962	0.061
18	79.954	79.958	79.992	79.935	79.953	79.959	0.057
19	79.910	79.950	79.947	79.915	79.994	79.943	0.083
20	79.948	79.946	79.943	79.935	79.920	79.939	0.028
21	79.917	79.949	79.957	79.971	79.968	79.952	0.054
22	79.973	79.959	79.971	79.947	79.949	79.960	0.026
23	79.920	79.961	79.937	79.935	79.934	79.937	0.041
24	79.937	79.934	79.931	79.934	79.964	79.940	0.032
25	79.945	79.954	79.957	79.935	79.961	79.950	0.026
					Average	**79.951**	**0.0402**

height of the steer support component. The data show five observations for each day over a 25-day period. Based on the above definitions of $\overline{X}$ and R, we can compute the last two columns of the table.

For example, for day 14, $\overline{X}$ is computed as

$$\overline{X} = (79.973 + 79.986 + 79.942 + 79.978 + 79.979)/5 = 79.972$$

Similarly, for day 14, R is computed as

$$\begin{aligned} R = {} & \max\{79.973,\ 79.986,\ 79.942,\ 79.978,\ 79.979\} \\ & - \min\{79.973,\ 79.986,\ 79.942,\ 79.978,\ 79.979\} = 0.044 \end{aligned}$$

After computing the mean and the range for every period, we proceed to compute the average range and the average $\overline{X}$ across all days. The average across all $\overline{X}$s is frequently called $\overline{\overline{X}}$ (pronounced "X-double bar"), reflecting that it is an average across averages, and the average range is called $\overline{R}$ (pronounced "R-bar"). As we can see at the bottom of Table 10.1, we have

$$\overline{\overline{X}} = 79.951 \quad \text{and} \quad \overline{R} = 0.0402$$

In creating the $\overline{X}$ chart, we use the computed value of $\overline{\overline{X}}$ as a center line and plot the values of $\overline{X}$ for each day. For the R-chart, we plot the value of R in a chart that uses the average range, $\overline{R}$, as the center line.

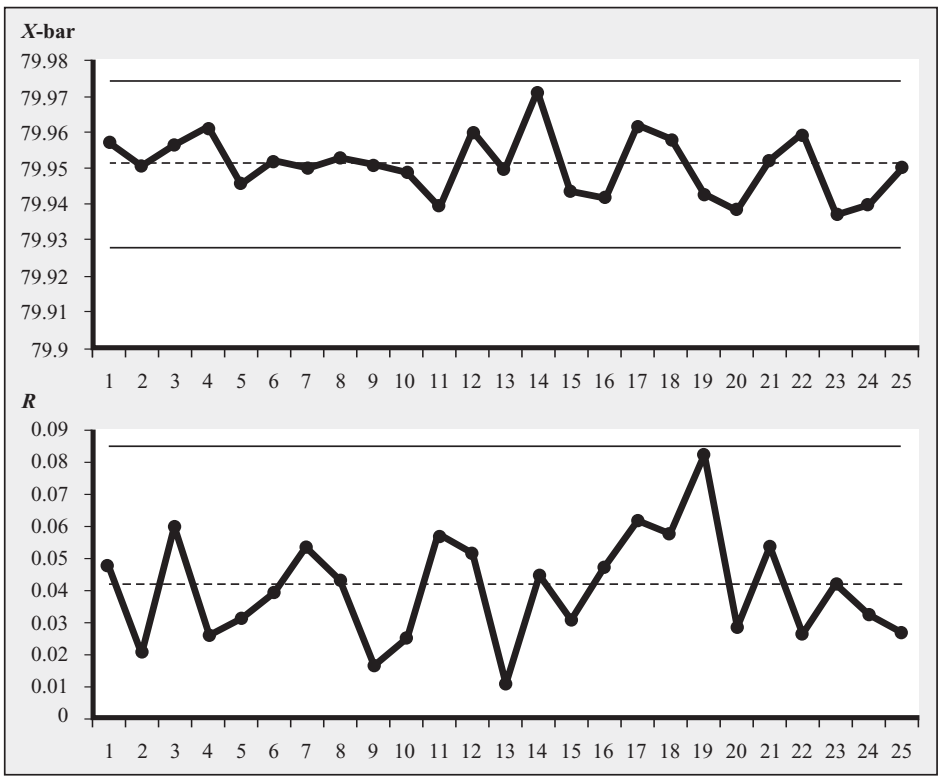
Finally, we have to include the control limits in the charts. We set the control limits such that when we observe an entry for $\overline{X}$ or R outside the control limits (i.e., above the upper control or below the lower control), we can say with 99.7 percent confidence that the process has gone "out of control." Fortunately, we do not have to statistically derive the control limits. Instead, we can use a set of precomputed parameters (summarized in Table 10.2) to compute the control limits based on the following equations:

$$\begin{aligned} \text{Upper control limit for } \overline{X} &= \overline{\overline{X}} + A_2 \times \overline{R} = 79.951 + 0.58 \times 0.0402 = 79.974 \\ \text{Lower control limit for } \overline{X} &= \overline{\overline{X}} - A_2 \times \overline{R} = 79.951 - 0.58 \times 0.0402 = 79.928 \\ \text{Upper control limit for } R &= D_4 \times \overline{R} = 2.11 \times 0.0402 = 0.0848 \\ \text{Lower control limit for } R &= D_3 \times \overline{R} = 0 \times 0.0402 = 0 \end{aligned}$$

TABLE 10.2
Control Chart Parameters for 99.7 Percent Confidence

Number of Observations in Subgroup (n)	Factor for X-Bar Chart (A_2)	Factor for Lower Control Limit in R Chart (D_3)	Factor for Upper Control Limit in R chart (D_4)	Factor to Estimate Standard Deviation (d_2)
2	1.88	0	3.27	1.128
3	1.02	0	2.57	1.693
4	0.73	0	2.28	2.059
5	0.58	0	2.11	2.326
6	0.48	0	2.00	2.534
7	0.42	0.08	1.92	2.704
8	0.37	0.14	1.86	2.847
9	0.34	0.18	1.82	2.970
10	0.31	0.22	1.78	3.078

FIGURE 10.5
***X*-bar Chart and *R* Chart**



The control charts obtained this way allow for a visual assessment of the variation of the process. The definition of control limits implies that 99.7 percent of the sample points are expected to fall between the upper and lower control limits. Thus, if any point falls outside the control limits, we can claim with a 99.7 percent confidence level that the process has gone "out of control," that is, that an assignable cause has occurred.

In addition to an observation $\overline{X}$ falling above the upper control limit or below the lower control limit, a sequence of eight subsequent points above (or below) the center line also should be seen as a warning sign justifying further investigation (in the presence of only common causes of variation, the probability of this happening is simply $(0.5)^8 = 0.004$, which corresponds to a very unlikely event).

Figure 10.5 shows the control charts for the Xootr. We observe that the production process for the steer support is well in control. There seems to be an inherent randomness in the exact size of the component. Yet, there is no systematic pattern such as a drift or a sudden jump outside the control limits.

10.4 Control Chart Example from a Service Setting

To illustrate an application of control charts in a service setting, we turn back to the case of the An-ser call center, the answering service in Wisconsin that we discussed in conjunction with the waiting time formula in Chapter 8. An-ser is interested in an analysis of call durations for a particular type of incoming call, as both mean and variance of call durations impact the customer waiting time (see Chapter 8).

To analyze call durations for this particular type of incoming call, An-ser collected the data displayed in Table 10.3 over a period of 27 days. Similar to the Xootr case, we

TABLE 10.3
Data for a Control Chart at An-ser

Period	X_1	X_2	X_3	X_4	X_5	Mean	Range
1	1.7	1.7	3.7	3.6	2.8	2.7	2
2	2.7	2.3	1.8	3.0	2.1	2.38	1.2
3	2.1	2.7	4.5	3.5	2.9	3.14	2.4
4	1.2	3.1	7.5	6.1	3.0	4.18	6.3
5	4.4	2.0	3.3	4.5	1.4	3.12	3.1
6	2.8	3.6	4.5	5.2	2.1	3.64	3.1
7	3.9	2.8	3.5	3.5	3.1	3.36	1.1
8	16.5	3.6	2.1	4.2	3.3	5.94	14.4
9	2.6	2.1	3.0	3.5	2.1	2.66	1.4
10	1.9	4.3	1.8	2.9	2.1	2.6	2.5
11	3.9	3.0	1.7	2.1	5.1	3.16	3.4
12	3.5	8.4	4.3	1.8	5.4	4.68	6.6
13	29.9	1.9	7.0	6.5	2.8	9.62	28.0
14	1.9	2.7	9.0	3.7	7.9	5.04	7.1
15	1.5	2.4	5.1	2.5	10.9	4.48	9.4
16	3.6	4.3	2.1	5.2	1.3	3.3	3.9
17	3.5	1.7	5.1	1.8	3.2	3.06	3.4
18	2.8	5.8	3.1	8.0	4.3	4.8	5.2
19	2.1	3.2	2.2	2.0	1.0	2.1	2.2
20	3.7	1.7	3.8	1.2	3.6	2.8	2.6
21	2.1	2.0	17.1	3.0	3.3	5.5	15.1
22	3.0	2.6	1.4	1.7	1.8	2.1	1.6
23	12.8	2.4	2.4	3.0	3.3	4.78	10.4
24	2.3	1.6	1.8	5.0	1.5	2.44	3.5
25	3.8	1.1	2.5	4.5	3.6	3.1	3.4
26	2.3	1.8	1.7	11.2	4.9	4.38	9.5
27	2.0	6.7	1.8	6.3	1.6	3.68	5.1
					Average	**3.81**	**5.85**

can compute the mean and the range for each of the 27 days. From this, we can then compute the overall mean:

$$\overline{\overline{X}} = 3.81 \text{ minutes}$$

and the average range

$$\overline{R} = 5.85 \text{ minutes}$$

We then compute the control limits using the constants from Table 10.2:

$$\text{Upper control limit for } \overline{X} = \overline{\overline{X}} + A_2 \times \overline{R} = 3.81 + 0.58 \times 5.85 = 7.20$$

$$\text{Lower control limit for } \overline{X} = \overline{\overline{X}} - A_2 \times \overline{R} = 3.81 - 0.58 \times 5.85 = 0.42$$

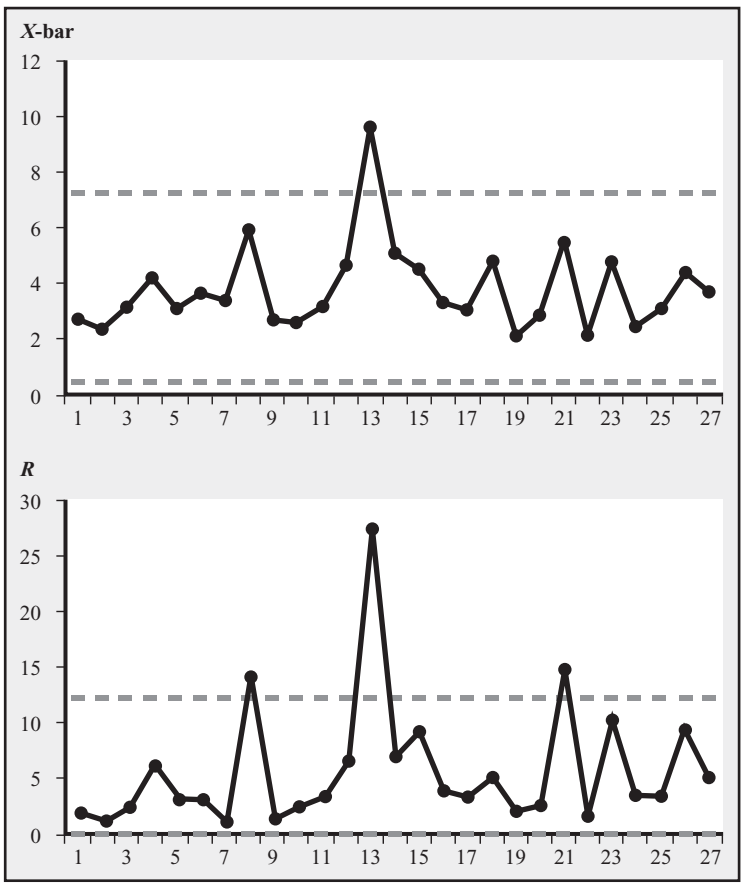
$$\text{Upper control limit for } R = D_4 \times \overline{R} = 2.11 \times 5.85 = 12.34$$

$$\text{Lower control limit for } R = D_3 \times \overline{R} = 0 \times 5.85 = 0$$

Combining the control limits with the values of the mean, $\overline{X}$, and the range, *R,* we obtain the control charts shown in Figure 10.6.

As we can see in Figure 10.6, the call durations exhibit a fair amount of variation. This leads to a large average range, *R*-bar (lower part of Figure 10.6), and explains the

FIGURE 10.6
Control Charts for the An-ser Case



large interval between the upper and lower control limits. Despite these relatively "forgiving" control limits, we observe that the process moves out of control on day 13, when the mean, $\bar{X}$, jumps up to 9.62 (upper part of Figure 10.6). There are also two additional days when the *R* chart indicates an abnormally large variation in the process.

Going back to the data we collected (Table 10.3), we see that this exceptionally large mean is driven by one long call duration of almost half an hour. Despite having one observation drive the result, we know with 99.7 percent confidence that this long duration was not just "bad luck" but indeed reflects an assignable cause. Thus, further investigation is warranted.

In this particular case, An-ser management discovered that several calls on the days in question were handled by an operator that typically handled different types of calls. Further data analysis revealed large operator-to-operator variation for the exact same type of call. This is visible in Table 10.4. Note that all calls are of the same type, so that the duration difference can fully be attributed to the operator. Table 10.4 indicates that customer service

TABLE 10.4
Comparison of Operators

	CSR 1	CSR 2	CSR 3	CSR 4	CSR 5
Mean	2.95	3.23	7.63	3.08	4.26
Standard deviation	0.96	2.36	7.33	1.87	4.41

representative 1 (CSR 1) has the lowest mean call durations. She also has the lowest standard deviation, indicating that she has the most control over her calls. In fact, listening to a sample of randomly recorded calls indicated that CSR 1 fully complied with the script for the call type. In contrast, CSR 3 took more than twice as long when answering the same calls. Moreover, her standard deviation was seven times as large. Listening to a sample of recorded calls from CSR 3 confirmed a lack of consistency and large deviations with respect to the established script.

10.5 Design Specifications and Process Capability

So far, we have focused our discussion on the question to what extent the process is "in control." However, it is important to understand that a process that is in control might still fail to deliver the quality demanded from the customer or a downstream operation in the process. The reason for this lies in the definition of the control limits. Consider again the Xootr example. Since we set the control limits of 79.928 and 79.974 according to how the process performed in the past (25 days in the case above), we only measure to what extent the process is operating in line with its historical behavior (in the spirit of the letter *R* in Figure 10.3, the first two rows were "in control," despite the poor handwriting in the second row). This, however, contains little information about the degree to which the process is meeting the *design specification*s of 79.900 mm to 80.000 mm.

The consistency requirement from the customer typically takes the form of a design specification. A design specification includes

- A *target value* (79.950 mm in the case of the steer support component).
- A *tolerance level,* describing the range of values that are acceptable from the customer's perspective, [79.900 mm, 80.000 mm] for the steer support.

Again, note that design specifications are driven by the needs of the downstream process or by the end customer, while control limits are driven by how the process step has been operating in the past. Thus, it is very well possible that a process is "in control" yet incapable of providing sufficiently tight tolerances demanded by the customer. Vice versa, we say that a process, while being "in control," is capable if it can produce output according to the design specifications.

So, how do we know if a given process is capable of meeting the tolerance level established by the design specifications? This depends on

- The tightness of the design specification, which we can quantify as the difference between the upper specification level (USL) and the lower specification level (LSL).
- The amount of variation in the current process, which we can estimate based on the range *R.* For small sample sizes, we can translate the range *R* into an estimator of the standard deviation using the following equation:

$$\hat{\sigma} = \overline{R}/d_2$$

where $\hat{\sigma}$ stands for the estimated standard deviations and the values of d_2 are summarized in Table 10.2. For the steer support point, we have:

$$\begin{aligned}\hat{\sigma} &= \overline{R}/d_2 \\ &= \frac{0.0402}{2.326} = 0.017283\end{aligned}$$

Note that one also can estimate the standard deviation using a traditional statistical approach.

Thus, to increase the capability of the process in meeting a given set of design specifications, we either have to increase the tolerance level or decrease the variability in the process. We can combine these two measures into a single score, which is frequently referred to as the *process capability* index:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}$$

Thus, the process capability index C_p measures the allowable tolerance relative to the actual variation of the process. Figure 10.7 compares different values of C_p for a given set of design specifications. As we can see, the much lower variation (σ_B) of the process in the lower part of the figure will make it less likely that a defect will occur; that is, that the process creates a flow unit that falls above the upper specification limit or below the lower specification limit.

For the steer support component, we compute the process capability measure as follows:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} = \frac{80.000 - 79.900}{6 \times 0.017283} = 0.964345$$

A capability index of $C_p = 1$ would correspond to a process that meets the quality requirements 99.7 percent of the time. In other words, the process would have 28 defects per 10,000 units.

Traditionally, quality experts have recommended a minimum process capability index of 1.33. However, Motorola, as part of its six-sigma program, now postulates that all efforts should be made to obtain a process capability C_p of 2.0 at every individual step.

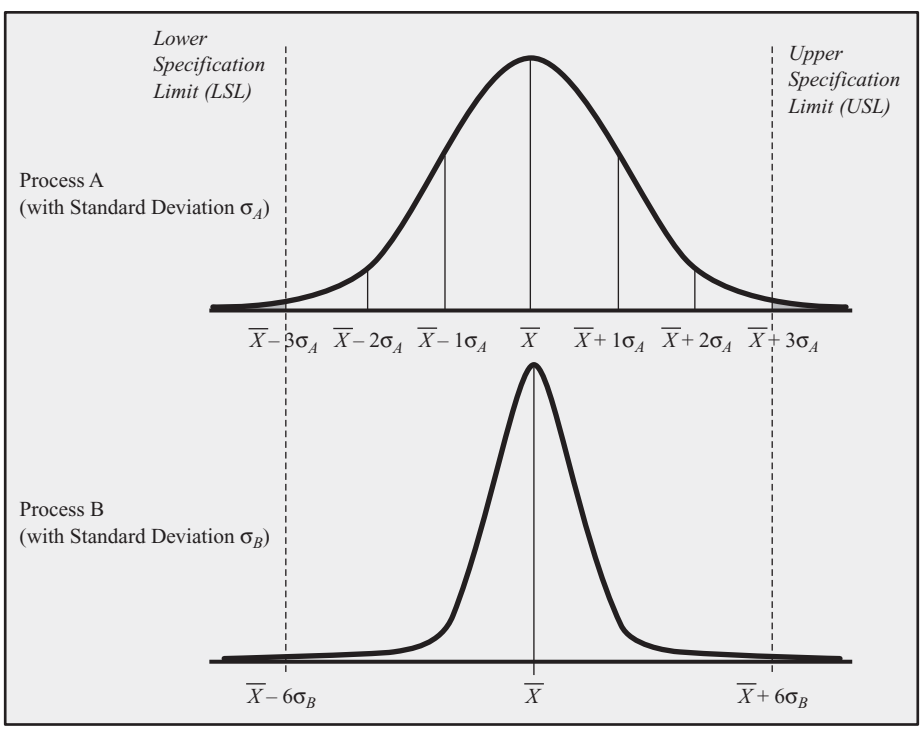


FIGURE 10.7
Comparison of Three-Sigma and Six-Sigma Process Capability

This is statistically equivalent to requiring that the USL is six standard deviations above the mean and the LSL is six standard deviations below the mean. This explains the name "six-sigma" (see Figure 10.7).

Xootr LLC uses process capability scores to compare different production technologies. For example, recently, the company considered streamlining its production process for the steer support component. Instead of extruding the part and then machining it, management suggested eliminating the machining step and using the extruded part directly for production.

Xootr LLC conducted a formal analysis of this proposal based on the process capability index C_p. Collecting data similar to Table 10.1, the company found that eliminating the machining step would lead to a dramatic increase in defects, reflecting a much lower process capability index (the design specifications have not changed and there is a much higher variation in height in absence of the machining step), and hence decided not to pursue this potentially cheaper production process.

10.6 Attribute Control Charts

Rather than collecting data concerning a specific variable and then comparing this variable with specification limits to determine if the associated flow unit is defective or not, it is frequently desirable to track the percentage of defective items in a given sample. This is especially the case if it is difficult to come up with a single variable, such as length or duration, that captures the degree of specification conformance. This is the idea behind *attribute control charts.*

To construct an attribute control chart, we need to be able to distinguish defective from nondefective flow units. In contrast to variable control charts, this distinction does not have to be made based on a single dimension. It could be the result of many variables with specification limits and even qualitative factors, as long as they can be measured consistently. For example, an airline tracking defects corresponding to lost luggage, a pharmacy trying to reduce the number of patients that were provided the wrong drugs, or a data entry operation struggling with handwriting recognition all would likely use an attribute control chart.

Sample sizes for attribute control charts tend to be larger, typically ranging from 50 to 200. Larger sample sizes are needed in particular if defects are relatively rare events. Samples are collected over several periods, just as in the case of variable control charts. Within each sample we evaluate the percentage of defective items. Let p denote this percentage. We then compute the average percentage of defects over all samples, which we call $\bar{p}$. This "average across averages" is the center line in our attribute control chart, just as we used $\bar{\bar{X}}$ as the center line for variable control charts.

To compute the control limits, we first need to obtain an estimate of the standard deviation of defects. This estimate is given by the following equation:

$$\text{Estimated standard deviation} = \sqrt{\frac{\bar{p}(1-\bar{p})}{\text{Sample size}}}$$

We then compute the upper and lower control limits:

$$\text{UCL} = \bar{p} + 3 \times \text{Estimated standard deviation}$$
$$\text{LCL} = \bar{p} - 3 \times \text{Estimated standard deviation}$$

Thus, we again set control limits such that the process is allowed to vary three standard deviations in each direction from the mean.

Whether one should use a variable control chart or an attribute control chart depends on the type of problem at hand.

- If there exists a single, measurable variable that determines if a unit is defective or not, one should always use variable control charts. The advantage of the variable control chart is that it makes use of valuable information that is discarded in attribute control charts. For example, if three sampled units were all very close to (yet still below) the upper specification limit, they would be classified as "nondefective" in the spirit of attribute control charts. In contrast, the variable control chart would use this information as leading to an increased estimated probability that a future unit might be above the upper specification limit.
- If there are many potential causes of defects, variable-based control charts are difficult to implement. Thus, when measuring defects in activities such as order entry in a call center, baggage handling for an airline, or drug handling in a pharmacy, attribute-based control charts should be used.

Given the multiple potential root causes of a defect, it is frequently desirable to find which of these root causes accounts for the majority of the problems. The Pareto diagram is a graphical way to identify the most important causes of process defects. To create a Pareto diagram, we need to collect data on the number of defect occurrences as well as the associated defect types. We can then plot simple bars with heights indicating the relative occurrences of the defect types. It is also common to plot the cumulative contribution of the defect types. An example of a Pareto diagram is shown in Figure 10.8. The figure categorizes defects related to customer orders at Xootr LLC.

Pareto charts were introduced to quality management by J. M. Juran, who observed that managers spent too much time trying to fix "small" problems while not paying enough attention to "big" problems. The Pareto principle, also referred to as the 80-20 rule, postulates that 20 percent of causes account for 80 percent of the problems. In the context of quality, the Pareto principle implies that a few defect types account for the majority of defects.

10.7 Robust Process Design

As discussed above, variation in a process parameter such as the geometry of a part or the duration of a service activity is at the root of all quality problems. So identifying the sources of variation and eliminating them should always be the first priority when aiming for a quality improvement.

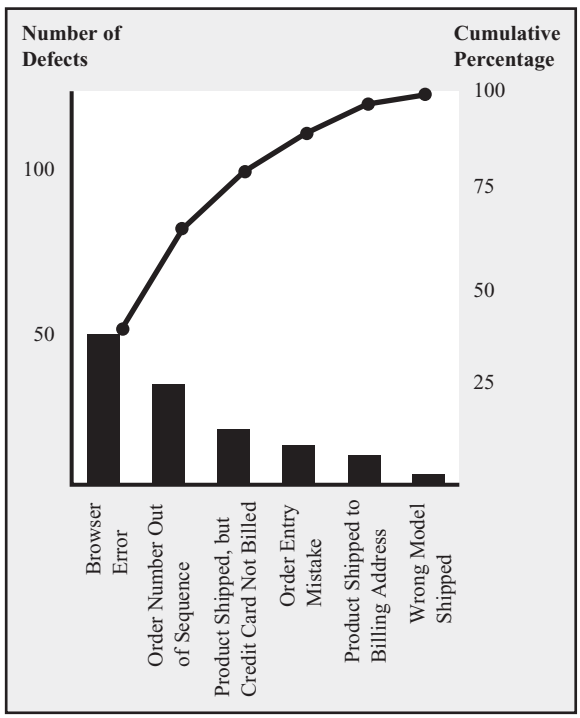
However, eliminating variation is not always possible. Especially when dealing with human resources (e.g., assembly-line workers) or human flow units (patients, calls in a call center), we are always exposed to variation that is beyond our control. Moreover, often the sources of variation might be under our control, yet their elimination might be prohibitively expensive.

For these reasons, instead of just fighting variation, we also need to be able to accommodate it. We need to design processes that are robust, that is, that do not fall apart and produce defects the moment they are exposed to variation. A good tennis player should always aim to hit the ball with the sweet spot of her racket, yet a good tennis racket also should be "forgiving" in that it does not lead to a poor shot the moment the hit is less than perfect.

To understand the concept of robust process design, consider the following illustrative example. Many universities and business schools are blessed (or cursed) with on-site restaurants, coffee shops, or cafeterias. As part of their baking operations, a large coffee shop needs to define an operating procedure to bake chocolate chip cookies.

FIGURE 10.8
Sources of Problems with Customer Orders at Xootr

Cause of Defect	Absolute Number	Percentage	Cumulative Percentage
Browser error	43	39%	39%
Order number out of sequence	29	26%	65%
Product shipped, but credit card not billed	16	15%	80%
Order entry mistake	11	10%	90%
Product shipped to billing address	8	7%	97%
Wrong model shipped	3	3%	100%
Total	**110**		



There are many important product attributes customers care about when it comes to the chocolate chip cookies that they purchase for $1.19 a piece. Yet, probably the most important one is the cookie's chewiness—is it too soft and still tastes like it is "half-baked" or is it too hard and crunches like a brick. The two key parameters that determine the chewiness are the bake time (the duration that the cookies are in the oven) and the oven temperature.

To state it formally:

$$\text{Chewiness of cookie} = F_1(\text{Bake time}) + F_2(\text{Oven temperature})$$

where F_1 and F_2 are two functions illustrated in Figure 10.9.

Note that there exists more than one way to obtain any given chewiness value. We can bake the cookie for 24 minutes at 240 degrees (process design A) or we can bake them for 21 minutes at 280 degrees (process design B). For the sake of argument, say that, from the customer's perspective, these two are identical.

A reality of baking cookies and selling them fresh is that this type of process is often exposed to a fair bit of variation. The typical operator involved in this process has received

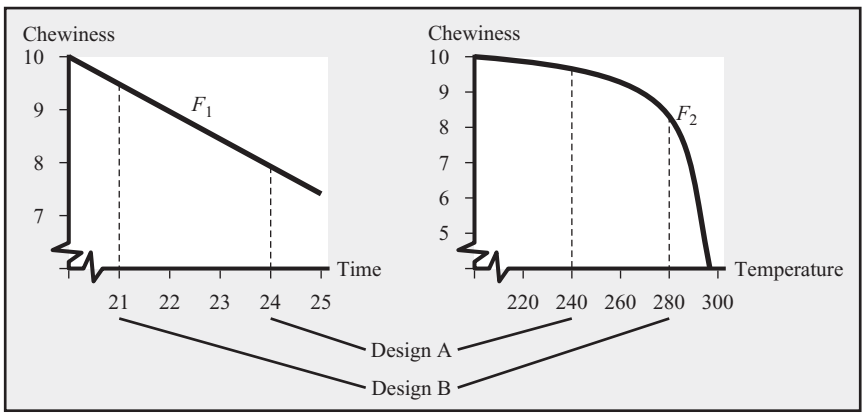


FIGURE 10.9
Two Different Process Recipes for Making Chocolate Chip Cookies

little training and is paid a relatively low wage rate. Often, the baking is carried out by operators who also have other responsibilities (i.e., they don't sit next to the oven during the baking time), making it likely that the baking time can vary ± 1 minute from the recipe and/or the actual oven temperature can vary ± 10 degrees.

So, which process recipe should we use? Process design A or process design B? From the customer's perspective and ignoring the effect of variation, it seems as if this choice does not matter. However, keeping the effect of variation in the actual baking time in mind, a second look at Figure 10.9 reveals that going for the 21-minute baking time can be a risky endeavor. The shorter baking time requires us to use a higher oven temperature. And, at this high oven temperature, even small variations in baking time and oven temperature (especially too high temperatures) can lead to bad outcomes.

For this reason, we say that process design A is more robust—it is more tolerant of variation in the process parameters. We can formalize the concept of robustness by looking at a two-dimensional plot such as shown in Figure 10.10. The figure shows a set of lines that

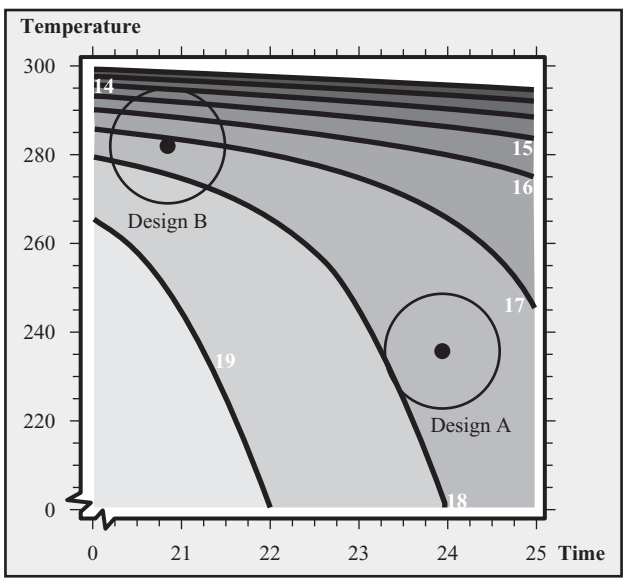


FIGURE 10.10
Cookie Example (continued)

correspond to time–temperature combinations yielding the same chewiness. Each shade of grey in the figure hence roughly corresponds to the same expected chewiness. Now, compare design A with design B in the figure by including

- A dot for the time–temperature combination.
- A circle around the dot that corresponds to all possible time–temperature combinations that might be chosen as a result of variation.

What makes process recipe A more robust is that the circle capturing all likely variations of the recipe stays entirely in one area of chewiness: we can afford a variation in the input parameters without suffering a variation in the output.

10.8 Impact of Yields and Defects on Process Flow

Defects, as described in previous sections, have a profound impact on the process flow. In this section, we discuss processes consisting of a sequence of process steps, of which at least one step suffers from detectable quality problems. In other words, there exists at least one step at which units are separated into "good units" and "defective units." Whereas good items can continue processing at the next operation, defective units either have to be *reworked* or are *eliminated from the process* (known as scrapped in the manufacturing context).

- In the case of the Xootr, the company scraps all steer support parts that do not meet the specifications as discussed previously.
- In contrast, Xootr LLC reworks Xootrs that require adjustments in the brake assembly. These Xootrs are rerouted to a separate operator in charge of rework. This (highly skilled) operator disassembles the brake (typically scrapping the brake cable) and adjusts the brake as needed, thereby creating a sellable Xootr.

The following examples help illustrate that the ideas of rework and flow unit elimination are by no means restricted to manufacturing:

- Following heart surgery, patients typically spend time recovering in the intensive care unit. While most patients can then be moved to a regular unit (and ultimately be sent home), some patients are readmitted to the intensive care unit in case of complications. From the perspective of the ICU, patients who have been discharged to regular units but then are readmitted to the ICU constitute rework.
- The recruitment process of large firms, most prominently the one of consulting companies, also exhibits a large percentage of flow units that are eliminated before the end of the process. For every offer made, consulting firms process hundreds of résumés and interview dozens of job candidates (possibly staged in several rounds). Typically, job candidates are eliminated from the applicant pool—rework (a job candidate asked to repeat her first-round interviews) is very rare.
- Pharmaceutical development analyzes thousands of chemical compounds for every new drug that enters the market. The initial set of compounds is reduced through a series of tests, many of which are very costly. After a test, some units are allowed to proceed to the next phase, while others are eliminated from the set of potential compounds for the clinical indication the company is looking for.

We define the *yield* of a resource as:

$$\text{Yield of resource} = \frac{\text{Flow rate of units processed successfully at the resource}}{\text{Flow rate}} = 1 - \frac{\text{Flow rate of defects at the resource}}{\text{Flow rate}}$$

Thus, the yield of a resource measures the percentage of good units that are processed at this resource. Similarly, we can define yields at the level of the overall process:

$$\text{Process yield} = \frac{\text{Flow rate of units processed successfully}}{\text{Flow rate}} = 1 - \frac{\text{Flow rate of defects}}{\text{Flow rate}}$$

Obviously, the words *defects* and *rework* sound harsh in some of the examples described above, especially if we are dealing with human flow units. However, the following concepts and calculations apply equally well for disk drives that have to be reworked because they did not meet the specifications of final tests and patients that have to be readmitted to intensive care because they did not recover as quickly as required to safely stay in a regular hospital unit.

It also should be pointed out that a defect does not always reflect the failure of a process step, but can reflect inherent randomness (common cause variation) in the process or differences with respect to the flow units at the beginning of the process. For example, dismissing a chemical compound as a potential cure for a given disease, does not imply that previous development steps did not do their job correctly. Instead, the development steps have simply revealed a (previously unknown) undesirable property of the chemical compound. Similarly, it lies in the nature of a recruiting process that its yield (percentage of applications resulting in a job) is well below 100 percent.

Rework

Rework means that some steps prior to the detection of the problem must be redone, or some additional process steps are required to transform a defective unit into a good unit. Two examples of rework are shown in Figure 10.11 (inventory locations are left out for simplicity).

In the upper part of the figure, defective units are taken out of the regular process and moved to a separate rework operation. This is common in many production processes

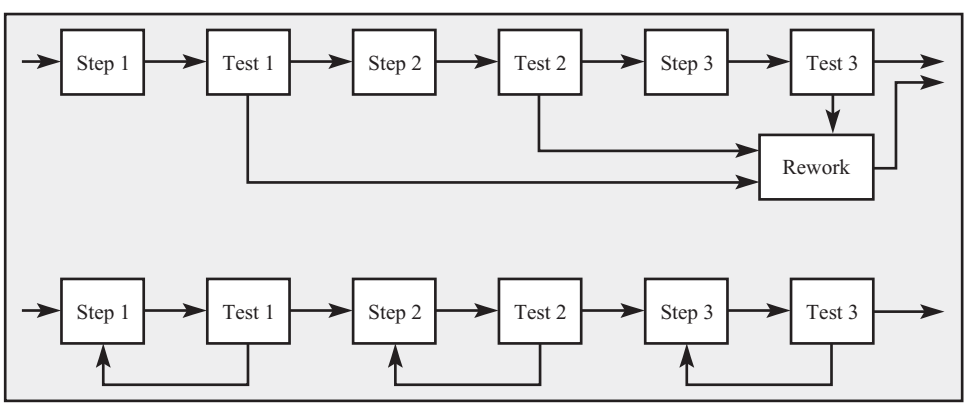


FIGURE 10.11
Two Processes with Rework

such as in the Xootr example discussed above. If the rework step is always able to turn a defective unit into a good unit, the process yield would return to 100 percent. In the lower part of the figure, defective units are reworked by the same resource that previously processed the unit. The readmission of a patient to the intensive care unit corresponds to such a case.

Rework changes the utilization profile of the process. Compared to the case of no defects, rework means that a resource has additional work flowing to it, which in turn increases utilization. As a consequence, rework can potentially change the location of the bottleneck.

Thus, when analyzing the influence of yields (and rework) on process capacity, we need to distinguish between bottleneck and nonbottleneck resources. If rework involves only nonbottleneck machines with a large amount of idle time, it has a negligible effect on the overall process capacity (note that it will still have cost implications, reflecting costs of material and extra labor at the rework step).

In many cases, however, rework is severe enough to make a resource a bottleneck (or, even worse, rework needs to be carried out on the bottleneck). As the capacity of the bottleneck equals the capacity of the overall process, all capacity invested in rework at the bottleneck is lost from the perspective of the overall process.

Eliminating Flow Units from the Process

In many cases, it is not possible or not economical to rework a flow unit and thereby transform a defective unit into a good unit. Once the Xootr machine has produced a defective steer support unit, it is almost impossible to rework this unit into a nondefective unit. Instead, despite an approximate material cost of $12 for the unit, the company scraps the unit and produces a replacement for it.

Similarly, a consulting firm searching for a new hire will prefer to simply reject the application, instead of investing in training to improve the job candidate's skills. If defective units are eliminated from the process, final output of good units is correspondingly reduced.

Strictly speaking, eliminating flow units from the process is a special form of rework, where all operations between the step where the defective unit leaves the process and the beginning of the process have to be reworked. Given that all operations up to the point of defect detection have to be reworked, the earlier we can detect and eliminate the corresponding flow unit, the less we waste capacity. This wasted capacity reflects that more units need to be started in the process than are finished. For example, to get 100 good units at the end of the process, we have to start with

$$\text{Number of units started to get 100 good units} = 100/\text{Process yield}$$

at the beginning of the process.

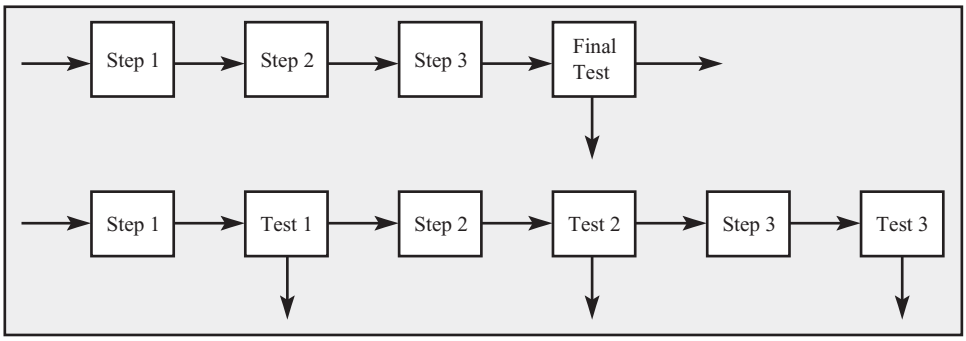
Two examples of processes in which defective units are eliminated are shown in Figure 10.12. In the upper part of the figure, defects are only detected at the end, and thereby have wasted capacity of every resource in the process. In the lower part of the figure, a test is conducted after every process step, which allows for the early elimination of defective parts, leading to less wasted capacity.

In a process in which defective units are eliminated, we can write the process yield as

$$\text{Process yield} = y_1 \times y_2 \times \cdots \times y_m$$

where m is the number of resources in the sequence and y_i is the yield of the ith resource.

FIGURE 10.12
Process with Scrap



Cost Economics and Location of Test Points

In addition to their effect on capacity, yields determine the value that a good unit has at various stages in the process. What is the value of a good unit in the process? The answer to this question will differ depending on whether we are capacity constrained or whether we are constrained by demand.

Consider the demand-constrained case first. At the beginning of the process, the value of a good item equals its input cost (the cost of raw material in the case of production). The value of a good unit increases as it moves through the process, even if no additional material is being added. Again, let y_n be the yield at the nth stage. The value leaving resource n is approximately $1/y_n$ times the sum of the value entering stage n plus any variable costs we incur at stage n.

The capacity-constrained case is fundamentally different. At the end of the process, the marginal extra revenue of the unit determines the value of a good unit. Yet, at the beginning of the process, the value of a good unit still equals its input costs. So should the valuation of a good unit be cost-based working forward or price-based working backwards? The discontinuity between these two approaches comes at the bottleneck operation. After the bottleneck, value is based on selling price; before the bottleneck, it is based on cost.

For example, assume that Xootr LLC is currently demand-constrained and we want to value a flow unit as it moves through the process. We should do this using a cost-based calculation, as—independent of a defect in this flow unit—we will achieve the same sales rate (i.e., we fulfill demand). In contrast, if Xootr LLC is capacity-constrained, we have to factor in the marginal extra revenue for those flow units that have already passed the bottleneck.

As a consequence of this, the costs that arise with detecting a defect dramatically increase as a flow unit moves through the process to market. Consider the case of a nonreworkable defect occurring at a prebottleneck resource, as depicted in Figure 10.13. If the defect is detected before the bottleneck, the costs of this defect are simply the costs of the materials that went into the unit up to the detection of the defect. However, if the defect is detected after the bottleneck and the process is currently capacity-constrained, the unit is almost as valuable as a complete unit. In the extreme case, if the defect is detected on the market, we are likely to incur major costs related to warranty, field repair, liability, and so forth. For this reason, in a capacity-constrained process, it is essential to have an inspection step prior to the bottleneck.

At a more conceptual level, Figure 10.13 relates to an idea referred to as *quality at the source,* an element of the Toyota Production System emphasizing that defects should be

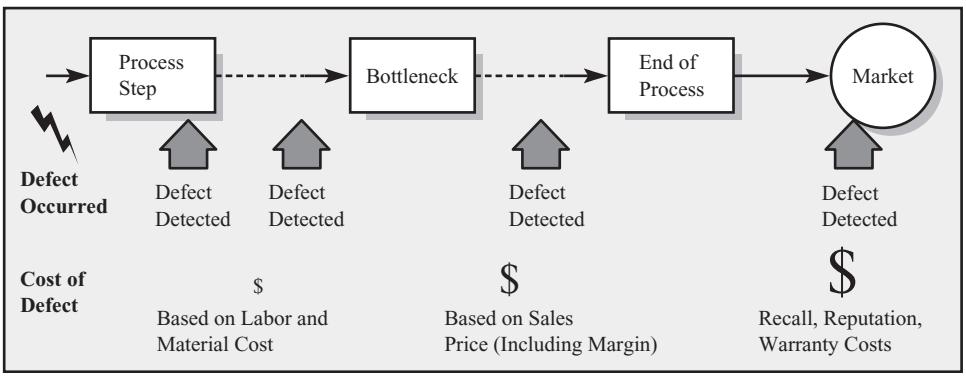


FIGURE 10.13
Cost of a Defect as a Function of Its Detection Location, Assuming a Capacity-Constrained Process

detected right when and where they occur, as opposed to being detected in a remote final inspection step. In addition to the cost benefits discussed above, another advantage of quality at the source is that the correction of the root cause that led to the defect is typically much easier to identify at the place and time when the defect is made. While a worker in charge of a process step that leads to a defect is likely to remember the context of the defect, figuring out what went wrong with a unit at a final inspection step is typically much harder.

Defects and Variability

Quality losses and yield-related problems not only change the capacity profile of a process, but they also cause variability. A yield of 90 percent means not that every tenth flow unit is defective, but that there is a 10 percent probability of a defect occurring. Thus, yield losses increase variability, which—as we have seen in Chapters 8 and 9—is the enemy of capacity.

Consider again the process flow diagram in the lower part of Figure 10.11, that is, a process where defective units are immediately reworked by repeating the operation. Even if the actual activity time is deterministic, yield losses force items into multiple visits at the same resource, and thus make the effective activity time for a *good* item a random variable.

Capacity losses due to variability can be partially compensated by allowing inventory after each operation with yields below 100 percent. The larger these buffers, the more the capacity-reducing impact of variability is reduced. However, additional inventory increases costs and flow times; it also can hurt the detection and solution of quality problems, as we discussed in Chapter 9.

10.9 A Process for Improvement

The strength of the statistical process control techniques discussed in this chapter results from their combination of collecting actual data with using professional analysis techniques.

The importance of data collection cannot be overemphasized. In many industries, collecting data about process performance is the exception rather than the norm. Once you have collected data, process improvement meetings turn fact-based and objective as opposed to being largely subjective. While most manufacturing facilities by now routinely collect data about their processes, most service processes are lagging behind. Only in the

last couple of years have service providers in banking or health care started to systemically track process data. This is somewhat surprising given that services are often blessed with loads of data because of their electronic workflow management systems.

But a successful process improvement project needs more than data. It is important to statistically analyze data. Otherwise, every small, random change in the process (including common cause variation) is interpreted as meaningful and acted upon. The tools outlined above help to separate the important from the unimportant.

In addition to statistical tools, it is also essential to have a clear action plan on how to organize a project aiming at process improvement. A well executed process improvement project tends to go through the following steps:

- You sense a problem and explore it broadly.
- You formulate a specific problem to work on/state a specific improvement theme.
- You collect data and analyze the situation.
- You find the root causes.
- You plan a solution and implement it.
- You evaluate the effects of the solution.
- You standardize the process to include the new solution if it is good.
- Then you take on the next problem.

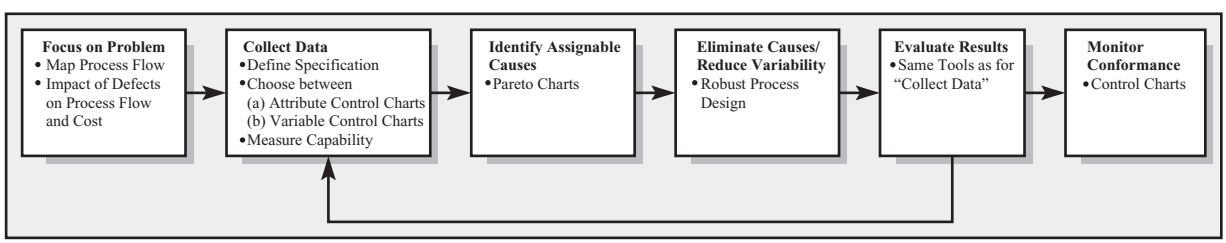
Figure 10.14 summarizes the tools introduced in this chapter by outlining a systematic process to achieve quality improvement.

The focus of the improvement project is guided by where defects are most costly and hence improvements have the biggest economic impact. Typically, this involves the bottleneck resource. We then collect data and analyze it, determining process capabilities and exact yields. This helps us understand the impact of defects on the process flow and ultimately on the economics of the process.

We have a choice between thinking of defects in a binary way (defect versus no defect) or based on a specific set of customer specifications (upper and lower specification limits). In the former case, we use attribute control charts; otherwise we use regular control charts as introduced previously in this chapter. This analysis lets us determine our current process capability. By classifying the defects and assigning them to causes (Pareto analysis), we also can find out the most significant root causes.

We then either eliminate these root causes or, using the robust process design logic, attempt to minimize their sensitivity to variation in process parameters. The resulting improved process is monitored and analyzed in the same way as previously, which either confirms or disconfirms the usefulness of our action. This is an iterative process, reflecting that there are multiple (potentially interacting) causes and a potentially limited understanding of the process.

FIGURE 10.14



Finally, control charts help with respect to standardizing a solution and in determining the degree of conformance with the new process design. They will also alert us of an emergence of any new assignable causes.

10.10 Further Reading

Wadsworth, Stephens, and Godfrey (1986) provide an excellent overview of various control charting methods. Their book also includes several examples of implementation. Breyfogle (1999) provides a detailed overview of many tools and definitions underlying six sigma. Interested readers also should look at the initial Motorola document about six sigma, which is summarized in Motorola (1987).

Six-sigma training is often done using a catapult to help illustrate that it often is better to consistently hit a spot that is slightly off target as opposed to occasionally hitting the target, yet hit a wide range of different points as well. See www.xpult.com for more details on six sigma and catapults.

More details on quality can be found in the earlier work by Juran (1951) or the more recent work Juran (1989).

Bohn and Terwiesch (1999) provide a framework for analyzing the economics of yield-driven processes, which we used as the foundation for the discussion of rework and scrap.

Ulrich and Eppinger (2011) is an excellent source for more details about robust process design and the design of experiments to improve products and processes.

Finally, the small booklet "Memory Jogger" is a highly effective manual for the quality improvement tools discussed in this chapter and beyond.

10.11 Practice Problems

Q10.1 **(Quality)** Consider the following potential quality problems:

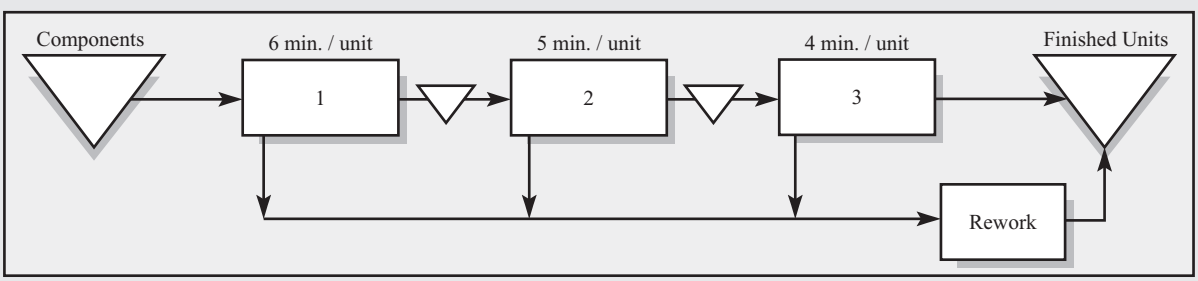
- Wine that is served in a restaurant sometimes is served too warm, while at other times it is served too cold.
- A surgeon in a hospital follows the hygiene procedures in place on most days, but not all days.
- A passenger traveling with an airline might be seated at a seat with a defective audio system.
- An underwriter in a bank might sometimes accidentally approve loans to consumers that are not creditworthy.

For each of these potential problems:

a. What type of data would you collect?

b. What type of control charts would you use?

Q10.2 **(Process with Rework)** Consider the following three-stage production process of glass ceramics, which is operated as a worker-paced line.



The process is experiencing severe quality problems related to insufficiently trained workers. Specifically, 20 percent of the parts going through operation 1 are badly processed by the operator. Rather than scrapping the unit, it is moved to a highly skilled rework operator, who can correct the mistake and finish up the unit completely within 15 minutes.

The same problem occurs at station 2, where 10 percent of the parts are badly processed, requiring 10 minutes of rework. Station 3 also has a 10 percent ratio of badly processed parts, each of them requiring 5 minutes by the rework operator.

a. What is the utilization of station 2 if work is released into the process at a rate of 5 units per hour?
b. Where in the process is the bottleneck? Why? (Remember, the bottleneck is the resource with the lowest capacity, independent of demand.)
c. What is the process capacity?

Chapter 11

Lean Operations and the Toyota Production System

Toyota is frequently associated with high quality as well as overall operational excellence, and, as we will discuss in this chapter, there are good reasons for this association—Toyota has enjoyed decades of economic success while changing the history of operations management.

- Various elements of the company's famous Toyota Production System (TPS) are covered throughout this book, but in this chapter we will review and summarize the components of TPS, as well as a few that have not been discussed in earlier chapters.
- We also will illustrate how the various elements of TPS are intertwined, thereby making it difficult to adapt some elements while not adapting others.

As we will discuss, one of the key objectives of TPS is the elimination of "waste" from processes such as idle time, unnecessary inventory, defects, and so forth. As a result, people often refer to (parts of) TPS as "lean operations." The expression "lean operations" has been especially popular in service industries.

11.1 The History of Toyota

To appreciate the elegance and success of the Toyota Production System, it is helpful to go back in time and compare the history of the Toyota Motor Company with the history of the Ford Motor Corporation.

Inspired by moving conveyor belts at slaughterhouses, Henry Ford pioneered the use of the assembly line in automobile production. The well-known Model T was the first mass-produced vehicle that was put together on an assembly line using interchangeable parts. Working with interchangeable parts allowed Ford to standardize assembly tasks, which had two important benefits. First, it dramatically reduced variability, and thereby increased quality. Second, it streamlined the production process, thereby making both manual and automated assembly tasks faster.

With the luxury of hindsight, it is fair to say that Ford's focus was on running his automotive production process with the goal of utilizing his expensive production equipment

as much as possible, thereby allowing him to crunch out the maximum number of vehicles. Ford soon reached an unmatched production scale—in the early days of the Model T, 9 out of 10 automotive vehicles in the world were produced by Ford! Benefiting from his scale economies, Ford drove the price of a Model T down, which made it affordable to the American middle class, an enormous market that was well suited to be served by mass production.

The Toyota Motor Corporation grew out of Toyota Industries, a manufacturer of automated looms, just prior to World War II. Toyota supported the Japanese army by supplying it with military trucks. Given the shortages of most supplies in Japan at that time, Toyota trucks were equipped with only one headlight and had an extremely simplistic design. As we will see, both the heritage as a loom maker as well as the simplicity of its first vehicle product had consequences for the future development of Toyota.

Following the war, shortages in Japan were even more severe. There existed virtually no domestic market for vehicles and little cash for the acquisition of expensive production equipment. The United States had an active role in the recovery process of Japan and so it is not surprising that the American production system had a strong influence on the young automaker. Toyota's early vehicles were in part produced using secondhand U.S. equipment and also otherwise had significant resemblances with the U.S. brands of Dodge and Chevrolet.

As inspiring as the Western industrial engineering must have been to Toyota, replicating it was out of the question. Mass production, with its emphasis on scale economies and large investments in machinery, did not fit Toyota's environment of a small domestic market and little cash.

Out of this challenging environment of scarcity, Toyota's management created the various elements of a system that we now refer to as the Toyota Production System (TPS). TPS was not invented overnight—it is the outcome of a long evolution that made Toyota the most successful automaker in the world and the gold standard for operations management.

Following a long period of growth, Toyota became the world's top automaker in the year 2008. Since then, Toyota experienced two crises. First, in the fourth quarter of 2009 and first quarter of 2010, Toyota recalled several million vehicles in response to reports of unintended vehicle acceleration. Toyota executives were questioned by the U.S. Congress, and the numerous reasons for a set of fatal accidents were widely discussed in the U.S. media. Early in 2011, the National Highway Traffic Safety Administration, in collaboration with NASA, released a report that identified driver error, as the main root cause behind the accidents (in most instances, drivers confused the gas pedal with the brake pedal). Despite the negative publicity associated with the recalls, Toyota was able to keep its position as the world's top automaker.

Second, following the Japanese earthquake of March 2011, Toyota was forced to shut down several of its assembly plants. Moreover, the company (and others) faced supply shortages of important automotive parts. The full impact of the earthquake on Toyota's 2011 production and its relative impact compared to other automakers is still unclear as we write this third edition.

But enough about Toyota—this chapter is not about Toyota, but it is about TPS. Many other industries are implementing TPS, with examples ranging from health care to banking. You can use TPS in your organization, whether you work for Toyota or for the German government. And, even Toyota does not always follow TPS. Thus, the power of TPS does not depend on Toyota's position in the ranking of the world's top automakers.

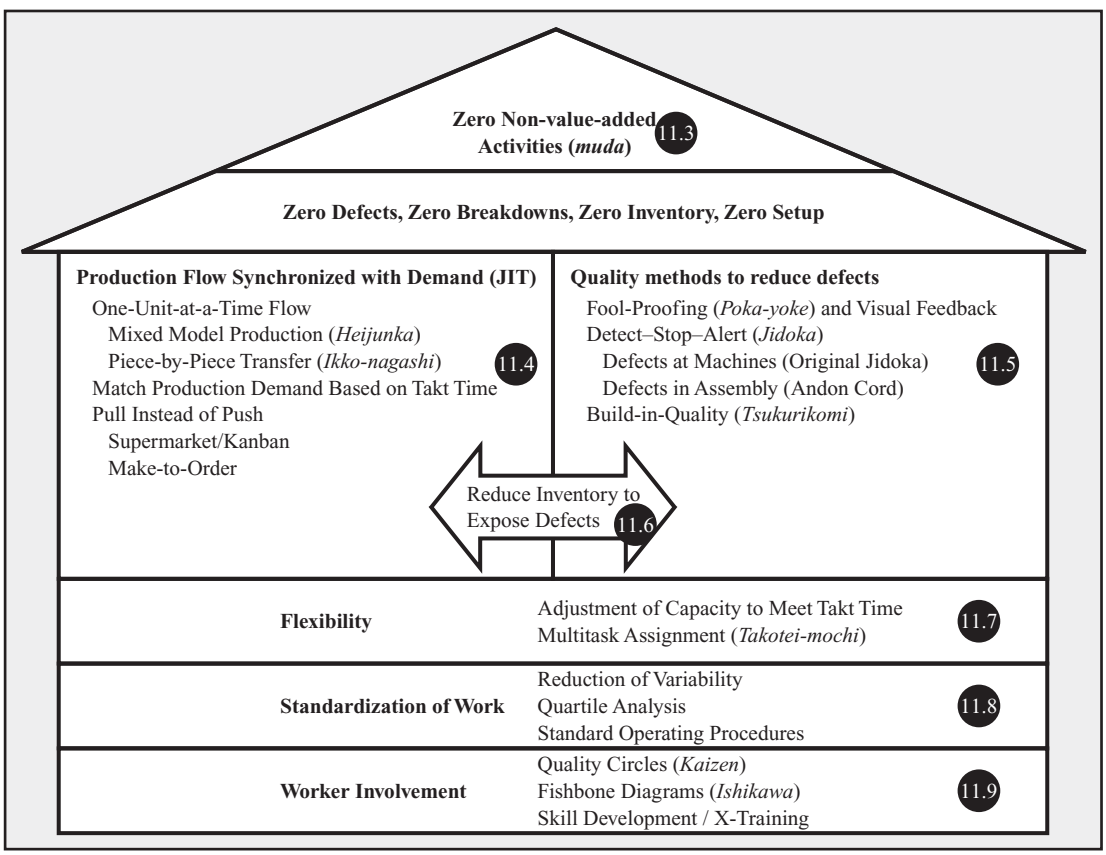
11.2 TPS Framework

While TPS is frequently associated with certain buzzwords such as JIT, kanban, and kaizen, one should not assume that simply implementing any of these concepts would lead to the level of operational excellence at Toyota. TPS is not a set of off-the-shelf solutions for various operational problems, but instead a complex configuration of various routines ranging from human resource management to the management of production processes.

Figure 11.1 summarizes the architecture of TPS. At the top, we have the principle of waste reduction. Below, we have a set of methods that help support the goal of waste reduction. These methods can be grouped into JIT methods (JIT stands for just-in-time) and quality improvement methods. There exist strong interdependencies among the various methods. We will discuss some of these interdependencies throughout this chapter, especially the interaction between JIT and quality.

Collectively, these methods help the organization to attack the various sources of waste that we will define in the next section. Among them are overproduction, waiting, transport, overprocessing, and inventory, all of which reflect a mismatch between supply and demand. So the first set of methods that we will discuss (Section 11.4) relate to synchronizing the

FIGURE 11.1 **The Basic Architecture of TPS**
(The numbers in the black circles correspond to the related section numbers of this chapter.)



production flow with demand. Output should be produced exactly when the customer wants it and in the quantity demanded. In other words, it should be produced just in time.

If we want to obtain a flow rate of the process that reliably matches demand while also following the just-in-time idea, we have to operate a process with no defects and no breakdowns. This is a direct consequence of our discussion in Chapters 8 and 9 (buffer or suffer): defects create variability and the only way we can obtain our target flow rate in a process with variability is to use buffers.

Toyota's strong emphasis on quality lets the company overcome the buffer-or-suffer tension: by producing with zero defects and zero breakdowns, the company neither has to suffer (sacrifice flow rate) nor to buffer (hold inventory). For this reason, and the fact that defects are associated with the waste of rework, quality management is the second pillar around which TPS is built.

Both JIT and quality management require some foundational methods such as the standardization of work (which eliminates variability), the flexibility to scale up and down process capacity in response to fluctuations in demand, and a set of human resource management practices.

11.3 The Seven Sources of Waste

In the late 1980s, a research consortium known as the International Motor Vehicle Program (IMVP) conducted a global benchmarking of automotive plants. The study compared quality and productivity data from plants in Asia, Europe, and North America. The results were a clear indication of how far Toyota already had journeyed in redesigning the historical concept of mass production.

Consider the data displayed in Table 11.1, which compares the General Motors Framingham assembly plant with the Toyota Takaoka assembly plant. The Toyota plant was about twice as productive and had three times fewer defects compared to the GM plant making a comparable vehicle. Moreover, it used its manufacturing space more efficiently and turned its components and parts inventory dramatically faster.

While the data underlying this exhibit are already 25 years old, they are still of high relevance today. First, the IMVP study in many ways was the first true proof of the superiority of TPS. For that reason, it constituted a milestone in the history of industrialization. Second, while all large automotive manufacturers have made substantial improvements since the initial data collection, two more recent rounds of benchmarking (see Holweg and Pil 2004) documented that the productivity of Japanese manufacturers has been a moving target. While U.S. and European manufacturers could improve their productivity, the Japanese producers have continued to improve theirs so that Toyota still enjoys a substantial competitive advantage today.

TABLE 11.1
General Motors Framingham Assembly Plant versus Toyota Takaoka Assembly Plant
(Based on 1986 benchmarking data from the IMVP Assembly Plant Survey.)

	GM Framingham	Toyota Takaoka
Gross Assembly Hours per Car	40.7	18
Assembly Defects per 100 Cars	130	45
Assembly Space per Car	8.1	4.8
Inventories of Parts (average)	2 weeks	2 hours

Notes: Gross assembly hours per car are calculated by dividing total hours of effort in the plant by the total number of cars produced.
Defects per car were estimated from the JD Power Initial Quality Survey for 1987.
Assembly Space per Car is square feet per vehicle per year, corrected for vehicle size.
Inventories of Parts are a rough average for major parts.

Source: Womack, Jones, and Roos (1991).

What accounts for the difference in productivity between the GM and the Toyota plant? Both processes end up with a very comparable car after all. The difference in productivity is accounted for by all the things that GM did that did not contribute to the production of the vehicle: non-value-added activities. TPS postulates the elimination of such non-value-added activities, which are also referred to as *muda.*

There are different types of muda. According to T. Ohno, one of the thought leaders with respect to TPS, there are seven sources of waste:

1. Overproduction. Producing too much, too soon, leads to additional waste in the forms of material handling, storage, and transportation. The Toyota Production System seeks to produce only what the customer wants and when the customer wants it.
2. Waiting. In the spirit of "matching supply with demand," there exist two types of waiting. In some cases, a resource waits for flow units, leading to idle time at the resource. Utilization measures the amount of waiting of this type—a low utilization indicates the resource is waiting for flow units to work on. In other cases, flow units wait for resources to become available. As a consequence, the flow time is longer than the value-added time. A good measure for this second type of waiting is the percentage of flow time that is value-added time (in the language of Chapter 8, this is the processing time, p, relative to the flow time, $T = T_q + p$). Both types of waiting reflect a poorly balanced process and can be reduced by using the tools outlined in Chapter 4.
3. Transport. Internal transport, be it carrying around half-finished computers, wheeling patients through the hospital, or carrying around folders with insurance claims, corresponds to the third source of waste. Processes should be laid out such that the physical layout reflects the process flow to minimize the distances flow units must travel through a process.
4. Overprocessing. A close analysis of activity times reveals that workers often spend more time on a flow unit than necessary. A worker might excessively polish the surface of a piece of metal he just processed or a doctor might ask a patient the same questions that a nurse has asked five minutes earlier.
5. Inventory. In the spirit of matching supply with demand, any accumulation of inventory has the potential to be wasteful. Inventory is closely related to overproduction and often indicates that the JIT methods have not (yet) been implemented correctly. Not only is inventory often non-value-adding, it often hides other problems in the process as it leads to long information turnaround times and eases the pressure to find and eliminate underlying root causes (see Section 11.6 for more details).
6. Rework. A famous saying in the Toyota Production System and the associated quality movement has been "Do it right the first time." As we have discussed in the previous chapter, rework increases variability and consumes capacity from resources. Not only does rework exist in manufacturing plants, it is also (unfortunately) common in service operations. For example, hospitals all too frequently repeat X-rays because of poor image quality or readmit patients to the intensive care unit.
7. Motion. There are many ways to perform a particular task such as the tightening of a screw on the assembly line or the movement of a patient from a wheelchair into a hospital bed. But, according to the early pioneers of the industrial revolution, including Frederick Taylor and Frank and Lillian Gilbreth, there is only one "right way." Every task should be carefully analyzed and should be optimized using a set of tools that today is known as ergonomics. To do otherwise is wasteful.

Just as we have seen in the context of line balancing, the objective of waste reduction is to maximize the percentage of time a resource is engaged in value-adding activity by reducing the non-value-added (wasteful) activities as much as possible.

At this point, a clarification of wording is in order. TPS's objective is to achieve zero waste, including zero inventory and zero defects. However, this objective is more an aspirational one than it is a numerical one. Consider the objective of zero inventory and recall from Little's Law: Inventory = Flow rate × Flow time. Thus, unless we are able to produce at the speed of light (flow time equal to zero), the only way to achieve zero inventory is by operating at zero flow rate—arguably, not a desirable outcome. So, of course, Toyota's factories don't operate at zero inventory, but they operate at a low level of inventory and keep on decreasing this low level. The same holds for zero defects. Defects happen in each of Toyota's assembly plants many, many times a shift. But they happen less often than elsewhere and are always thought of as a potential for process improvement.

It is important to emphasize that the concept of waste is not unique to manufacturing. Consider, for example, the day of a nurse in a large hospital. In an ideal world, a nurse is there to care for patients. Independent of managed care, this is both the ambition of the nurse and the desire of the patient. However, if one carefully analyzes the workday of most nurses, a rather different picture emerges. Most nurses spend less than half of their time helping patients and waste the other time running around in the hospital, doing paperwork, searching for medical supplies, coordinating with doctors and the hospital administration, and so on. (See Tucker [2004] for an excellent description of nursing work from an operations management perspective.) This waste is frustrating for the nurse, leads to poor care for the patient, and is expensive for the health care provider.

Once we have reduced waste, we can perform the same work, yet at lower costs. In a process that is currently capacity constrained, waste reduction is also a way to increase output (flow rate) and hence revenues. As we have discussed in Chapter 6, the economic impact of these improvements can be dramatic.

A useful way to analyze and describe the effects of waste is the Overall Equipment Effectiveness (OEE) framework, used by McKinsey and other consulting firms. The objective of the framework is to identify what percentage of a resource's time is true, value-added time and what percentage is wasted. This provides a good estimate for the potential for process improvement before engaging in waste reduction.

As is illustrated by Figure 11.2, we start the OEE analysis by documenting the total available time of the resource. From this total time (100 percent), some time is wasted on machine breakdowns (or, in the case of human resources, absenteeism) and setup times, leading to an available time that is substantially less than the total planned time (in this

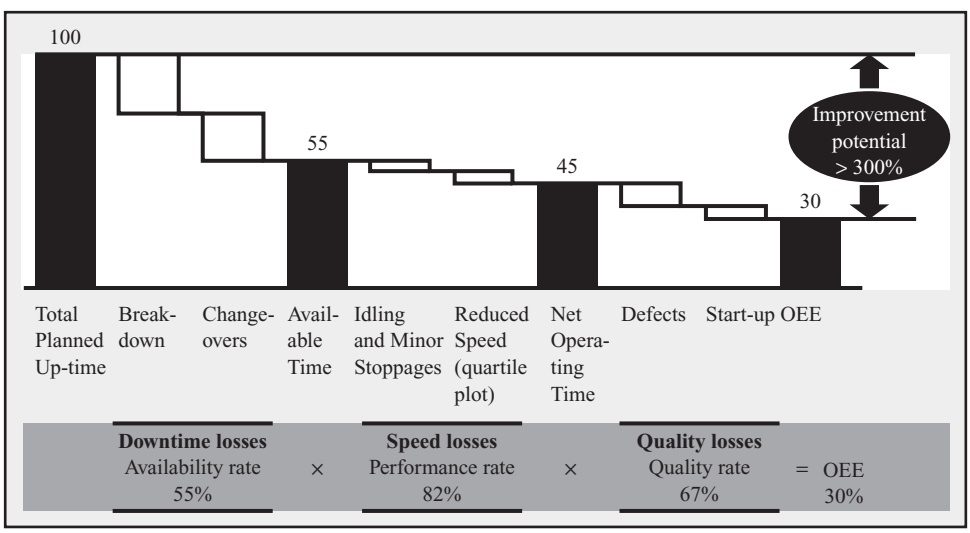


FIGURE 11.2
The Overall Equipment Effectiveness Framework

case, only 55 percent of the total planned time is available for production). However, not all of the remaining 55 percent is value-added time. Because of poor process balance, the resource is likely to be occasionally idle. Also, the resource might not operate at an optimum speed, as the activity time includes some waste and some incidental work that does not add direct customer value. In the case of Figure 11.2, 82 percent of the available time is used for operation, which leaves a total of 45 percent (= 55% × 82%). If one then factors in a further waste of capacity resulting from defects, rework, and start-ups (67 percent), we see that only 30 percent (55% × 82% × 67%) of the available capacity is used to really add value!

The following two examples illustrate the usefulness of the OEE framework in non-manufacturing settings. They also illustrate that wasting as much as half of the capacity of an expensive resource is much more common than one might expect:

- In the loan underwriting process of a major consumer bank, a recent case study documented that a large fraction of the underwriting capacity is not used productively. Unproductive time included (a) working on loans that are unlikely to be accepted by customers because the bank has already taken too long to get a response back to the customer, (b) idle time, (c) processing loans that resources preceding underwriting already could have rejected because of an obviously low creditworthiness of the application, (d) incidental activities of paper handling, and (e) attempting to reach customers on the phone but failing to do so. The study estimates that only 40 percent of the underwriting capacity is used in a value-adding way.
- In the operating rooms of a major hospital, the capacity is left unused because of (a) gaps in the schedule, (b) procedure cancellation, (c) room cleaning time, (d) patient preparation time, and (e) procedure delays because of the doctor or the anesthesiologist arriving late. After completing waste identification, the hospital concluded that only 60 percent of its operating room time was used productively. One might argue that patient preparation is a rather necessary and hence value-adding step prior to surgery. Yet, it is not clear that this step has to happen in the operating room. In fact, some hospitals are now using the tools of setup time reduction discussed in Chapter 7 and preparing the patient for surgery outside of the operating room so that the changeover from one surgical procedure to another is reduced.

11.4 JIT: Matching Supply with Demand

Just-in-time (JIT) is about matching supply with demand. The goal is to create a supply process that forms a smooth flow with its demand, thereby giving customers exactly what they need, when they need it.

In this section, we discuss three steps toward achieving a JIT process. The three steps build on each other and hence should be taken in the order they are presented. They presume that the process is already in-control (see Chapter 10) using standardized tasks and is able to achieve reliable quality:

1. Achieve a *one-unit-at-a-time* flow.
2. Produce at the rate of customer demand.
3. Implement a *pull system* using *kanban* or *make-to-order production.*

Achieve One-Unit-at-a-Time Flow

Compare the following two technologies that move people from one level of a building to another: an escalator and an elevator. Most of us associate plenty of waiting with elevators—we wait for the elevator to arrive and we wait stuck between dozens of people as the elevator stops at seemingly every floor. Escalators, in contrast, keep people moving toward their destination, no waiting and no jamming of people.

People waiting for and standing in elevators are like batches in a production setting. Chapter 7 already has discussed the concepts of SMED, the reduction of setup times that makes small production batches economically possible. In TPS, production plans are designed to avoid large batches of the same variant. Instead, product variants are mixed together on the assembly line (mixed-model production, which is also known as *heijunka*), as discussed in Chapter 7.

In addition to reducing setup times, we also should attempt to create a physical layout for our resources that closely mirrors the process flow. In other words, two resources that are close to each other in the process flow diagram also should be co-located in physical space. This avoids unnecessary transports and reduces the need to form transport batches. This way flow units can flow one unit at a time from one resource to the next (*ikko-nagashi*).

Produce at the Rate of Customer Demand

Once we have created a one-unit-at-a-time flow, we should make sure that our flow rate is in line with demand. Historically, most large-scale operations have operated their processes based on forecasts. Using planning software (often referred to as MRP, for materials requirement planning, and ERP, for enterprise resource planning), work schedules were created for the various subprocesses required to create the final product.

Forecasting is a topic for itself (see Chapter 12), but most forecasts have the negative property of not being right. So at the end of a planning period (e.g., one month), the ERP system would update its next production plan, taking the amount of inventory in the process into account. This way, in the long run, production more or less matches demand. Yet, in the day-to-day operations, extensive periods of substantial inventories or customer backorders exist.

TPS aims at reducing finished goods inventory by operating its production process in synchronization with customer orders. This is true for both the overall number of vehicles produced as well as with respect to the mix of vehicles across various models.

We translate customer demand into production rate (flow rate) using the concept of takt time. Takt time is derived from the German word *takt,* which stands for "tact" or "clock." Just like an orchestra needs to follow a common tact imposed by the conductor, a JIT process should follow the tact imposed by demand. Takt time calculations are identical to what we have seen with demand rate and flow rate calculations in earlier chapters.

Implement Pull Systems

The synchronization with the aggregate level of demand through takt time is an important step toward the implementation of JIT. However, inventory not only exists at the finished-goods level, but also throughout the process (work-in-process inventory). Some parts of the process are likely to be worker paced with some (hopefully modest) amount of inventory between resources. We now have to design a coordination system that coordinates these resources by controlling the amount of inventory in the process. We do this by implementing a pull system.

In a pull system, the resource furthest downstream (i.e., closest to the market) is paced by market demand. In addition to its own production, it also relays the demand information to the next station upstream, thus ensuring that the upstream resource also is paced by demand. If the last resource assembles two electronics components into a computer, it relays the demand for two such components to the next resource upstream. This way, the external demand is transferred step by step through the process, leading to an information flow moving in the opposite direction relative to the physical flow of the flow units.

Such a demand-driven pull system is in contrast to a *push system* where flow units are allowed to enter the process independent of the current amount of inventory in process. Especially if the first resources in the process have low levels of utilization—and are thereby likely to flood the downstream with inventory—push systems can lead to substantial inventory in the process.

To implement a pull system, TPS advocates two forms of process control:

- In kanban-based pull (also known as fill-up or supermarket pull), the upstream replenishes what demand has withdrawn from the downstream.
- Make-to-order refers to the release of work into a system only when a customer order has been received for that unit.

Consider the kanban system first. *Kanban* refers to a production and inventory control system in which production instructions and parts delivery instructions are triggered by the consumption of parts at the downstream step (Fujimoto 1999).

In a kanban system, standardized returnable parts containers circulate between the upstream and the downstream resources. The upstream resource is authorized to produce a unit when it receives an empty container. In other words, the arrival of an empty container triggers a production order. The term *kanban* refers to the card that is attached to each container. Consequently, kanban cards are frequently called work authorization forms.

A simplified description of a kanban system is provided by Figure 11.3. A downstream resource (right) consumes some input component that it receives from its upstream resource (left). The downstream resource empties containers of these input components—the downstream resource literally takes the part out of the container for its own use, thereby creating an empty container, which in turn, as already mentioned, triggers a production order for the upstream resource. Thus, the use of kanban cards between all resources in the process provides an effective and easy-to-implement mechanism for tying the demand of the process (downstream) with the production of the resources (upstream). They therefore enforce a match between supply and demand.

The main advantage of a kanban system is that there can never be more inventory between two resources than what has been authorized by the kanban cards—the upstream resource can only produce when it has an empty container, so production stops when all of the containers are full, thereby limiting the inventory to the number of containers. In contrast, with a push system, the upstream resource continues to produce as long as it has work. For example, suppose the upstream resource is a lathe that produces the legs for a wood chair. With a push system, the lathe keeps producing legs as long as it has blocks of wood to work on. With a kanban system, the lathe produces a set of chair legs only if it has an empty kanban. Hence, with a kanban system, the lathe stops working only when it runs out of kanbans, whereas with a push system the lathe only stops working when it runs out of raw materials. The distinction can lead to very different behavior. In a push system, inventory can simply "happen" to management because there is theoretically no limit to the amount of inventory that can pile up after a resource (e.g., think of the plant manager walking through

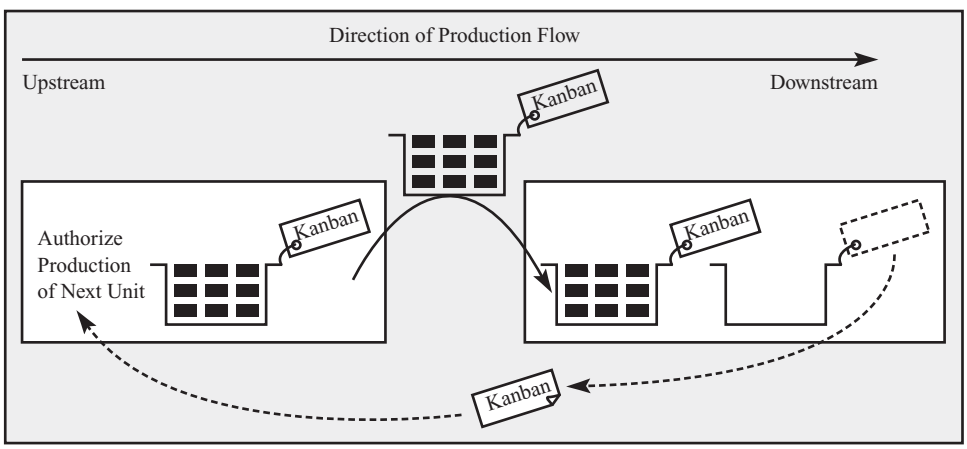


FIGURE 11.3
The Operation of a Kanban System

the process and saying, "Wow, we have a lot of inventory at this step today"). In contrast, in a kanban system the amount of inventory becomes a managerial decision variable—the maximum inventory is controlled via the number of kanban cards in the process.

As an alternative to a kanban system, we also can implement a pull system using a make-to-order process. As is suggested by the term "make-to-order," resources in such a process only operate after having received an explicit customer order. Typically, the products corresponding to these orders then flow through the process on a first-in, first-out (FIFO) basis. Each flow unit in the make-to-order process is thereby explicitly assigned to one specific customer order. Consider the example of a rear-view mirror production in an auto plant to see the difference between kanban and make-to-order. When the operator in charge of producing the interior rear-view mirror at the plant receives the work authorization through the kanban card, it has not yet been determined which customer order will be filled with this mirror. All that is known is that there are—at the aggregate—a sufficient number of customer orders such that production of this mirror is warranted. Most likely, the final assembly line of the same auto plant (including the mounting of the rear-view mirror) will be operated in a make-to-order manner, that is, the operator putting in the mirror can see that it will end up in the car of Mr. Smith.

Many organizations use both forms of pull systems. Consider computer maker Dell. Dell's computers are configured in work cells. Processes supplying components are often operated using kanban. Thus, rear-view mirrors at Toyota and power supplies at Dell flow through the process in sufficient volume to meet customer demand, yet are produced in response to a kanban card and have not yet been assigned to a specific order.

When considering which form of a pull system one wants to implement, the following should be kept in mind:

- Kanban should be used for products or parts (a) that are processed in high volume and limited variety, (b) that are required with a short lead time so that it makes economic sense to have a limited number of them (as many as we have kanban cards) preproduced, and (c) for which the costs and efforts related to storing the components are low.
- Make-to-order should be used when (a) products or parts are processed in low volume and high variety, (b) customers are willing to wait for their order, and (c) it is expensive or difficult to store the flow units. Chapter 13 will explain the costs and benefits of a make-to-order production system.

11.5 Quality Management

If we operate with no buffers and want to avoid the waste of rework, operating at zero defects is a must. To achieve zero defects, TPS relies on defect prevention, rapid defect detection, and a strong worker responsibility with respect to quality.

Defects can be prevented by "fool-proofing" many assembly operations, that is, by making mistakes in assembly operations physically impossible (*poka-yoke*). Components are designed in a way that there exists one single way of assembling them.

If, despite defect prevention, a problem occurs, TPS attempts to discover and isolate this problem as quickly as possible. This is achieved through the *jidoka* concept. The idea of jidoka is to stop the process immediately whenever a defect is detected and to alert the line supervisor. This idea goes back to the roots of Toyota as a maker of automated looms. Just like an automated loom should stop operating in the case of a broken thread, a defective machine should shut itself off automatically in the presence of a defect.

Shutting down the machine forces a human intervention in the process, which in turn triggers process improvement (Fujimoto 1999). The jidoka concept has been generalized to include any mechanism that stops production in response to quality problems, not just for automated machines. The most well-known form of jidoka is the *Andon cord,* a cord

running adjacent to assembly lines that enables workers to stop production if they detect a defect. Just like the jidoka automatic shut-down of machines, this procedure dramatizes manufacturing problems and acts as a pressure for process improvements.

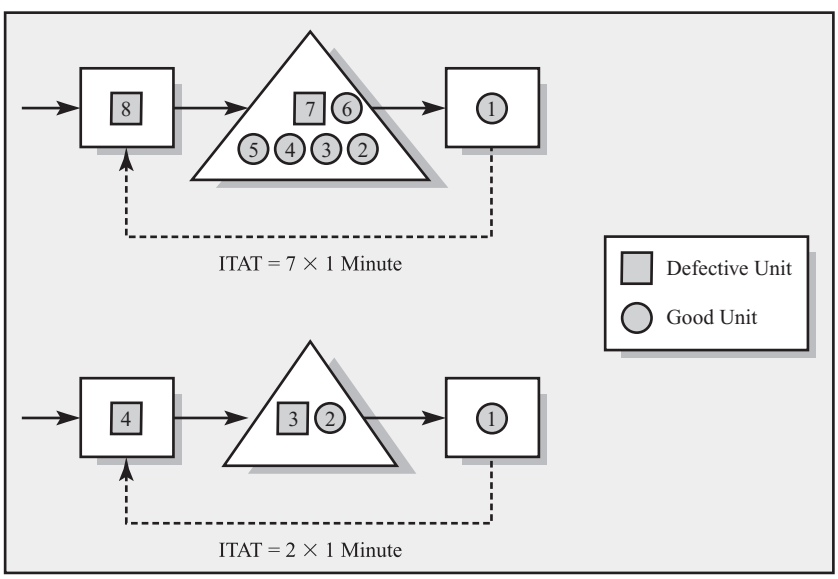
A worker pulling the Andon cord upon detecting a quality problem is in sharp contrast to Henry Ford's historical assembly line that would leave the detection of defects to a final inspection step. In TPS, "the next step is the customer" and every resource should only let those flow units move downstream that have been inspected and evaluated as good parts. Hence, quality inspection is "built in" (*tsukurikomi*) and happens at every step in the line, as opposed to relying on a final inspection step alone.

The idea of detect–stop–alert that underlies the jidoka principle is not just a necessity to make progress towards implementing the zero inventory principle. Jidoka also benefits from the zero inventory principle, as large amounts of work-in-process inventory achieve the opposite of jidoka: they delay the detection of a problem, thereby keeping a defective process running and hiding the defect from the eyes of management. This shows how the various TPS principles and methods are interrelated, mutually strengthening each other.

To see how work-in-process inventory is at odds with the idea of jidoka, consider a sequence of two resources in a process, as outlined in Figure 11.4. Assume the activity times at both resources are equal to one minute per unit. Assume further that the upstream resource (on the left) suffers quality problems and—at some random point in time—starts producing bad output. In Figure 11.4, this is illustrated by the resource producing squares instead of circles. How long will it take until a quality problem is discovered? If there is a large buffer between the two resources (upper part of Figure 11.4), the downstream resource will continue to receive good units from the buffer. In this example, it will take seven minutes before the downstream resource detects the defective flow unit. This gives the upstream resource seven minutes to continue producing defective parts that need to be either scrapped or reworked.

Thus, the time between when the problem occurred at the upstream resource and the time it is detected at the downstream resource depends on the size of the buffer between the two resources. This is a direct consequence of Little's Law. We refer to the time between creating a defect and receiving the feedback about the defect as the *information turnaround time* (*ITAT*). Note that we assume in this example that the defect is detected in the next resource downstream. The impact of inventory on quality is much worse if defects only get detected at the end of the process (e.g., at a final inspection step). In this case, the

FIGURE 11.4
Information Turnaround Time and Its Relationship with Buffer Size



ITAT is driven by all inventory downstream from the resource producing the defect. This motivates the built-in inspection we mentioned above.

11.6 Exposing Problems through Inventory Reduction

Our discussion on quality reveals that inventory covers up problems. So to improve a process, we need to turn the "inventory hiding quality problems" effect on its head: we want to reduce inventory to expose defects and then fix the underlying root cause of the defect.

Recall that in a kanban system, the number of kanban cards—and hence the amount of inventory in the process—is under managerial control. So we can use the kanban system to gradually reduce inventory and thereby expose quality problems. The kanban system and its approach to buffers can be illustrated with the following metaphor. Consider a boat sailing on a canal that has numerous rocks in it. The freight of the boat is very valuable, so the company operating the canal wants to make sure that the boat never hits a rock. Figure 11.5 illustrates this metaphor.

One approach to this situation is to increase the water level in the canal. This way, there is plenty of water over the rocks and the likelihood of an accident is low. In a production setting, the rocks correspond to quality problems (defects), setup times, blocking or starving, breakdowns, or other problems in the process and the ship hitting a rock corresponds to lost throughput. The amount of water corresponds to the amount of inventory in the process (i.e., the number of kanban cards), which brings us back to our previous "buffer-or-suffer" discussion.

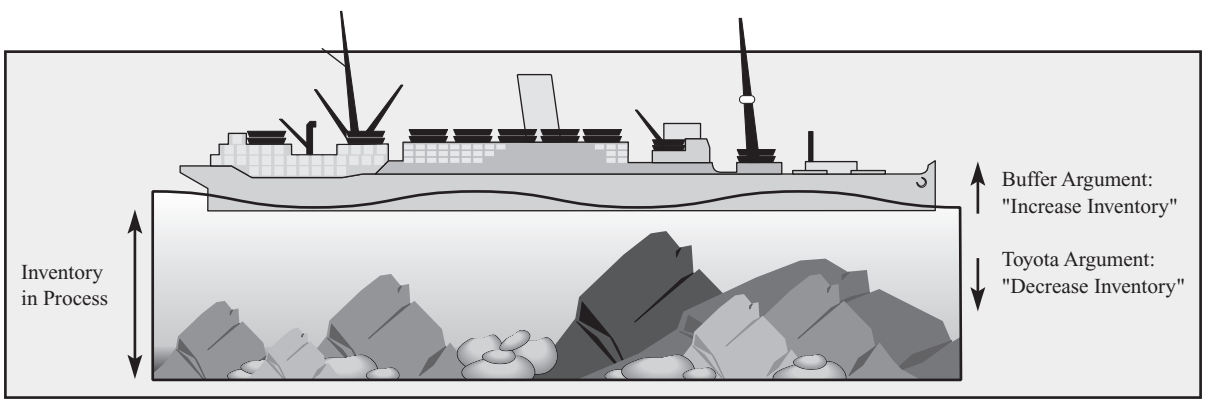
An alternative way of approaching the problem is this: instead of covering the rocks with water, we also could consider reducing the water level in the canal (reduce the number of kanban cards). This way, the highest rocks are exposed (i.e., we observe a process problem), which provides us with the opportunity of removing them from the canal. Once this has been accomplished, the water level is lowered again, until—step by step—all rocks are removed from the canal. Despite potential short-term losses in throughput, the advantage of this approach is that it moves the process to a better frontier (i.e., it is better along multiple dimensions).

This approach to inventory reduction is outlined in Figure 11.6. We observe that we first need to accept a short-term loss in throughput reflecting the reduction of inventory (we stay on the efficient frontier, as we now have less inventory). Once the inventory level is lowered, we are able to identify the most prominent problems in the process (rocks in the water). Once identified, these problems are solved and thereby the process moves to a more desirable frontier.

Both in the metaphor and in our ITAT discussion above, inventory is the key impediment to learning and process improvement. Since with kanban cards, management is in

FIGURE 11.5 **More or Less Inventory? A Simple Metaphor**

Source: Stevenson 2006.



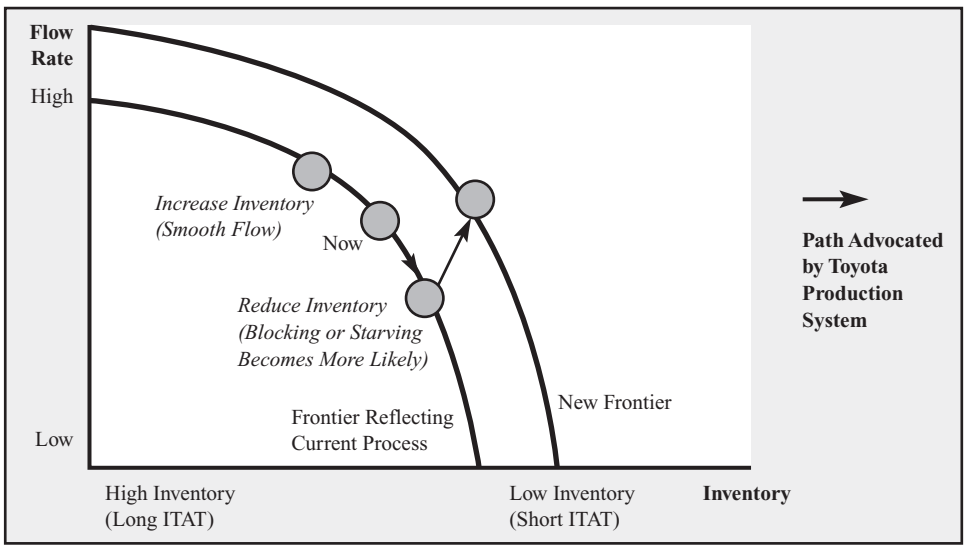


FIGURE 11.6
Tension between Flow Rate and Inventory Levels/ITAT

control of the inventory level, it can proactively manage the tension between the short-term need of a high throughput and the long-term objective of improving the process.

11.7 Flexibility

Given that there typically exist fluctuations in demand from the end market, TPS attempts to create processes with sufficient flexibility to meet such fluctuations. Since forecasts are more reliable at the aggregate level (across models or components, see discussion of pooling in Chapter 8 and again in Chapter 15), TPS requests workers to be skilled in handling multiple machines.

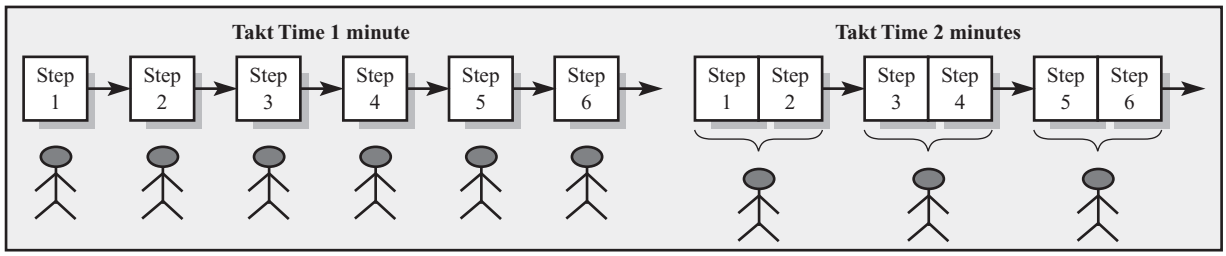
- When production volume has to be decreased for a product because of low demand, TPS attempts to assign some workers to processes creating other products and to have the remaining workers handle multiple machines simultaneously for the process with the low-demand product.
- When production volume has to be increased for a product because of high demand, TPS often uses a second pool of workers (temporary workers) to help out with production. Unlike the first pool of full-time employees (typically with lifetime employment guarantee and a broad skill set), these workers are less skilled and can only handle very specific tasks.

Consider the six-step operation shown in Figure 11.7. Assume all activities have an activity time of one minute per unit. If demand is low (right), we avoid idle time (low average labor utilization) by running the process with only three operators (typically, full-time employees). In this case, each operator is in charge of two minutes of work, so we would achieve a flow rate of 0.5 unit per minute. If demand is high (left in the Figure 11.7), we assign one worker to each step, that is, we bring in additional (most likely temporary) workers. Now, the flow rate can be increased to one unit per minute.

This requires that the operators are skilled in multiple assembly tasks. Good training, job rotation, skill-based payment, and well-documented standard operating procedures are essential requirements for this. This flexibility also requires that we have a multitiered workforce consisting of highly skilled full-time employees and a pool of temporary workers (who do not need such a broad skill base) that can be called upon when demand is high.

Such multitask flexibility of workers also can help decrease idle time in cases of activities that require some worker involvement but are otherwise largely automated. In these

FIGURE 11.7 Multi-task Flexibility
(Note: The figure assumes a 1 minute/unit activity time at each station.)

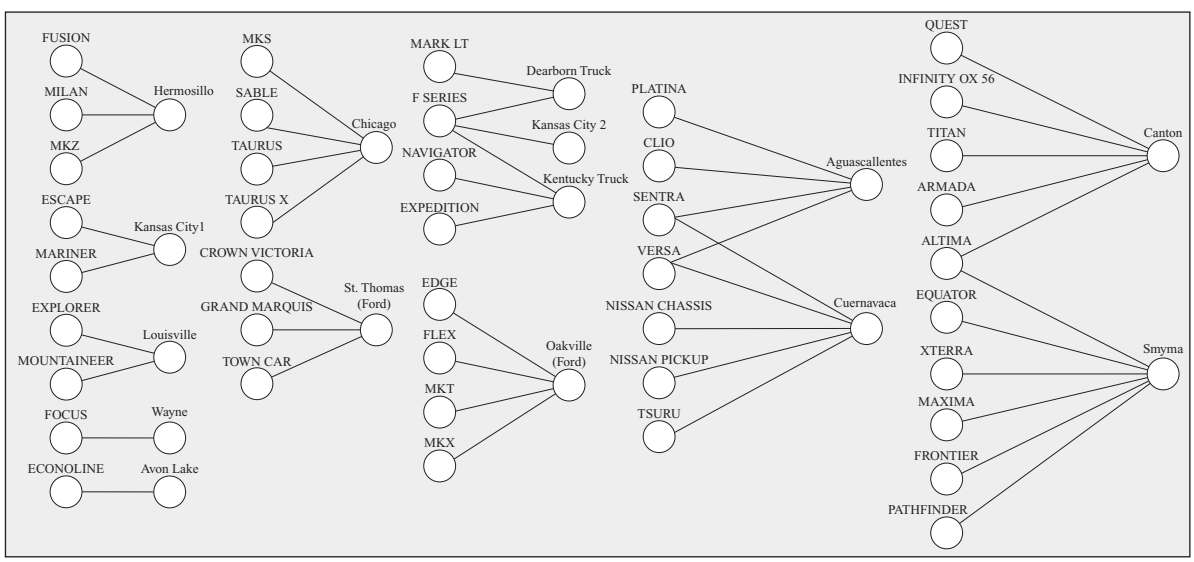


cases, a worker can load one machine and while this machine operates, the worker—instead of being idle—operates another machine along the process flow (*takotei-mochi*). This is facilitated if the process flow is arranged in a U-shaped manner, in which case a worker can share tasks not only with the upstream and the downstream resource, but also with another set of tasks in the process. Another important form of flexibility relates to the ability of one plant to produce more than one vehicle model. Consider the data displayed in Figure 11.8. The left part of the figure shows how Ford's vehicles are allocated to Ford's production plants. As we can see, many vehicles are dedicated to one plant and many of the plants can only produce a small set of vehicles. Consequently, if demand increases relative to the plant's capacity, that plant is unlikely to have sufficient capacity to fulfill it. If demand decreases, the plant is likely to have excess capacity.

In an ideal world, the company would be able to make every model in every plant. This way, high demand from one model would cancel out with low demand from another one, leading to better plant utilization and more sales. However, such capacity pooling would require the plants to be perfectly flexible—requiring substantial investments in production tools and worker skills. An interesting alternative to such perfect flexibility is the concept of partial flexibility, also referred to as *chaining.* The idea of chaining is that every car can be made in two plants and that the vehicle-to-plant assignment creates a chain that connects as many vehicles and plants as possible. As we will see in Chapter 15, such partial flexibility results in almost the same benefits of full flexibility, yet at dramatically lower costs. The right side of Figure 11.8 shows the vehicle-to-plant assignment of

FIGURE 11.8 Vehicle-to-Plant Assignments at Ford (Left) and at Nissan (right).

Source: Moreno and Terwiesch (2011).



Nissan (North America) and provides an illustrative example of partial flexibility. In an environment of volatile demand, this partial flexibility has allowed Nissan to keep its plants utilized without providing the hefty discounts offered by its competitors.

11.8 Standardization of Work and Reduction of Variability

As we have seen in Chapters 8 and 9, variability is a key inhibitor in our attempt to create a smooth flow. In the presence of variability, either we need to buffer (which would violate the zero inventory philosophy) or we suffer occasional losses in throughput (which would violate the principle of providing the customer with the requested product when demanded). For this reason, the Toyota Production System explicitly embraces the concepts of variability measurement, control, and reduction discussed in Chapter 10.

The need for stability in a JIT process and the vulnerability of an unbuffered process were visible in the computer industry following the 1999 Taiwanese earthquake. Several of the Taiwanese factories that were producing key components for computer manufacturers around the world were forced to shut down their production due to the earthquake. Such an unpredicted shutdown was more disruptive for computer manufacturers with JIT supply chains than those with substantial buffers (e.g., in the form of warehouses) in their supply chains (Papadakis 2002).

Besides earthquakes, variability occurs because of quality defects (see above) or because of differences in activity times for the same or for different operators. Figure 11.9 shows performance data from a large consumer loan processing organization. The figure compares the performance of the top-quartile operator (i.e., the operator who has 25 percent of the other operators achieving a higher performance and 75 percent of the operators achieving a lower performance) with the bottom quartile operator (the one who has 75 percent of the operators achieving a higher performance). As we can see, there can exist dramatic differences in the productivity across employees.

A quartile analysis is a good way to identify the presence of large differences across operators and to estimate the improvement potential. For example, we could estimate what would happen to process capacity if all operators would be trained so that they achieve a performance in line with the current top-quartile performance.

11.9 Human Resource Practices

We have seen seven sources of waste, but the Toyota Production System also refers to an eighth source—the waste of the human intellect. For this reason, a visitor to an operation that follows the Toyota Production System philosophy often encounters signs with expressions like "In our company, we all have two jobs: (1) to do our job and (2) to improve it."

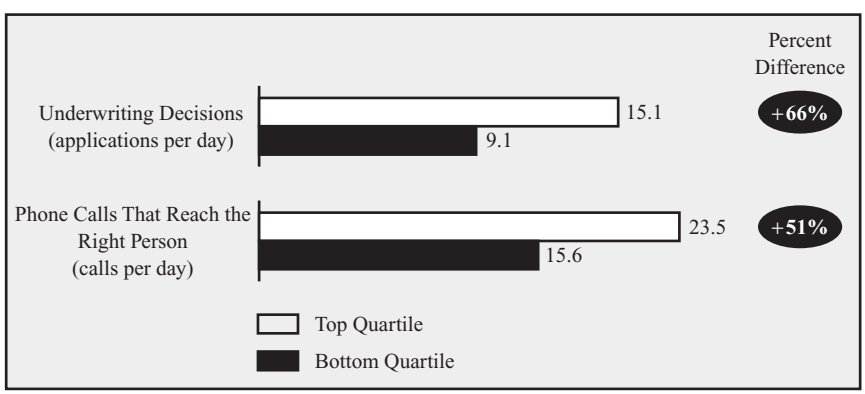


FIGURE 11.9
Productivity Comparison across Underwriters

To illustrate different philosophies toward workers, consider the following two quotes. The first one comes from the legendary book *Principles of Scientific Management* written by Frederick Taylor, which still makes an interesting read almost a century after its first appearance (once you have read the quote below, you will at least enjoy Taylor's candid writing style). The second quote comes from Konosuka Matsushita, the former chairman of Panasonic.

Let us look at Taylor's opinion first and consider his description of pig iron shoveling, an activity that Taylor studied extensively in his research. Taylor writes: "This work is so crude and elementary that the writer firmly believes that it would be possible to train an intelligent gorilla so as to become a more efficient pig-iron handler than any man can be."

Now, consider Matsushita, whose quote almost reads like a response to Taylor:

> We are going to win and you are going to lose. There is nothing you can do about it, because the reasons for failure are within yourself. With you, the bosses do the thinking while the workers wield the screw drivers. You are convinced that this is the way to run a business. For you, the essence of management is getting the ideas out of the heads of the bosses and in to the hands of the labour. [. . .] Only by drawing on the combined brainpower of all its employees can a firm face up to the turbulence and constraints of today's environment.

TPS, not surprisingly, embraces Matsushita's perspective of the "combined brainpower." We have already seen the importance of training workers as a source of flexibility.

Another important aspect of the human resource practices of Toyota relates to process improvement. Quality circles bring workers together to jointly solve production problems and to continuously improve the process (*kaizen*). Problem solving is very data driven and follows a standardized process, including control charts, fishbone (Ishikawa) diagrams, the "Five Whys," and other problem-solving tools. Thus, not only do we standardize the production process, we also standardize the process of improvement.

Ishikawa diagrams (also known as *fishbone diagrams* or cause–effect diagrams) graphically represent variables that are causally related to a specific outcome, such as an increase in variation or a shift in the mean. When drawing a fishbone diagram, we typically start with a horizontal arrow that points at the name of the outcome variable we want to analyze. Diagonal lines then lead to this arrow representing main causes. Smaller arrows then lead to these causality lines, creating a fishbonelike shape. An example of this is given by Figure 11.10. Ishikawa diagrams are simple yet powerful problem-solving tools that can be used to structure brainstorming sessions and to visualize the causal structure of a complex system.

A related tool that also helps in developing causal models is known as the "Five Whys." The tool is prominently used in Toyota's organization when workers search for the root cause of a quality problem. The basic idea of the "Five Whys" is to continually question ("Why did this happen?") whether a potential cause is truly the root cause or is merely a symptom of a deeper problem.

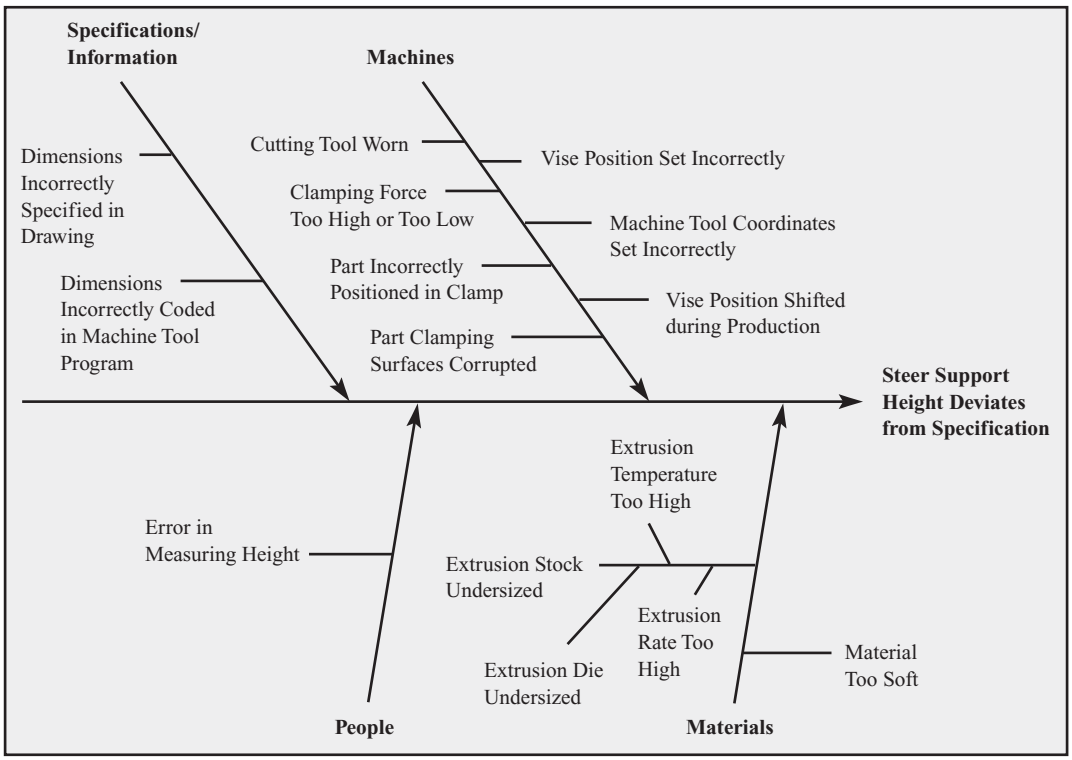
In addition to these operational principles, TPS includes a range of human resource management practices, including stable employment ("lifetime employment") for the core workers combined with the recruitment of temporary workers; a strong emphasis on skill development, which is rewarded financially through skill-based salaries; and various other aspects relating to leadership and people management.

11.10 Lean Transformation

How do you turn around an existing operation to achieve operational excellence as we have discussed it above? Clearly, even an operations management textbook has to acknowledge that there is more to a successful operational turnaround than the application of a set of tools.

McKinsey, as a consulting firm with a substantial part of its revenues resulting from operations work, refers to the set of activities required to improve the operations of a client

FIGURE 11.10 **Example of an Ishikawa Diagram**



as a *lean transformation.* There exist three aspects to such a lean transformation: the operating system, a management infrastructure, and the mindsets and behaviors of the employees involved.

With the operating system, the firm refers to various aspects of process management as we have discussed in this chapter and throughout this book: an emphasis on flow, matching supply with demand, and a close eye on the variability of the process.

But technical solutions alone are not enough. So the operating system needs to be complemented by a management infrastructure. A central piece of this infrastructure is performance measurement. Just as we discussed in Chapter 6, defining finance-level performance measures and then cascading them into the operations is a key struggle for many companies. Moreover, the performance measures should be tracked over time and be made transparent throughout the organization. The operator needs to understand which performance measures he or she is supposed to achieve and how these measures contribute to the bigger picture. Management infrastructure also includes the development of operator skills and the establishment of formal problem-solving processes.

Finally, the mindset of those involved in working in the process is central to the success of a lean transformation. A nurse might get frustrated from operating in an environment of waste that is keeping him or her from spending time with patients. Yet, the nurse, in all likelihood, also will be frustrated by the implementation of a new care process that an outsider imposes on his or her ward. Change management is a topic well beyond the scope of this book: open communication with everyone involved in the process, collecting and discussing process data, and using some of the tools discussed in Chapter 10 as well as with respect to kaizen can help make the transformation a success.

11.11 Further Reading

Readers who want to learn more about TPS are referred to excellent reading, such as Fujimoto (1999) or Ohno (1988), from which many of the following definitions are taken.

Fujimoto (1999) describes the evolution of the Toyota Production System. While not a primary focus of the book, it also provides excellent descriptions of the main elements of the Toyota Production System. The results of the benchmarking studies are reported in Womack, Jones, and Roos (1991) and Holweg and Pil (2004).

Bohn and Jaikumar (1992) is a classic reading that challenges the traditional, optimization-focused paradigm of operations management. Their work stipulates that companies should not focus on optimizing decisions for their existing business processes, but rather should create new processes that can operate at higher levels of performance.

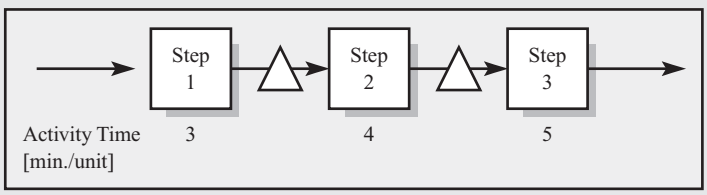
Drew, McCallum, and Roggenhofer (2004) describe the "Journey to Lean," a description of the steps constituting a lean transformation as described by a group of McKinsey consultants.

Tucker (2004) provides a study of TPS-like activities from the perspective of nurses who encounter quality problems in their daily work. Moreno and Terwiesch discuss flexibility strategies in the U.S. automotive industry and analyze if and to what extent firms with flexible production systems are able to achieve higher plant utilization and lower price discounts.

The Wikipedia entries for Toyota, Ford, Industrial Revolution, Gilbreth, and Taylor are also interesting summaries and were helpful in compiling the historical reviews presented in this chapter.

11.12 Practice Problems

Q11.1 **(Three Step)** Consider a worker-paced line with three process steps, each of which is staffed with one worker. The sequence of the three steps does not matter for the completion of the product. Currently, the three steps are operated in the following sequence.

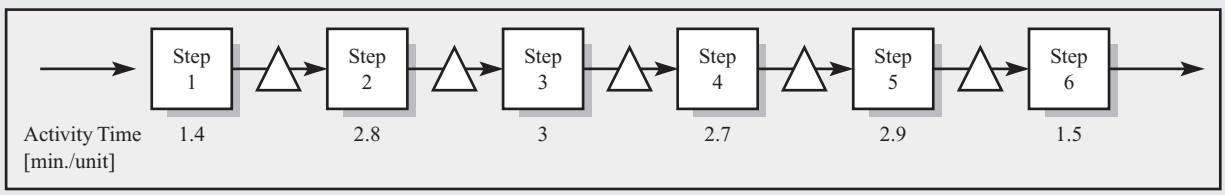


a. What would happen to the inventory in the process if the process were operated as a push system?

b. Assuming you would have to operate as a push system, how would you resequence the three activities?

c. How would you implement a pull system?

Q11.2 **(Six Step)** Consider the following six-step worker-paced process. Each resource is currently staffed by one operator. Demand is 20 units per hour. Over the past years, management has attempted to rebalance the process, but given that workers can only complete tasks that are adjacent to each other, no further improvement has been found.



a. What would you suggest to improve this process? (*Hint:* Think "out of the box.")

Chapter 12

Betting on Uncertain Demand: The Newsvendor Model[1]

Matching supply and demand is particularly challenging when supply must be chosen before observing demand and demand is stochastic (uncertain). To illustrate this point, suppose you are the owner of a simple business: selling newspapers. Each morning you purchase a stack of papers with the intention of selling them at your newsstand at the corner of a busy street. Even though you have some idea regarding how many newspapers you can sell on any given day, you never can predict demand for sure. Some days you sell all of your papers, while other days end with unsold newspapers to be recycled. As the newsvendor, you must decide how many papers to buy at the start of each day. Because you must decide how many newspapers to buy before demand occurs, unless you are very lucky, you will not be able to match supply to demand. A decision tool is needed to make the best out of this difficult situation. The *newsvendor model* is such a tool.

You will be happy to learn that the newsvendor model applies in many more settings than just the newsstand business. The essential issue is that you must take a firm bet (how much inventory to order) before some random event occurs (demand) and then you learn that you either bet too much (demand was less than your order) or you bet too little (demand exceeded your order). This trade-off between "doing too much" and "doing too little" occurs in other settings. Consider a technology product with a long lead time to source components and only a short life before better technology becomes available. Purchase too many components and you risk having to sell off obsolete technology. Purchase too few and you may forgo sizable profits. Cisco is a company that can relate to these issues: In 2000 they estimated that they were losing 10 percent of their potential orders to rivals due to long lead times created by shortages of parts; but by early 2001, the technology bubble had burst and they had to write off $2.5 billion in inventory.

This chapter begins with a description of the production challenge faced by O'Neill Inc., a sports apparel manufacturer. O'Neill's decision also closely resembles the newsvendor's task. We then describe the newsvendor model in detail and apply it to O'Neill's problem. We also show how to use the newsvendor model to forecast a number of performance measures relevant to O'Neill.

[1] Data in this chapter have been disguised to protect confidential information.

12.1 O'Neill Inc.

O'Neill Inc. is a designer and manufacturer of apparel, wetsuits, and accessories for water sports: surf, dive, waterski, wake-board, triathlon, and wind surf. Their product line ranges from entry-level products for recreational users, to wetsuits for competitive surfers, to sophisticated dry suits for professional cold-water divers (e.g., divers that work on oil platforms in the North Sea). O'Neill divides the year into two selling seasons: Spring (February through July) and Fall (August through January). Some products are sold in both seasons, but the majority of their products sell primarily in a single season. For example, waterski is active in the Spring season whereas recreational surf products sell well in the Fall season. Some products are not considered fashionable (i.e., they have little cosmetic variety and they sell from year to year), for example, standard neoprene black booties. With product names like "Animal," "Epic," "Hammer," "Inferno," and "Zen," O'Neill clearly also has products that are subject to the whims of fashion. For example, color patterns on surf suits often change from season to season to adjust to the tastes of the primary user (15–30-year-old males from California).

O'Neill operates its own manufacturing facility in Mexico, but it does not produce all of its products there. Some items are produced by the TEC Group, O'Neill's contract manufacturer in Asia. While TEC provides many benefits to O'Neill (low cost, sourcing expertise, flexible capacity, etc.), they do require a three-month lead time on all orders. For example, if O'Neill orders an item on November 1, then O'Neill can expect to have that item at its distribution center in San Diego, California, ready for shipment to customers, only on January 31.

To better understand O'Neill's production challenge, let's consider a particular wetsuit used by surfers and newly redesigned for the upcoming spring season, the Hammer 3/2. (The "3/2" signifies the thickness of the neoprene on the suit: 3 mm thick on the chest and 2 mm everywhere else.) Figure 12.1 displays the Hammer 3/2 and O'Neill's logo. O'Neill has decided to let TEC manufacture the Hammer 3/2. Due to TEC's three-month lead time, O'Neill needs to submit an order to TEC in November before the start of the spring season. Using past sales data for similar products and the judgment of its designers and sales representatives, O'Neill developed a forecast of 3,200 units for total demand during the spring season for the Hammer 3/2. Unfortunately, there is considerable uncertainty in that forecast despite the care and attention placed on the formation of the forecast. For example, it is O'Neill's experience that 50 percent of the time the actual demand deviates from their initial forecast by more than 25 percent of the forecast. In other words, only 50 percent of the time is the actual demand between 75 percent and 125 percent of their forecast.

Although O'Neill's forecast in November is unreliable, O'Neill will have a much better forecast for total season demand after observing the first month or two of sales. At that time, O'Neill can predict whether the Hammer 3/2 is selling slower than forecast, in which case O'Neill is likely to have excess inventory at the end of the season, or whether the Hammer 3/2 is more popular than predicted, in which case O'Neill is likely to stock out. In the latter case, O'Neill would love to order more Hammers, but the long lead time from Asia prevents O'Neill from receiving those additional Hammers in time to be useful. Therefore, O'Neill essentially must "live or dive" with its single order placed in November.

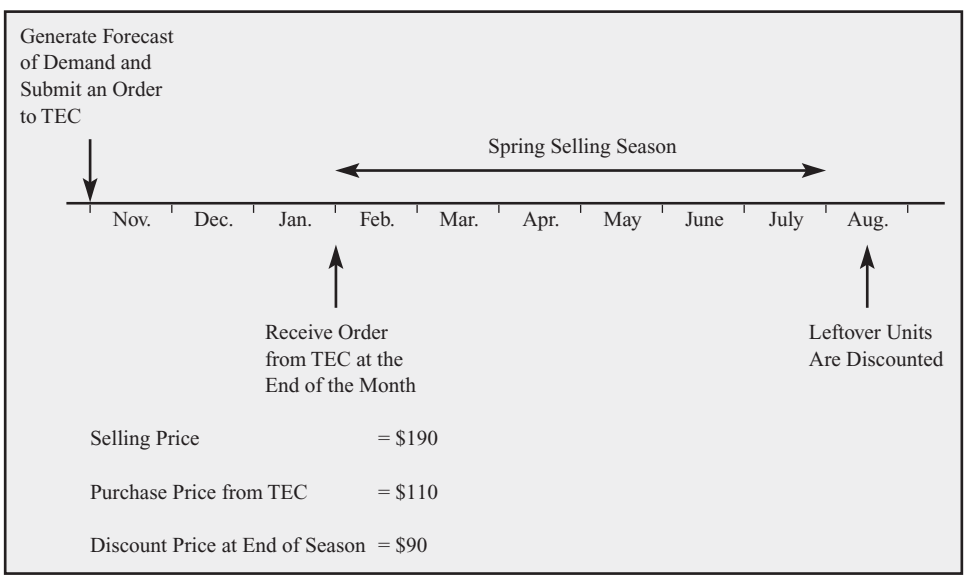
Fortunately for O'Neill, the economics on the Hammer are pretty good. O'Neill sells the Hammer to retailers for $190 while it pays TEC $110 per suit. If O'Neill has leftover inventory at the end of the season, it is O'Neill's experience that they are able to sell that inventory for $90 per suit. Figure 12.2 summarizes the time line of events and the economics for the Hammer 3/2.

So how many units should O'Neill order from TEC? You might argue that O'Neill should order the forecast for total demand, 3,200, because 3,200 is the most likely outcome.

FIGURE 12.1
O'Neill's Hammer 3/2 Wetsuit and Logo for the Surf Market

The forecast is also the value that minimizes the expected absolute difference between the actual demand and the production quantity; that is, it is likely to be close to the actual demand. Alternatively, you may be concerned that forecasts are always biased and therefore suggest an order quantity less than 3,200 would be more prudent. Finally, you might argue that because

FIGURE 12.2
Time Line of Events and Economics for O'Neill's Hammer 3/2 Wetsuit



the gross margin on the Hammer is more than 40 percent ((190 − 110)/190 = 0.42), O'Neill should order more than 3,200 in case the Hammer is a hit. We next define the newsvendor model and then discuss what the newsvendor model recommends for an order quantity.

12.2 An Introduction to the Newsvendor Model

The newsvendor model considers a setting in which you have only one production or procurement opportunity. Because that opportunity occurs well in advance of a single selling season, you receive your entire order just before the selling season starts. Stochastic demand occurs during the selling season. If demand exceeds your order quantity, then you sell your entire order. But if demand is less than your order quantity, then you have leftover inventory at the end of the season.

There is a fixed cost per unit ordered: for the Hammer 3/2, Cost = 110. It is important that Cost includes only costs that depend on the number of units ordered; amortized fixed costs should not be included, because they are unaffected by our order quantity decision. In other words, this cost figure should include all costs that vary with the order quantity and no costs that do not vary with the order quantity. There is a fixed price for each unit you sell; in this case, Price = 190.

If there is leftover inventory at the end of the season, then there is some value associated with that inventory. To be specific, there is a fixed *Salvage value* that you earn on each unit of leftover inventory: with the Hammer, the Salvage value = 90. It is possible that leftover inventory has no salvage value whatsoever, that is, Salvage value = 0. It is also possible leftover inventory is costly to dispose, in which case the salvage value may actually be a salvage cost. For example, if the product is a hazardous chemical, then there is a cost for disposing of leftover inventory; that is, Salvage value < 0 is possible.

To guide your production decision, you need a forecast for demand. O'Neill's initial forecast for the Hammer is 3,200 units for the season. But it turns out (for reasons that are explained later) you need much more than just a number for a forecast. You need to have a sense of how accurate your forecast is; you need a forecast on your forecast error! For example, in an ideal world, there would be absolutely no error in your forecast: if the forecast is 3,200 units, then 3,200 units is surely the demand for the season. In reality, there will be error in the forecast, but forecast error can vary in size. For example, it is better to be 90 percent sure demand will be between 3,100 and 3,300 units than it is to be 90 percent sure demand will be between 2,400 and 4,000 units. Intuition should suggest that you might want to order a different amount in those two situations.

To summarize, the newsvendor model represents a situation in which a decision maker must make a single bet (e.g., the order quantity) before some random event occurs (e.g., demand). There are costs if the bet turns out to be too high (e.g., leftover inventory that is salvaged for a loss on each unit). There are costs if the bet turns out to be too low (the opportunity cost of lost sales). The newsvendor model's objective is to bet an amount that correctly balances those opposing forces. To implement the model, we need to identify our costs and how much demand uncertainty we face. We have already identified our costs, so the next section focuses on the task of identifying the uncertainty in Hammer 3/2 demand.

12.3 Constructing a Demand Forecast

The newsvendor model balances the cost of ordering too much against the cost of ordering too little. To do this, we need to understand how much demand uncertainty there is for the Hammer 3/2, which essentially means we need to be able to answer the following question:

What is the probability demand will be less than or equal to Q units?

for whatever Q value we desire. In short, we need a *distribution function.* Recall from statistics, every random variable is defined by its distribution function, $F(Q)$, which is the probability the outcome of the random variable is Q or lower. In this case the random variable is demand for the Hammer 3/2 and the distribution function is

$$F(Q) = \text{Prob}\{\text{Demand is less than or equal to } Q\}$$

For convenience, we refer to the distribution function, $F(Q)$, as our demand forecast because it gives us a complete picture of the demand uncertainty we face. The objective of this section is to explain how we can use a combination of intuition and data analysis to construct our demand forecast.

Distribution functions come in two forms. *Discrete distribution functions* can be defined in the form of a table: There is a set of possible outcomes and each possible outcome has a probability associated with it. The following is an example of a simple discrete distribution function with three possible outcomes:

Q	*F(Q)*
2,200	0.25
3,200	0.75
4,200	1.00

The Poisson distribution is an example of a discrete distribution function that we will use extensively. With *continuous distribution functions* there are an unlimited number of possible outcomes. Both the exponential and the normal are continuous distribution functions. They are defined with one or two parameters. For example, the normal distribution is defined by two parameters: its mean and its standard deviation. We use μ to represent the mean of the distribution and σ to represent the standard deviation. (μ is the Greek letter mu and σ is the Greek letter sigma.) This notation for the mean and the standard deviation is quite common, so we adopt it here.

In some situations, a discrete distribution function provides the best representation of demand, whereas in other situations a continuous distribution function works best. Hence, we work with both types of distribution functions.

Now let's turn to the complex task of actually creating the forecast. As mentioned in Section 12.1, the Hammer 3/2 has been redesigned for the upcoming spring season. As a result, actual sales in the previous season might not be a good guide for expected demand in the upcoming season. In addition to the product redesign, factors that could influence expected demand include the pricing and marketing strategy for the upcoming season, changes in fashion, changes in the economy (e.g., is demand moving toward higher or lower price points), changes in technology, and overall trends for the sport. To account for all of these factors, O'Neill surveyed the opinion of a number of individuals in the organization on their personal demand forecast for the Hammer 3/2. The survey's results were averaged to obtain the initial 3,200-unit forecast. This represents the "intuition" portion of our demand forecast. Now we need to analyze O'Neill's available data to further develop the demand forecast.

Table 12.1 presents data from O'Neill's previous spring season with wetsuits in the surf category. Notice that the data include both the original forecasts for each product as well as its actual demand. The original forecast was developed in a process that was comparable to the one that led to the 3,200-unit forecast for the Hammer 3/2 for this season. For example, the forecast for the Hammer 3/2 in the previous season was 1,300 units, but actual demand was 1,696 units.

TABLE 12.1
Forecasts and Actual Demand Data for Surf Wetsuits from the Previous Spring Season

Product Description	Forecast	Actual Demand	Error*	A/F Ratio**
JR ZEN FL 3/2	90	140	−50	1.56
EPIC 5/3 W/HD	120	83	37	0.69
JR ZEN 3/2	140	143	−3	1.02
WMS ZEN-ZIP 4/3	170	163	7	0.96
HEATWAVE 3/2	170	212	−42	1.25
JR EPIC 3/2	180	175	5	0.97
WMS ZEN 3/2	180	195	−15	1.08
ZEN-ZIP 5/4/3 W/HOOD	270	317	−47	1.17
WMS EPIC 5/3 W/HD	320	369	−49	1.15
EVO 3/2	380	587	−207	1.54
JR EPIC 4/3	380	571	−191	1.50
WMS EPIC 2MM FULL	390	311	79	0.80
HEATWAVE 4/3	430	274	156	0.64
ZEN 4/3	430	239	191	0.56
EVO 4/3	440	623	−183	1.42
ZEN FL 3/2	450	365	85	0.81
HEAT 4/3	460	450	10	0.98
ZEN-ZIP 2MM FULL	470	116	354	0.25
HEAT 3/2	500	635	−135	1.27
WMS EPIC 3/2	610	830	−220	1.36
WMS ELITE 3/2	650	364	286	0.56
ZEN-ZIP 3/2	660	788	−128	1.19
ZEN 2MM S/S FULL	680	453	227	0.67
EPIC 2MM S/S FULL	740	607	133	0.82
EPIC 4/3	1,020	732	288	0.72
WMS EPIC 4/3	1,060	1,552	−492	1.46
JR HAMMER 3/2	1,220	721	499	0.59
HAMMER 3/2	1,300	1,696	−396	1.30
HAMMER S/S FULL	1,490	1,832	−342	1.23
EPIC 3/2	2,190	3,504	−1,314	1.60
ZEN 3/2	3,190	1,195	1,995	0.37
ZEN-ZIP 4/3	3,810	3,289	521	0.86
WMS HAMMER 3/2 FULL	6,490	3,673	2,817	0.57

*Error = Forecast − Actual demand
**A/F ratio = Actual demand divided by Forecast

So how does O'Neill know actual demand for a product that stocks out? For example, how does O'Neill know that actual demand was 1,696 for last year's Hammer 3/2 if they only ordered 1,500 units? Because retailers order via phone or fax, O'Neill can keep track of each retailer's initial order, that is, the retailer's demand before the retailer knows a product is unavailable. (However, life is not perfect: O'Neill's phone representatives do not always record a customer's initial order into the computer system, so there is even some uncertainty with that figure. We'll assume this is a minor issue and not address it in our analysis.) In other settings, a firm may not be able to know actual demand with that level of precision. For example, a retailer of O'Neill's products probably does not get to observe what demand could be for the Hammer 3/2 once the Hammer is out of stock at the retailer. However, that retailer would know when during the season the Hammer stocked out and, hence, could use that information to forecast how many additional units could have been sold during the remainder of the season. Therefore, even if a firm cannot directly observe lost sales, a firm should be able to obtain a reasonable estimate for what demand could have been.

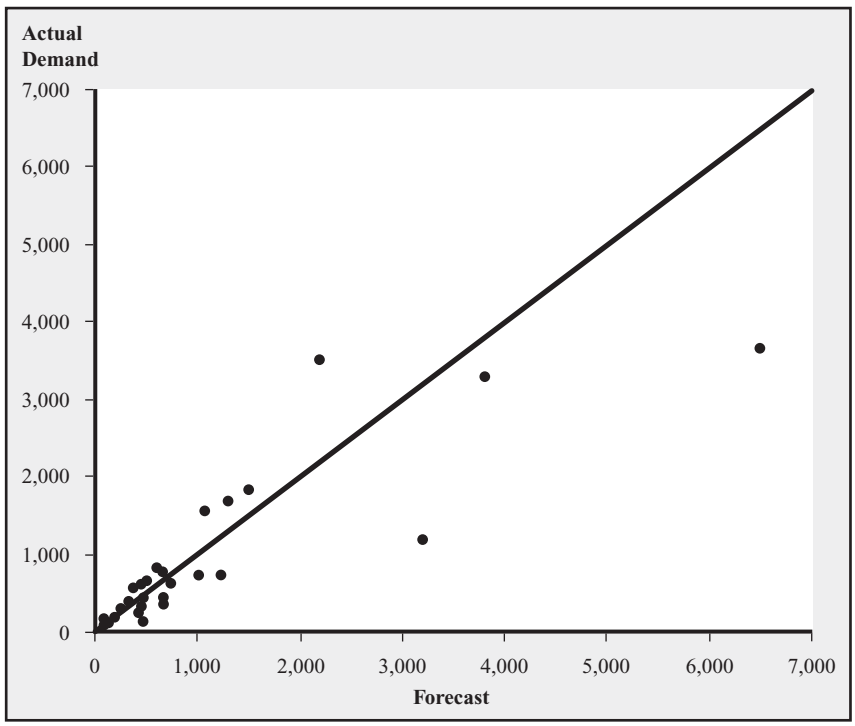


FIGURE 12.3
Forecasts and Actual Demand for Surf Wetsuits from the Previous Season

As can be seen from the data, the forecasts ranged from a low of 90 units to a high of 6,490 units. There was also considerable forecast error: O'Neill goofed with the Women's Hammer 3/2 Full suit with a forecast nearly 3,000 units above actual demand, while the forecast for the Epic 3/2 suit was about 1,300 units too low. Figure 12.3 gives a scatter plot of forecasts and actual demand. If forecasts were perfect, then all of the observations would lie along the diagonal line.

While the absolute errors for some of the bigger products are dramatic, the forecast errors for some of the smaller products are also significant. For example, the actual demand for the Juniors Zen Flat Lock 3/2 suit was more than 150 percent greater than forecast. This suggests that we should concentrate on the relative forecast errors instead of the absolute forecast errors.

Relative forecast errors can be measured with the *A/F ratio:*

$$\text{A/F ratio} = \frac{\text{Actual demand}}{\text{Forecast}}$$

An accurate forecast has an A/F ratio = 1, while an A/F ratio above 1 indicates the forecast was too low and an A/F ratio below 1 indicates the forecast was too high. Table 12.1 displays the A/F ratios for our data in the last column.

Those A/F ratios provide a measure of the forecast accuracy from the previous season. To illustrate this point, Table 12.2 sorts the data in ascending A/F order. Also included in the table is each product's A/F rank in the order and each product's percentile, the fraction of products that have that A/F rank or lower. (For example, the product with the fifth A/F ratio has a percentile of 5/33 = 15.2 percent because it is the fifth product out of 33 products in the data.) We see from the data that actual demand is less than 80 percent of the forecast for one-third of the products (the A/F ratio 0.8 has a percentile of 33.3) and

TABLE 12.2
Sorted A/F Ratios for Surf Wetsuits from the Previous Spring Season

Product Description	Forecast	Actual Demand	A/F Ratio*	Rank	Percentile**
ZEN-ZIP 2MM FULL	470	116	0.25	1	3.0
ZEN 3/2	3,190	1,195	0.37	2	6.1
ZEN 4/3	430	239	0.56	3	9.1
WMS ELITE 3/2	650	364	0.56	4	12.1
WMS HAMMER 3/2 FULL	6,490	3,673	0.57	5	15.2
JR HAMMER 3/2	1,220	721	0.59	6	18.2
HEATWAVE 4/3	430	274	0.64	7	21.2
ZEN 2MM S/S FULL	680	453	0.67	8	24.2
EPIC 5/3 W/HD	120	83	0.69	9	27.3
EPIC 4/3	1,020	732	0.72	10	30.3
WMS EPIC 2MM FULL	390	311	0.80	11	33.3
ZEN FL 3/2	450	365	0.81	12	36.4
EPIC 2MM S/S FULL	740	607	0.82	13	39.4
ZEN-ZIP 4/3	3,810	3,289	0.86	14	42.4
WMS ZEN-ZIP 4/3	170	163	0.96	15	45.5
JR EPIC 3/2	180	175	0.97	16	48.5
HEAT 4/3	460	450	0.98	17	51.5
JR ZEN 3/2	140	143	1.02	18	54.5
WMS ZEN 3/2	180	195	1.08	19	57.6
WMS EPIC 5/3 W/HD	320	369	1.15	20	60.6
ZEN-ZIP 5/4/3 W/HOOD	270	317	1.17	21	63.6
ZEN-ZIP 3/2	660	788	1.19	22	66.7
HAMMER S/S FULL	1,490	1,832	1.23	23	69.7
HEATWAVE 3/2	170	212	1.25	24	72.7
HEAT 3/2	500	635	1.27	25	75.8
HAMMER 3/2	1,300	1,696	1.30	26	78.8
WMS EPIC 3/2	610	830	1.36	27	81.8
EVO 4/3	440	623	1.42	28	84.8
WMS EPIC 4/3	1,060	1,552	1.46	29	87.9
JR EPIC 4/3	380	571	1.50	30	90.9
EVO 3/2	380	587	1.54	31	93.9
JR ZEN FL 3/2	90	140	1.56	32	97.0
EPIC 3/2	2,190	3,504	1.60	33	100.0

*A/F ratio = Actual demand divided by Forecast
**Percentile = Rank divided by total number of wetsuits (33)

actual demand is greater than 125 percent of the forecast for 27.3 percent of the products (the A/F ratio 1.25 has a percentile of 72.7).

Given that the A/F ratios from the previous season reflect forecast accuracy in the previous season, maybe the current season's forecast accuracy will be comparable. Hence, we want to find a distribution function that will match the accuracy we observe in Table 12.2. We will use the normal distribution function to do this. Before getting there, we need a couple of additional results.

Take the definition of the A/F ratio and rearrange terms to get

$$\text{Actual demand} = \text{A/F ratio} \times \text{Forecast}$$

For the Hammer 3/2, the forecast is 3,200 units. Note that the forecast is not random, but the A/F ratio is random. Hence, the randomness in actual demand is directly related

to the randomness in the A/F ratio. Using standard results from statistics and the above equation, we get the following results:

$$\text{Expected actual demand} = \text{Expected A/F ratio} \times \text{Forecast}$$

and

$$\text{Standard deviation of demand} = \text{Standard deviattion of A/F ratios} \times \text{Forecast}$$

Expected actual demand, or *expected demand* for short, is what we should choose for the mean for our normal distribution, μ. The average A/F ratio in Table 12.2 is 0.9976. Therefore, expected demand for the Hammer 3/2 in the upcoming season is $0.9976 \times 3{,}200 = 3{,}192$ units. In other words, if the initial forecast is 3,200 units and the future A/F ratios are comparable to the past A/F ratios, then the mean of actual demand is 3,192 units. So let's choose 3,192 units as our mean of the normal distribution.

This decision may raise some eyebrows: If our initial forecast is 3,200 units, why do we not instead choose 3,200 as the mean of the normal distribution? Because 3,192 is so close to 3,200, assigning 3,200 as the mean probably would lead to a good order quantity as well. However, suppose the average A/F ratio were 0.90, that is, on average, actual demand is 90 percent of the forecast. It is quite common for people to have overly optimistic forecasts, so an average A/F ratio of 0.90 is possible. In that case, expected actual demand would only be $0.90 \times 3{,}200 = 2{,}880$. Because we want to choose a normal distribution that represents actual demand, in that situation it would be better to choose a mean of 2,880 even though our initial forecast is 3,200. (Novice golfers sometimes adopt an analogous strategy. If a golfer consistently hooks the ball to the right on her drives, then she should aim to the left of the flag. In an ideal world, there would be no hook to her shot nor a bias in the forecast. But if the data say there is a hook, then it should not be ignored. Of course, the golfer and the forecaster also should work on eliminating the bias.)

Now that we have a mean for our normal distribution, we need a standard deviation. The second equation above tells us that the standard deviation of actual demand equals the standard deviation of the A/F ratios times the forecast. The standard deviation of the A/F ratios in Table 12.2 is 0.369. (Use the "stdev()" function in Excel.) So the standard deviation of actual demand is the standard deviation of the A/F ratios times the initial forecast: $0.369 \times 3{,}200 = 1{,}181$. Hence, to express our demand forecast for the Hammer 3/2, we can use a normal distribution with a mean of 3,192 and a standard deviation of 1,181. See Exhibit 12.1 for a summary of the process of choosing a mean and a standard deviation for a normal distribution forecast.

Now that we have the parameters of a normal distribution that will express our demand forecast, we need to be able to find $F(Q)$. There are two ways this can be done. The first way is to use spreadsheet software. For example, in Excel use the function Normdist(Q, 3192, 1181, 1). The second way, which does not require a computer, is to use the Standard Normal Distribution Function Table in Appendix B.

The *standard normal* is a particular normal distribution: its mean is 0 and its standard deviation is 1. To introduce another piece of common Greek notation, let $\Phi(z)$ be the distribution function of the standard normal. Even though the standard normal is a continuous distribution, it can be "chopped up" into pieces to make it into a discrete distribution. The Standard Normal Distribution Function Table is exactly that; that is, it is the discrete version of the standard normal distribution. The full table is in Appendix B, but Table 12.3 reproduces a portion of the table.

The format of the Standard Normal Distribution Function Table makes it somewhat tricky to read. For example, suppose you wanted to know the probability that the outcome of a standard normal is 0.51 or lower. We are looking for the value of $\Phi(z)$ with $z = 0.51$. To find that value, pick the row and column in the table such that the first number in the row and the first number in the column add up to the z value you seek. With $z = 0.51$, we are looking for the row that begins with 0.50 and the column that begins with 0.01, because the sum of those two

Exhibit 12.1

A PROCESS FOR USING HISTORICAL A/F RATIOS TO CHOOSE A MEAN AND STANDARD DEVIATION FOR A NORMAL DISTRIBUTION FORECAST

Step 1. Assemble a data set of products for which the forecasting task is comparable to the product of interest. In other words, the data set should include products that you expect would have similar forecast error to the product of interest. (They may or may not be similar products.) The data should include an initial forecast of demand and the actual demand. We also need forecast for the item for the upcoming season.

Step 2. Evaluate the A/F ratio for each product in the data set. Evaluate the average of the A/F ratios (that is, the expected A/F ratio) and the standard deviation of the A/F ratios. (In Excel use the average() and stdev() functions.)

Step 3. The mean and standard deviation of the normal distribution that we will use as the forecast can now be evaluated with the following two equations:

$$\text{Expected demand} = \text{Expected A/F ratio} \times \text{Forecast}$$

$$\text{Standard deviation of demand} = \text{Standard deviation of A/F ratios} \times \text{Forecast}$$

where the forecast in the above equations is the forecast for the item for the upcoming season.

values equals 0.51. The intersection of that row with that column gives $\Phi(z)$; from Table 12.3 we see that $\Phi(0.51) = 0.6950$. Therefore, there is a 69.5 percent probability the outcome of a standard normal is 0.51 or lower.

But it is unlikely that our demand forecast will be a standard normal distribution. So how can we use the standard normal to find $F(Q)$; that is, the probability demand will be Q or lower given that our demand forecast is some other normal distribution? The answer is that we convert the quantity we are interested in, Q, into an equivalent quantity for the standard normal. In other words, we find a z such that $F(Q) = \Phi(z)$; that is, the probability demand is less than or equal to Q is the same as the probability the outcome of a standard normal is z or lower. That z is called the *z-statistic.* Once we have the appropriate z-statistics, we then just look up $\Phi(z)$ in the Standard Normal Distribution Function Table to get our answer.

To convert Q into the equivalent z-statistic, use the following equation:

$$z = \frac{Q - \mu}{\sigma}$$

For example, suppose we are interested in the probability that demand for the Hammer 3/2 will be 4,000 units or lower, that is, $Q = 4{,}000$. With a normal distribution that has mean 3,192 and standard deviation 1,181, the quantity $Q = 4{,}000$ has a z-statistic of

$$z = \frac{4{,}000 - 3{,}192}{1{,}181} = 0.68$$

TABLE 12.3
A Portion of the Standard Normal Distribution Function Table, $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8269	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621

Exhibit 12.2

A PROCESS FOR EVALUATING THE PROBABILITY DEMAND IS EITHER LESS THAN OR EQUAL TO Q (WHICH IS $F(Q)$) OR MORE THAN Q (WHICH IS $1 - F(Q)$)

If the demand forecast is a normal distribution with mean μ and standard deviation σ, then follow steps A and B:

A. Evaluate the z-statistic that corresponds to Q:

$$z = \frac{Q - \mu}{\sigma}$$

B. The probability demand is less than or equal to Q is $\Phi(z)$. With Excel $\Phi(z)$ can be evaluated with the function Normsdist(z); otherwise, look up $\Phi(z)$ in the Standard Normal Distribution Function Table in Appendix B. If you want the probability demand is greater than Q, then your answer is $1 - \Phi(z)$.

If the demand forecast is a discrete distribution function table, then look up $F(Q)$, which is the probability demand is less than or equal to Q. If you want the probability demand is greater than Q, then the answer is $1 - F(Q)$.

Therefore, the probability demand for the Hammer 3/2 is 4,000 units or lower is $\Phi(0.68)$; that is, it is the same as the probability the outcome of a standard normal is 0.68 or lower. According to the Standard Normal Distribution Function Table (see Table 12.3 for convenience), $\Phi(0.68) = 0.7517$. In other words, there is just over a 75 percent probability that demand for the Hammer 3/2 will be 4,000 or fewer units. Exhibit 12.2 summarizes the process of finding the probability demand will be less than or equal to some Q (or more than Q).

You may recall that it has been O'Neill's experience that demand deviated by more than 25 percent from their initial forecast for 50 percent of their products. We can now check whether that experience is consistent with our normal distribution forecast for the Hammer 3/2. Our initial forecast is 3,200 units. So a deviation of 25 percent or more implies demand is either less than 2,400 units or more than 4,000 units. The z-statistic for $Q = 2{,}400$ is $z = (2400 - 3192)/1181 = -0.67$, and from the Standard Normal Distribution Function Table, $\Phi(-0.67) = 0.2514$. (Find the row with -0.60 and the column with -0.07.) If there is a 25.14 percent probability demand is less than 2,400 units and a 75.17 percent probability that demand is less than 4,000 units, then there is a $75.17 - 25.14 = 50.03$ percent probability that demand is between 2,400 and 4,000 units. Hence, O'Neill's initial assertion regarding forecast accuracy is consistent with our normal distribution forecast of demand.

To summarize, the objective in this section is to develop a detailed demand forecast. A single "point forecast" (e.g., 3,200 units) is not sufficient. We need to quantify the amount of variability that may occur about our forecast; that is, we need a distribution function. We obtained this distribution function by fitting a normal distribution to our historical forecast accuracy data, Table 12.2.

12.4 The Expected Profit-Maximizing Order Quantity

The next step after assembling all of our inputs (selling price, cost, salvage value, and demand forecast) is to choose an order quantity. The first part in that process is to decide what is our objective. A natural objective is to choose our production/procurement quantity

to maximize our expected profit. This section explains how to do this. Section 12.6 considers other possible objectives.

Before revealing the actual procedure for choosing an order quantity to maximize expected profit, it is helpful to explore the intuition behind the solution. Consider again O'Neill's Hammer 3/2 ordering decision. Should we order one unit? If we do, then there is a very good chance we will sell the unit: With a forecast of 3,192 units, it is likely we sell at least one unit. If we sell the unit, then the gain from that unit equals \$190 − \$110 = \$80 (the selling price minus the purchase cost). The *expected* gain from the first unit, which equals the probability of selling the first unit times the gain from the first unit, is then very close to \$80. However, there is also a slight chance that we do not sell the first unit, in which case we incur a loss of \$110 − \$90 = \$20. (The loss equals the difference between the purchase cost and the discount price.) But since the probability we do not sell that unit is quite small, the *expected* loss on the first unit is nearly \$0. Given that the expected gain from the first unit clearly exceeds the expected loss, the profit from ordering that unit is positive. In this case it is a good bet to order at least one unit.

After deciding whether to order one unit, we can now consider whether we should order two units, and then three units, and so forth. Two things happen as we continue this process. First, the probability that we sell the unit we are considering decreases, thereby reducing the expected gain from that unit. Second, the probability we do not sell that unit increases, thereby increasing the expected loss from that unit. Now imagine we order the 6,400th unit. The probability of selling that unit is quite low, so the expected gain from that unit is nearly zero. In contrast, the probability of *not* selling that unit is quite high, so the expected loss is nearly \$20 on that unit. Clearly it makes no sense to order the 6,400th unit. This pattern is illustrated in Figure 12.4. We see that from some unit just above 4,000 the expected gain on that unit equals its expected loss.

Let's formalize this intuition some more. In the newsvendor model, there is a trade-off between ordering too much (which could lead to costly leftover inventory) and ordering too little (which could lead to the opportunity cost of lost sales). To balance these forces, it is useful to think in terms of a cost for ordering too much and a cost for ordering too little. Maximizing expected profit is equivalent to minimizing those costs. To be specific, let C_o be the *overage cost,* the loss incurred when a unit is ordered but not sold. In other words, the overage cost is the per-unit cost of overordering. For the Hammer 3/2, we have $C_o = 20$.

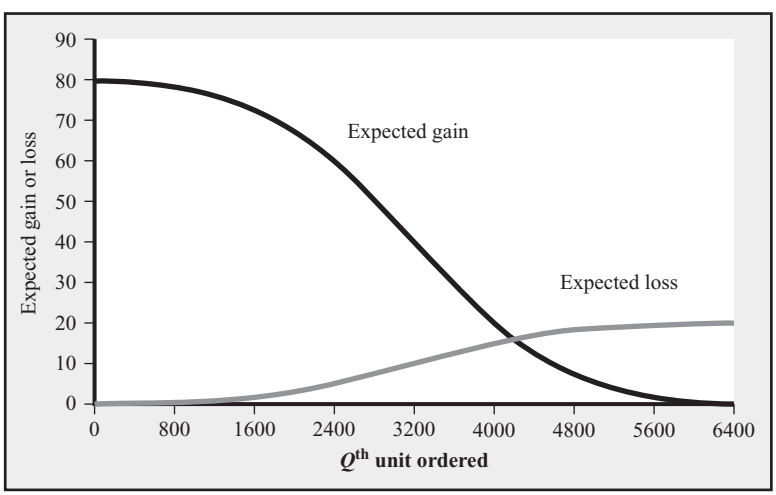


FIGURE 12.4
The Expected Gain and Expected Loss from the *Q*th Hammer 3/2 Ordered by O'Neill

In contrast to C_o, let C_u be the *underage cost,* the opportunity cost of not ordering a unit that could have been sold. The following is an equivalent definition for C_u: C_u is the gain from selling a unit. In other words, the underage cost is the per-unit opportunity cost of underordering. For the Hammer 3/2, $C_u = 80$. Note that the overage and underage costs are defined for a *single unit.* In other words, C_o is not the total cost of all leftover inventory; instead, C_o is the cost *per unit* of leftover inventory. The reason for defining C_o and C_u for a single unit is simple: We don't know how many units will be left over in inventory, or how many units of demand will be lost, but we do know the cost of each unit left in inventory and the opportunity cost of each lost sale.

Now that we have defined the overage and underage costs, we need to choose Q to strike the balance between them that results in the maximum expected profit. Based on our previous reasoning, we should keep ordering additional units until the expected loss equals the expected gain.

The expected loss on a unit is the cost of having the unit in inventory (the overage cost) times the probability it is left in inventory. For the Qth unit, that probability is $F(Q)$: It is left in inventory if demand is less than Q.[2] Therefore, the expected loss is $C_o \times F(Q)$. The expected gain on a unit is the benefit of selling a unit (the underage cost) times the probability the unit is sold, which in this case occurs if demand is greater than Q. The probability demand is greater than Q is $(1 - F(Q))$. Therefore, the expected gain is $C_u \times (1 - F(Q))$.

It remains to find the order quantity Q that sets the expected loss on the Qth unit equal to the expected gain on the Qth unit:

$$C_o \times F(Q) = C_u \times (1 - F(Q))$$

If we rearrange terms in the above equation, we get

$$F(Q) = \frac{C_u}{C_o + C_u} \qquad \textbf{(12.1)}$$

The profit-maximizing order quantity is the order quantity that satisfies the above equation. If you are familiar with calculus and would like to see a more mathematically rigorous derivation of the optimal order quantity, see Appendix D.

So how can we use equation (12.1) to actually find Q? Let's begin by just reading it. It says that the order quantity that maximizes expected profit is the order quantity Q such that demand is less than or equal to Q with probability $C_u/(C_o + C_u)$. That ratio with the underage and overage costs is called the *critical ratio.* We now have an explanation for why our forecast must be a distribution function. To choose the profit-maximizing order quantity, we need to find the quantity such that demand will be less than that quantity with a particular probability (the critical ratio). The mean alone (i.e., just a sales forecast) is insufficient to do that task.

Let's begin with the easy part. We know for the Hammer 3/2 that $C_u = 80$ and $C_o = 20$, so the critical ratio is

$$\frac{C_u}{C_o + C_u} = \frac{80}{20 + 80} = 0.8$$

[2] That statement might bother you. You might recall that $F(Q)$ is the probability demand is Q or lower. If demand is exactly Q, then the Qth unit will not be left in inventory. Hence, you might argue that it is more precise to say that $F(Q - 1)$ is the probability the Qth unit is left in inventory. However, the normal distribution assumes demand can be any value, including values that are not integers. If you are willing to divide each demand into essentially an infinite number of fractional pieces, as is assumed by the normal, then $F(Q)$ is indeed the probability there is leftover inventory. If you are curious about the details, see Appendix D.

We are making progress, but now comes the tricky part: We need to find the order quantity Q such that there is a 80 percent probability that demand is Q or lower.

There are two ways to find a Q such that there is an 80 percent probability that demand will be Q or smaller. The first is to use the Excel function, Normsinv(), and the second is to use the Standard Normal Distribution Function Table. If you have Excel available, the first method is the easiest, but they both follow essentially the same process, as we will see.

If we have Excel, to find the optimal Q, we begin by finding the z statistic such that there is an 80 percent probability the outcome of a standard normal is z or lower. Then we convert that z into the Q we seek. To find our desired z, use the following Excel function:

$$z = \text{Normsinv(Critical ratio)}$$

In our case, the critical ratio is 0.80 and Normsinv(0.80) returns 0.84. That means that there is an 80 percent chance the outcome of a standard normal will be 0.84 or lower. That would be our optimal order quantity if demand followed a standard normal distribution. But our demand is not standard normal. It is normal with mean 3192 and standard deviation 1181. To convert our z into an order quantity that makes sense for our actual demand forecast, we use the following equation:

$$Q = \mu + z \times \sigma$$

where μ = Mean of the normal distribution

σ = Standard deviation of the normal distribution

Hence, using our Excel method, the expected profit maximizing order quantity for the Hammer 3/2 is $Q = 3{,}192 + 0.84 \times 1{,}181 = 4{,}184$.

The second method to find Q is to use the Standard Normal Distribution Function Table. Again, we want to find the z such that the probability the standard normal is z or less is equal to the critical ratio, which in this case is 0.80. Looking at Table 12.3, we see that $\Phi(0.84) = 0.7995$ and $\Phi(0.85) = 0.8023$, neither of which is exactly the 0.80 probability we are looking for: $z = 0.84$ yields a slightly lower probability (79.95 percent) and $z = 0.85$ yields a slightly higher probability (80.23 percent). What should we do? The rule is simple, which we will call the *round-up rule:*

Round-up rule. Whenever you are looking up a target value in a table and the target value falls between two entries, choose the entry that leads to the larger order quantity.

In this case the larger quantity is $z = 0.85$, so we will go with 0.85. Now, like with our Excel process, we convert that z into a $Q = 3{,}192 + 0.85 \times 1{,}181 = 4{,}196$.

Why do our two methods lead to different answers? In short, Excel does not implement the round-up rule. But that raises the next question. Is it OK to use Excel to get our answer? The answer is "yes." To explain, when demand is normally distributed, there will be a small difference between the Excel answer, using the Normsinv() function, and the Standard Normal Distribution Function Table answer. In this case, the difference between the two is only 12 units, which is less than 0.3 percent away from 4,196.

Therefore, the expected profit with either of these order quantities will be essentially the same. Furthermore, Excel provides a convenient means to perform this calculation quickly.

So, if Excel is the quick and easy method, why should we bother with the Standard Normal Distribution Function Table and the round-up rule? Because when our demand forecast is a discrete distribution function, the round-up rule provides the more accurate answer. (Recall, a discrete distribution function assumes that the only possible outcomes

Exhibit 12.3

A PROCEDURE TO FIND THE ORDER QUANTITY THAT MAXIMIZES EXPECTED PROFIT IN THE NEWSVENDOR MODEL

Step 1: Evaluate the critical ratio: $\frac{C_u}{C_o + C_u}$. In the case of the Hammer 3/2, the underage cost is C_u = Price − Cost and the overage cost is C_o = Cost − Salvage value.

Step 2: If the demand forecast is a normal distribution with mean μ and standard deviation σ, then follow steps A and B:

A. Find the optimal order quantity if demand had a standard normal distribution. One method to achieve this is to find the z value in the Standard Normal Distribution Function Table such that

$$\Phi(z) = \frac{C_u}{C_o + C_u}$$

(If the critical ratio value does not exist in the table, then find the two z values that it falls between. For example, the critical ratio 0.80 falls between $z = 0.84$ and $z = 0.85$. Then choose the larger of those two z values.) A second method is to use the Excel function Normsinv: z = Normsinv(Critical ratio).

B. Convert z into the order quantity that maximizes expected profit, Q: $Q = \mu + z \times \sigma$

are integers.) This is particularly valuable when expected demand is small, say, 10 units, or 1 unit, or even 0.25 unit. In those cases, the normal distribution function does not model demand well (in part, because it is a continuous distribution function). Furthermore, it can make a big difference (in terms of expected profit) whether one or two units are ordered. Hence, the value of understanding the round-up rule.

This discussion probably has left you with one final question—Why is the round-up rule the right rule? The critical ratio is actually closer to 0.7995 (which corresponds to $z = 0.84$) than it is to 0.8023 (which corresponds to $z = 0.85$). That is why Excel chooses $z = 0.84$. Shouldn't we choose the z value that leads to the probability that is closest to the critical ratio? In fact, that is not the best approach. The critical ratio equation works with the following logic—keep ordering until you get to the first order quantity such that the critical ratio is less than the probability demand is that order quantity or lower. That logic leads the rule to "step over" the critical ratio and then stop, that is, the round-up rule. Excel, in contrast, use the "get as close to the critical ratio as possible" rule. If you are hungry for a more in-depth explanation and justification, see Appendix D. Otherwise, stick with the round-up rule, and you will be fine. Exhibit 12.3 summarizes these steps.

12.5 Performance Measures

The previous section showed us how to find the order quantity that maximizes our expected profit. This section shows us how to evaluate a number of relevant performance measures. As Figure 12.5 indicates, these performance measures are closely related. For example, to evaluate expected leftover inventory, you first evaluate expected lost sales (which has up to three inputs: the order quantity, the loss function table, and the standard deviation of demand), then expected sales (which has two inputs: expected lost sales and expected demand), and then expected leftover inventory (which has two inputs: expected sales and the order quantity).

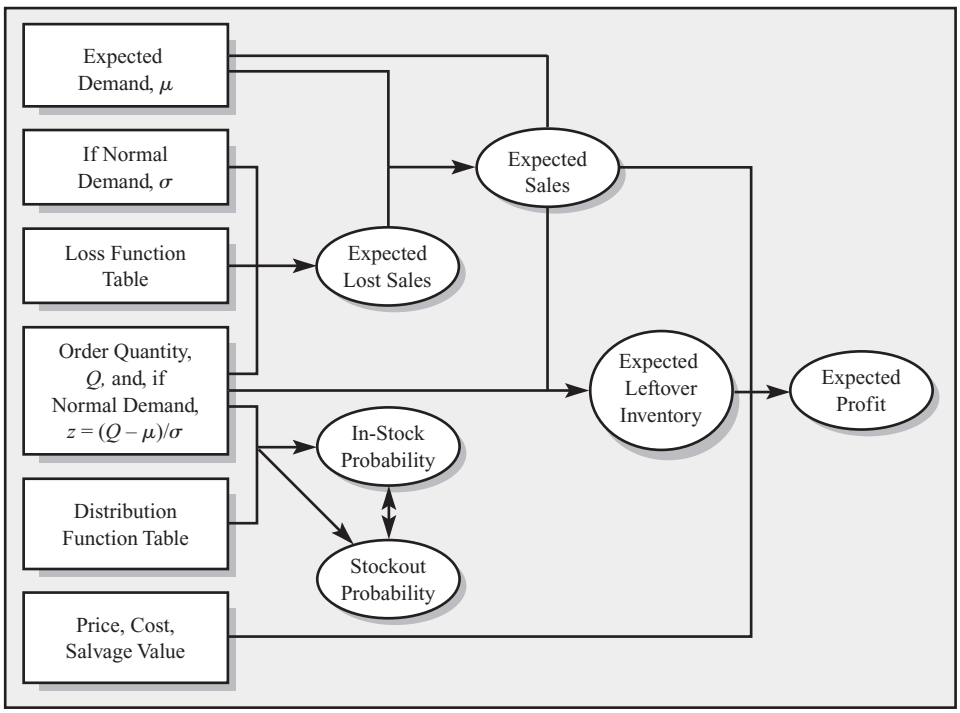


FIGURE 12.5
The Relationships between Initial Input Parameters (boxes) and Performance Measures (ovals)
Note: Some performance measures require other performance measures as inputs; for example, expected sales requires expected demand and expected lost sales as inputs.

These performance measures can be evaluated for any order quantity, not just the expected profit-maximizing order quantity. To emphasize this point, this section evaluates these performance measures assuming 3,500 Hammer 3/2s are ordered. See Table 13.1 for the evaluation of these measures with the optimal order quantity, 4,196 units.

Expected Lost Sales

Let's begin with *expected lost sales,* which is the expected number of units by which demand (a random variable) exceeds the order quantity (a fixed threshold). (Because order quantities are measured in physical units, sales and lost sales are measured in physical units as well, not in monetary units.) For example, if we order 3,500 units of the Hammer but demand could have been high enough to sell 3,821 units, then we would lose $3{,}821 - 3{,}500 = 321$ units of demand. Expected lost sales is the amount of demand that is not satisfied, which should be of interest to a manager even though the opportunity cost of lost sales does not show up explicitly on any standard accounting document.

Note that we are interested in the *expected* lost sales. Demand can be less than our order quantity, in which case lost sales is zero, or demand can exceed our order quantity, in which case lost sales is positive. Expected lost sales is the average of all of those events (the cases with no lost sales and all cases with positive lost sales).

How do we find expected lost sales for any given order quantity? When demand is normally distributed, use the following equation:

$$\text{Expected lost sales} = \sigma \times L(z)$$

where σ = Standard deviation of the normal distribution representing demand
$L(z)$ = Loss function with the standard normal distribution

We already know $\sigma = 1{,}181$ but what is *L(z)*? Like with the optimal order quantity, there are two methods to find *L(z)*, one using Excel and one using a table. With either method, we first find the *z*-statistic that corresponds to our chosen order quantity, $Q = 3{,}500$:

$$z = \frac{Q - \mu}{\sigma} = \frac{3{,}500 - 3{,}192}{1{,}181} = 0.26$$

The first method then uses the following Excel formula to evaluate the expected lost sales if demand were a standard normal distribution, *L(z)*:

$$L(z) = \text{Normdist}(z,0,1,0) - z*(1 - \text{Normsdist}(z))$$

(If you are curious about the derivation of the above function, see Appendix D.) In this case, Excel provides the following answer: $L(0.26) = \text{Normdist}(0.26,0,1,0) - 0.26*(1 - \text{Normsdist}(0.26)) = 0.2824$.

The second method uses the Standard Normal Loss Function Table in Appendix B to look up the expected lost sales. From that table we see that $L(0.26) = 0.2824$. In this case, our two methods yield the same value for *L(z),* which always is the case when we input into the Excel function a *z* value rounded to the nearest hundredth (e.g., 0.26 instead of 0.261). Therefore, if the order quantity is 3,500 Hammer 3/2s, then we can expect to lose $\sigma \times L(z) = 1{,}181 \times 0.2824 = 334$ units of demand.

How do we evaluate expected lost sales when we do not use a normal distribution to model demand? In that situation we need a table to tell us what expected lost sales is for our chosen order quantity. For example, Appendix B provides the loss function for the Poisson distribution with different means. Appendix C provides a procedure to evaluate the loss function for any discrete distribution function. We relegate this procedure to the appendix because it is computationally burdensome; that is, it is the kind of calculation you want to do on a spreadsheet rather than by hand.

Exhibit 12.4 summarizes the procedures for evaluating expected lost sales.

Expected Sales

Each unit of demand results in either a sale or a lost sale, so

$$\text{Expected sales} + \text{Expected lost sales} = \text{Expected demand}$$

We already know expected demand: It is the mean of the demand distribution, μ. Rearrange terms in the above equation and we get

$$\text{Expected sales} = \mu - \text{Expected lost sales}$$

Therefore, the procedure to evaluate expected sales begins by evaluating expected lost sales. See Exhibit 12.5 for a summary of this procedure.

Let's evaluate expected sales if 3,500 Hammers are ordered and the normal distribution is our demand forecast. We already evaluated expected lost sales to be 334 units. Therefore, Expected sales = 3,192 − 334 = 2,858 units.

Notice that expected sales is always less than expected demand (because expected lost sales is never negative). While you might get lucky and sell more than the mean demand, on average you cannot sell more than the mean demand.

Exhibit 12.4

EXPECTED LOST SALES EVALUATION PROCEDURE

If the demand forecast is a normal distribution with mean μ and standard deviation σ, then follow steps A through D:

A. Evaluate the z-statistic for the order quantity Q: $z = \dfrac{Q - \mu}{\sigma}$.

B. Use the z-statistic to look up in the Standard Normal Loss Function Table the expected lost sales, $L(z)$, with the standard normal distribution.

C. Expected lost sales $= \sigma \times L(z)$.

D. With Excel, expected lost sales can be evaluated with the following equation:

$$\text{Expected lost sales} = \sigma^*(\text{Normdist}(z,0,1,0) - z^*(1 - \text{Normsdist}(z)))$$

If the demand forecast is a discrete distribution function table, then expected lost sales equals the loss function for the chosen order quantity, $L(Q)$. If the table does not include the loss function, then see Appendix C for how to evaluate it.

Expected Leftover Inventory

Expected leftover inventory is the average amount that demand (a random variable) is less than the order quantity (a fixed threshold). (In contrast, expected lost sales is the average amount by which demand exceeds the order quantity.)

The following equation is true because every unit purchased is either sold or left over in inventory at the end of the season:

$$\text{Expected sales} + \text{Expected leftover inventory} = Q$$

Rearrange the above equation to obtain

$$\text{Expected leftover inventory} = Q - \text{Expected sales}$$

We know the quantity purchased, Q. Therefore, we can easily evaluate expected leftover inventory once we have evaluated expected sales. See Exhibit 12.5 for a summary of this procedure.

If the demand forecast is a normal distribution and 3,500 Hammers are ordered, then expected leftover inventory is $3{,}500 - 2{,}858 = 642$ units because we evaluated expected sales to be 2,858 units.

It may seem surprising that expected leftover inventory and expected lost sales can both be positive. While in any particular season there is either leftover inventory or lost sales, but not both, we are interested in the expectation of those measures over all possible outcomes. Therefore, each *expectation* can be positive.

Expected Profit

We earn Price − Cost on each unit sold and we lose Cost − Salvage value on each unit we do not sell, so our expected profit is

$$\begin{aligned}\text{Expected profit} = {} & [(\text{Price} - \text{Cost}) \times \text{Expected sales}] \\ & -[(\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory}]\end{aligned}$$

Therefore, we can evaluate expected profit after we have evaluated expected sales and leftover inventory. See Exhibit 12.5 for a summary of this procedure.

Exhibit 12.5

EXPECTED SALES, EXPECTED LEFTOVER INVENTORY, AND EXPECTED PROFIT EVALUATION PROCEDURES

Step 1. Evaluate expected lost sales (see Exhibit 12.4). All of these performance measures can be evaluated directly in terms of expected lost sales and several known parameters: μ = Expected demand; Q = Order quantity; Price; Cost; and Salvage value.

Step 2. Use the following equations to evaluate the performance measure of interest.

$$\begin{aligned}
\text{Expected sales} &= \mu - \text{Expected lost sales} \\
\text{Expected leftover inventory} &= Q - \text{Expected sales} \\
&= Q - \mu + \text{Expected lost sales} \\
\text{Expected profit} &= [(\text{Price} - \text{Cost}) \times \text{Expected sales}] \\
&\quad - [(\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory}]
\end{aligned}$$

With an order quantity of 3,500 units and a normal distribution demand forecast, the expected profit for the Hammer 3/2 is

$$\text{Expected profit} = (\$80 \times 2,858) - (\$20 \times 642) = \$215,800$$

In-Stock Probability and Stockout Probability

A common measure of customer service is the in-stock probability. The in-stock probability is the probability the firm ends the season having satisfied all demand. (Equivalently, the in-stock probability is the probability the firm has stock available for every customer.) That occurs if demand is less than or equal to the order quantity,

$$\text{In-stock probability} = F(Q)$$

The stockout probability is the probability the firm stocks out for some customer during the selling season (i.e., a lost sale occurs). Because the firm stocks out if demand exceeds the order quantity,

$$\text{Stockout probability} = 1 - F(Q)$$

(The firm either stocks out or it does not, so the stockout probability equals 1 minus the probability demand is Q or lower.) We also can see that the stockout probability and the in-stock probability are closely related:

$$\text{Stockout probability} = 1 - \text{In-stock probability}$$

See Exhibit 12.6 for a summary of the procedure to evaluate these probabilities. With an order quantity of 3,500 Hammers, the z-statistic is $z = (3,500 - 3,192)/1,181 = 0.26$. From the Standard Normal Distribution Function Table, we find $\Phi(0.26) = 0.6026$, so the in-stock probability is 60.26 percent. The stockout probability is $1 - 0.6026 = 39.74$ percent.

The in-stock probaility is not the only measure of customer service. Another popular measure is the *fill rate*. The fill rate is the probability a customer is able to purchase a unit (i.e., does not experience a stockout). Interestingly, this is not the same as the in-stock probability, which is the probability that all demand is satisfied. For example, if Q = 100 and demand turns out to be 101, then most customers were able to purchase a unit but the firm did not satisfy all demand. See Appendix D for more information regarding how to evaluate the fill rate.

Exhibit 12.6

IN-STOCK PROBABILITY AND STOCKOUT PROBABILITY EVALUATION

If the demand forecast is a normal distribution with mean μ and standard deviation σ, then follow steps A through D:

A. Evaluate the z-statistic for the order quantity: $z = \dfrac{Q - \mu}{\sigma}$.

B. Use the z-statistic to look up in the Standard Normal Distribution Function Table the probability the standard normal demand is z or lower, $\Phi(z)$.

C. In-stock probability $= \Phi(z)$ and Stockout probability $= 1 - \Phi(z)$.

D. In Excel, In-stock probability = Normsdist(z) and Stockout probability = 1 – Normsdist(z).

If the demand forecast is a discrete distribution function table, then In-stock probability $= F(Q)$ and Stockout probability $= 1 - F(Q)$, where $F(Q)$ is the probability demand is Q or lower.

12.6 Choosing an Order Quantity to Meet a Service Objective

Maximizing expected profit is surely a reasonable objective for choosing an order quantity, but it is not the only objective. As we saw in the previous section, the expected profit-maximizing order quantity may generate an unacceptable in-stock probability from the firm's customer service perspective. This section explains how to determine an order quantity that satisfies a customer service objective, in particular, a minimum in-stock probability.

Suppose O'Neill wants to find the order quantity that generates a 99 percent in-stock probability with the Hammer 3/2. The in-stock probability is $F(Q)$. So we need to find an order quantity such that there is a 99 percent probability that demand is that order quantity or lower. Given that our demand forecast is normally distributed, we first find the z-statistic that achieves our objective with the standard normal distribution. In the Standard Normal Distribution Function Table, we see that $\Phi(2.32) = 0.9898$ and $\Phi(2.33) = 0.9901$. Again, we choose the higher z-statistic, so our desired order quantity is now $Q = \mu + z \times \sigma = 3{,}192 + 2.33 \times 1{,}181 = 5{,}944$. You can use Excel to avoid looking up a probability in the Standard Normal Distribution Function Table to find z:

$$z = \text{Normsinv(In-stock probability)}$$

Notice that a substantially higher order quantity is needed to generate a 99 percent in-stock probability than the one that maximizes expected profit (4,196). Exhibit 12.7 summarizes the process for finding an order quantity to satisfy a target in-stock probability.

12.7 Managerial Lessons

Now that we have detailed the process of implementing the newsvendor model, it is worthwhile to step back and consider the managerial lessons it implies.

With respect to the forecasting process, there are three key lessons.

• For each product, it is insufficient to have just a forecast of expected demand. We also need a forecast for how variable demand will be about the forecast. That uncertainty in the forecast is captured by the standard deviation of demand.

Exhibit 12.7

A PROCEDURE TO DETERMINE AN ORDER QUANTITY THAT SATISFIES A TARGET IN-STOCK PROBABILITY

If the demand forecast is a normal distribution with mean μ and standard deviation σ, then follow steps A and B:

A. Find the *z*-statistic in the Standard Normal Distribution Function Table that satisfies the in-stock probability, that is,

$$\Phi(z) = \text{In-stock probability}$$

If the in-stock probability falls between two *z* values in the table, choose the higher *z*. In Excel, *z* can be found with the following formula:

$$z = \text{Normsinv(In-stock probability)}.$$

B. Convert the chosen *z*-statistic into the order quantity that satisfies our target in-stock probability,

$$Q = \mu + z \times \sigma$$

If the demand forecast is a discrete distribution function table, then find the order quantity in the table such that $F(Q) =$ In-stock probability. If the in-stock probability falls between two entries in the table, choose the entry with the larger order quantity.

• It is important to track actual demand. Two common mistakes are made with respect to this issue. First, do not forget that actual demand may be greater than actual sales due to an inventory shortage. If it is not possible to track actual demand after a stockout occurs, then you should attempt a reasonable estimate of actual demand. Second, actual demand includes potential sales only at the regular price. If you sold 1,000 units in the previous season, but 600 of them were at the discounted price at the end of the season, then actual demand is closer to 400 than 1,000.

• You need to keep track of past forecasts and forecast errors in order to assess the standard deviation of demand. Without past data on forecasts and forecast errors, it is very difficult to choose reasonable standard deviations; it is hard enough to forecast the mean of a distribution, but forecasting the standard deviation of a distribution is nearly impossible with just a "gut feel." Unfortunately, many firms fail to maintain the data they need to implement the newsvendor model correctly. They might not record the data because it is an inherently undesirable task to keep track of past errors: Who wants to have a permanent record of the big forecasting goofs? Alternatively, firms may not realize the importance of such data and therefore do not go through the effort to record and maintain it.

There are also a number of important lessons from the order quantity choice process.

• The profit-maximizing order quantity generally does not equal expected demand. If the underage cost is greater than the overage cost (i.e., it is more expensive to lose a sale than it is to have leftover inventory), then the profit-maximizing order quantity is larger than expected demand. (Because then the critical ratio is greater than 0.50.) On the other hand, some products may have an overage cost that is larger than the underage cost. For such products, it is actually best to order less than the expected demand.

• The order quantity decision should be separated from the forecasting process. The goal of the forecasting process is to develop the best forecast for a product's demand and therefore should proceed without regard to the order quantity decision. This can be frustrating for some firms. Imagine the marketing department dedicates considerable effort to develop a forecast and then the operations department decides to produce a quantity above the forecast. The marketing department may feel that their efforts are being ignored or their expertise is being second-guessed. In addition, they may be concerned that they would be responsible for ensuring that all of the production is sold even though their forecast was more conservative. The separation between the forecasting and the order quantity decision also implies that two products with the same mean forecast may have different expected profit-maximizing order quantities, either because they have different critical ratios or because they have different standard deviations.

• Explicit costs should not be overemphasized relative to opportunity costs. Inventory at the end of the season is the explicit cost of a demand–supply mismatch, while lost sales are the opportunity cost. Overemphasizing the former relative to the latter will cause you to order less than the profit-maximizing order quantity.

• It is important to recognize that choosing an order quantity to maximize expected profit is only one possible objective. It is also a very reasonable objective, but there can be situations in which a manager may wish to consider an alternative objective. For example, maximizing expected profit is wise if you are not particularly concerned with the variability of profit. If you are managing many different products so that the realized profit from any one product cannot cause undue hardship on the firm, then maximizing expected profit is a good objective to adopt. But if you are a startup firm with a single product and limited capital, then you might not be able to absorb a significant profit loss. In situations in which the variability of profit matters, it is prudent to order less than the profit-maximizing order quantity. The expected profit objective also does not consider customer service explicitly in its objective. With the expected profit-maximizing order quantity for the Hammer 3/2, the in-stock probability is about 80 percent. Some managers may feel this is an unacceptable level of customer service, fearing that unsatisfied customers will switch to a competitor. Figure 12.6 displays the trade-off between

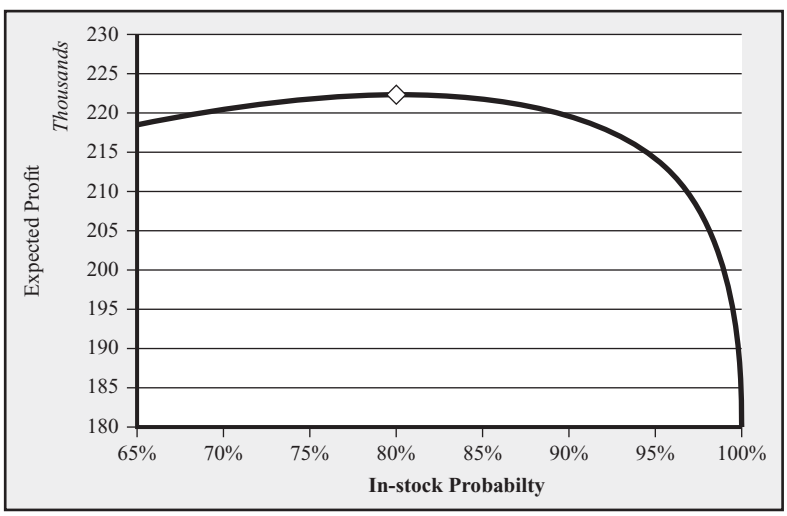


FIGURE 12.6
The Trade-off between Profit and Service with the Hammer 3/2
The circle indicates the in-stock probability and the expected profit of the optimal order quantity, 4,196 units.

service and expected profit. As we can see, the expected profit curve is reasonably flat around the maximum, which occurs with an in-stock probability that equals 80 percent. Raising the in-stock probability to 90 percent may be considered worthwhile because it reduces profits by slightly less than 1 percent. However, raising the in-stock dramatically, say, to 99 percent, may cause expected profits to fall too much—in that case by nearly 10 percent.

• Finally, while it is impossible to perfectly match supply and demand when supply must be chosen before random demand, it is possible to make a smart choice that balances the cost of ordering too much with the cost of ordering too little. In other words, uncertainty should not invite ad hoc decision making.

12.8 Summary

The newsvendor model is a tool for making a decision when there is a "too much–too little" challenge: Bet too much and there is a cost (e.g., leftover inventory), but bet too little and there is a different cost (e.g., the opportunity cost of lost sales). (See Table 12.4 for a summary of the key notation and equations.) To make this trade-off effectively, it is necessary to have a complete forecast of demand. It is not enough to just have a single sales forecast; we need to know the potential variation about that sales forecast. With a forecast model of demand (e.g., normal distribution with mean 3,192 and standard deviation 1,181), we can choose a quantity to maximize expected profit or to achieve a desired in-stock probability. For any chosen quantity, we can evaluate several performance measures, such as expected sales and expected profit.

TABLE 12.4
Summary of Key Notation and Equations in Chapter 12

Q = Order quantity C_u = Underage cost C_o = Overage cost Critical ratio $= \dfrac{C_u}{C_o + C_u}$

μ = Expected demand σ = Standard deviation of demand

$F(Q)$: Distribution function $\Phi(Q)$: Distribution function of the standard normal

Expected actual demand = Expected A/F ratio × Forecast

Standard deviation of actual demand = Standard deviation of A/F ratios × Forecast

Expected profit-maximizing order quantity: $F(Q) = \dfrac{C_u}{C_o + C_u}$

z-statistic or normalized order quantity: $z = \dfrac{Q - \mu}{\sigma}$.

$Q = \mu + z \times \sigma$

$L(z)$ = Expected lost sales with the standard normal distribution

Expected lost sales = $\sigma \times L(z)$ Expected sales = μ − Expected lost sales

Excel: Expected lost sales = σ * (Normdist(z,0,1,0) − z * (1 − Normsdist(z)))

Expected leftover inventory = Q − Expected sales

Expected profit = [(Price − Cost) × Expected sales]
− [(Cost − Salvage value) × Expected leftover inventory]

In-stock probability = $F(Q)$ Stockout probability = 1 − In-stock probability

Excel: z = Normsinv(Target in-stock probability)

Excel: In-stock probability = Normsdist(z)

12.9 Further Reading

The newsvendor model is one of the most extensively studied models in operations management. It has been extended theoretically along numerous dimensions (e.g., multiple periods have been studied, the pricing decision has been included, the salvage values could depend on the quantity salvaged, the decision maker's tolerance for risk can be incorporated into the objective function, etc.)

Several textbooks provide more technical treatments of the newsvendor model than this chapter. See Nahmias (2005), Porteus (2002), or Silver, Pyke, and Peterson (1998).

For a review of the theoretical literature on the newsvendor model, with an emphasis on the pricing decision in a newsvendor setting, see Petruzzi and Dada (1999).

12.10 Practice Problems

Q12.1* **(McClure Books)** Dan McClure owns a thriving independent bookstore in artsy New Hope, Pennsylvania. He must decide how many copies to order of a new book, *Power and Self-Destruction,* an exposé on a famous politician's lurid affairs. Interest in the book will be intense at first and then fizzle quickly as attention turns to other celebrities. The book's retail price is $20 and the wholesale price is $12. The publisher will buy back the retailer's leftover copies at a full refund, but McClure Books incurs $4 in shipping and handling costs for each book returned to the publisher. Dan believes his demand forecast can be represented by a normal distribution with mean 200 and standard deviation 80.

a. Dan will consider this book to be a blockbuster for him if it sells more than 400 units. What is the probability *Power and Self-Destruction* will be a blockbuster?

b. Dan considers a book a "dog" if it sells less than 50 percent of his mean forecast. What is the probability this exposé is a "dog"?

c. What is the probability demand for this book will be within 20 percent of the mean forecast?

d. What order quantity maximizes Dan's expected profit?

e. Dan prides himself on good customer service. In fact, his motto is "McClure's got what you want to read." How many books should Dan order if he wants to achieve a 95 percent in-stock probability?

f. If Dan orders the quantity chosen in part e to achieve a 95 percent in-stock probability, then what is the probability that "Dan won't have what some customer wants to read" (i.e., what is the probability some customer won't be able to purchase a copy of the book)?

g. Suppose Dan orders 300 copies of the book. What would Dan's expected profit be in this case?

Q12.2* (**EcoTable Tea**) EcoTable is a retailer of specialty organic and ecologically friendly foods. In one of their Cambridge, Massachusetts, stores, they plan to offer a gift basket of Tanzanian teas for the holiday season. They plan on placing one order and any leftover inventory will be discounted at the end of the season. Expected demand for this store is 4.5 units and demand should be Poisson distributed. The gift basket sells for $55, the purchase cost to EcoTable is $32, and leftover baskets will be sold for $20.

a. If they purchase only 3 baskets, what is the probability that some demand will not be satisfied?

b. If they purchase 10 baskets, what is the probability that they will have to mark down at least 3 baskets?

c. How many baskets should EcoTable purchase to maximize its expected profit?

d. Suppose they purchase 4 baskets. How many baskets can they expect to sell?

e. Suppose they purchase 6 baskets. How many baskets should they expect to have to mark down at the end of the season?

(* indicates that the solution is in Appendix E)

f. Suppose EcoTable wants to minimize its inventory while satisfying all demand with at least a 90 percent probability. How many baskets should they order?

g. Suppose EcoTable orders 8 baskets. What is its expected profit?

Q12.3* **(Pony Express Creations)** Pony Express Creations Inc. (www.pony-ex.com) is a manufacturer of party hats, primarily for the Halloween season. (80 percent of their yearly sales occur over a six-week period.) One of their popular products is the Elvis wig, complete with sideburns and metallic glasses. The Elvis wig is produced in China, so Pony Express must make a single order well in advance of the upcoming season. Ryan, the owner of Pony Express, expects demand to be 25,000 and the following is his entire demand forecast:

Q	Prob($D = Q$)	$F(Q)$	$L(Q)$
5,000	0.0181	0.0181	20,000
10,000	0.0733	0.0914	15,091
15,000	0.1467	0.2381	10,548
20,000	0.1954	0.4335	6,738
25,000	0.1954	0.6289	3,906
30,000	0.1563	0.7852	2,050
35,000	0.1042	0.8894	976
40,000	0.0595	0.9489	423
45,000	0.0298	0.9787	168
50,000	0.0132	0.9919	61
55,000	0.0053	0.9972	21
60,000	0.0019	0.9991	7
65,000	0.0006	0.9997	2
70,000	0.0002	0.9999	0
75,000	0.0001	1.0000	0

Prob($D = Q$) = Probability demand D equals Q
$F(Q)$ = Probability demand is Q or lower
$L(Q)$ = Expected lost sales if Q units are ordered

The Elvis wig retails for $25, but Pony Express's wholesale price is $12. Their production cost is $6. Leftover inventory can be sold to discounters for $2.50.

a. Suppose Pony Express orders 40,000 Elvis wigs. What is the chance they have to liquidate 10,000 or more wigs with a discounter?

b. What order quantity maximizes Pony Express's expected profit?

c. If Pony Express wants to have a 90 percent in-stock probability, then how many Elvis wigs should be ordered?

d. If Pony Express orders 50,000 units, then how many wigs can they expect to have to liquidate with discounters?

e. If Pony Express insists on a 100 percent in-stock probability for its customers, then what is its expected profit?

Q12.4* **(Flextrola)** Flextrola, Inc., an electronics systems integrator, is planning to design a key component for their next-generation product with Solectrics. Flextrola will integrate the component with some software and then sell it to consumers. Given the short life cycles of such products and the long lead times quoted by Solectrics, Flextrola only has one opportunity to place an order with Solectrics prior to the beginning of its selling season. Flextrola's demand during the season is normally distributed with a mean of 1,000 and a standard deviation of 600.

(* indicates that the solution is in Appendix E)

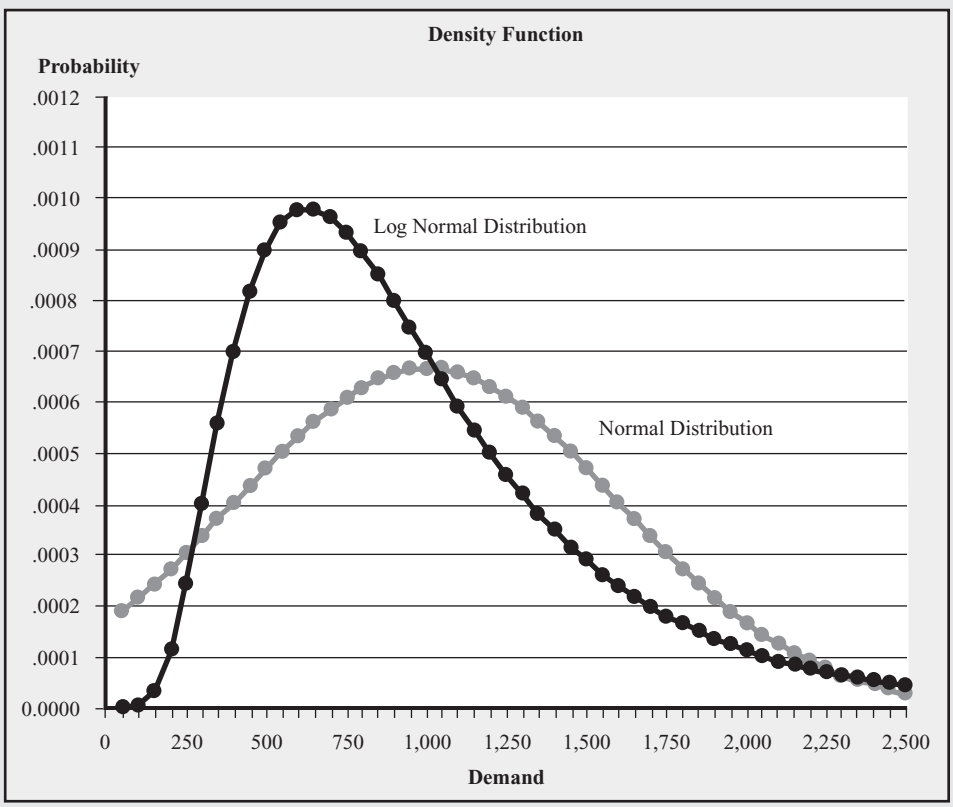
Solectrics' production cost for the component is $52 per unit and it plans to sell the component for $72 per unit to Flextrola. Flextrola incurs essentially no cost associated with the software integration and handling of each unit. Flextrola sells these units to consumers for $121 each. Flextrola can sell unsold inventory at the end of the season in a secondary electronics market for $50 each. The existing contract specifies that once Flextrola places the order, no changes are allowed to it. Also, Solectrics does not accept any returns of unsold inventory, so Flextrola must dispose of excess inventory in the secondary market.

a. What is the probability that Flextrola's demand will be within 25 percent of its forecast?
b. What is the probability that Flextrola's demand will be more than 40 percent greater than its forecast?
c. Under this contract, how many units should Flextrola order to maximize its expected profit?

For parts d through i, assume Flextrola orders 1,200 units.

d. What are Flextrola's expected sales?
e. How many units of inventory can Flextrola expect to sell in the secondary electronics market?
f. What is Flextrola's expected gross margin percentage, which is (Revenue − Cost)/Revenue?
g. What is Flextrola's expected profit?
h. What is Solectrics' expected profit?
i. What is the probability that Flextrola has lost sales of 400 units or more?
j. A sharp manager at Flextrola noticed the demand forecast and became wary of assuming that demand is normally distributed. She plotted a histogram of demands from previous seasons for similar products and concluded that demand is better represented by the log normal distribution. Figure 12.7 plots the density function for both the log normal

FIGURE 12.7
Density Function



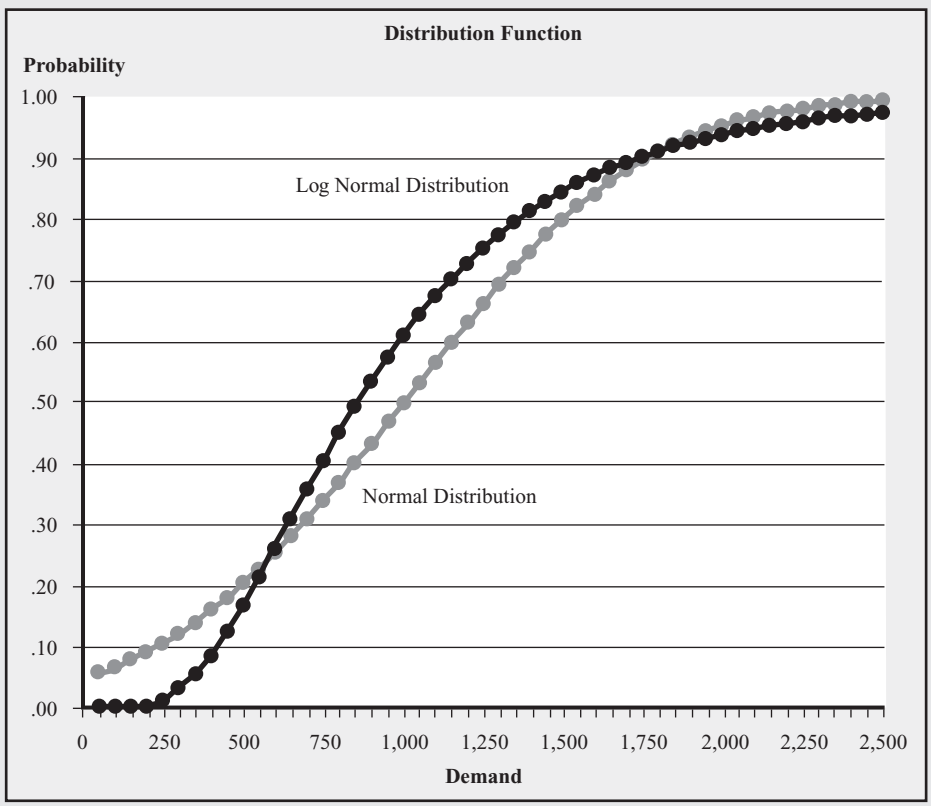
and the normal distribution, each with mean of 1,000 and standard deviation of 600. Figure 12.8 plots the distribution function for both the log normal and the normal. Using the more accurate forecast (i.e., the log normal distribution), approximately how many units should Flextrola order to maximize its expected profit?

Q12.5* **(Fashionables)** Fashionables is a franchisee of The Limited, the well-known retailer of fashionable clothing. Prior to the winter season, The Limited offers Fashionables the choice of five different colors of a particular sweater design. The sweaters are knit overseas by hand, and because of the lead times involved, Fashionables will need to order its assortment in advance of the selling season. As per the contracting terms offered by The Limited, Fashionables also will not be able to cancel, modify, or reorder sweaters during the selling season. Demand for each color during the season is normally distributed with a mean of 500 and a standard deviation of 200. Further, you may assume that the demands for each sweater are independent of those for a different color.

The Limited offers the sweaters to Fashionables at the wholesale price of $40 per sweater and Fashionables plans to sell each sweater at the retail price of $70 per unit. The Limited delivers orders placed by Fashionables in truckloads at a cost of $2,000 per truckload. The transportation cost of $2,000 is borne by Fashionables. Assume unless otherwise specified that all the sweaters ordered by Fashionables will fit into one truckload. Also assume that all other associated costs, such as unpacking and handling, are negligible.

The Limited does not accept any returns of unsold inventory. However, Fashionables can sell all of the unsold sweaters at the end of the season at the fire-sale price of $20 each.

FIGURE 12.8
Distribution Function



(* indicates that the solution is in Appendix E)

a. How many units of each sweater type should Fashionables order to maximize its expected profit?

b. If Fashionables wishes to ensure a 97.5 percent in-stock probability, what should its order quantity be for each type of sweater?

For parts c and d, assume Fashionables orders 725 of each sweater.

c. What is Fashionables' expected profit?

d. What is the stockout probability for each sweater?

e. Now suppose that The Limited announces that the unit of truckload capacity is 2,500 total units of sweaters. If Fashionables orders more than 2,500 units in total (actually, from 2,501 to 5,000 units in total), it will have to pay for two truckloads. What now is Fashionables' optimal order quantity for each sweater?

Q12.6** **(Teddy Bower Parkas)** Teddy Bower is an outdoor clothing and accessories chain that purchases a line of parkas at $10 each from its Asian supplier, TeddySports. Unfortunately, at the time of order placement, demand is still uncertain. Teddy Bower forecasts that its demand is normally distributed with mean of 2,100 and standard deviation of 1,200. Teddy Bower sells these parkas at $22 each. Unsold parkas have little salvage value; Teddy Bower simply gives them away to a charity.

a. What is the probability this parka turns out to be a "dog," defined as a product that sells less than half of the forecast?

b. How many parkas should Teddy Bower buy from TeddySports to maximize expected profit?

c. If Teddy Bower wishes to ensure a 98.5 percent in-stock probability, how many parkas should it order?

For parts d and e, assume Teddy Bower orders 3,000 parkas.

d. Evaluate Teddy Bower's expected profit.

e. Evaluate Teddy Bower's stockout probability

Q12.7 **(Teddy Bower Boots)** To ensure a full line of outdoor clothing and accessories, the marketing department at Teddy Bower insists that they also sell waterproof hunting boots. Unfortunately, neither Teddy Bower nor TeddySports has expertise in manufacturing those kinds of boots. Therefore, Teddy Bower contacted several Taiwanese suppliers to request quotes. Due to competition, Teddy Bower knows that it cannot sell these boots for more than $54. However, $40 per boot was the best quote from the suppliers. In addition, Teddy Bower anticipates excess inventory will need to be sold off at a 50 percent discount at the end of the season. Given the $54 price, Teddy Bower's demand forecast is for 400 boots, with a standard deviation of 300.

a. If Teddy Bower decides to include these boots in its assortment, how many boots should it order from its supplier?

b. Suppose Teddy Bower orders 380 boots. What would its expected profit be?

c. John Briggs, a buyer in the procurement department, overheard at lunch a discussion of the "boot problem." He suggested that Teddy Bower ask for a quantity discount from the supplier. After following up on his suggestion, the supplier responded that Teddy Bower could get a 10 percent discount if they were willing to order at least 800 boots. If the objective is to maximize expected profit, how many boots should it order given this new offer?

Q12.8 **(Land's End)** Geoff Gullo owns a small firm that manufactures "Gullo Sunglasses." He has the opportunity to sell a particular seasonal model to Land's End. Geoff offers Land's End two purchasing options:

- Option 1. Geoff offers to set his price at $65 and agrees to credit Land's End $53 for each unit Land's End returns to Geoff at the end of the season (because those units did not sell). Since styles change each year, there is essentially no value in the returned merchandise.

- Option 2. Geoff offers a price of $55 for each unit, but returns are no longer accepted. In this case, Land's End throws out unsold units at the end of the season.

This season's demand for this model will be normally distributed with mean of 200 and standard deviation of 125. Land's End will sell those sunglasses for $100 each. Geoff 's production cost is $25.

a. How much would Land's End buy if they chose option 1?

b. How much would Land's End buy if they chose option 2?

c. Which option will Land's End choose?

d. Suppose Land's End chooses option 1 and orders 275 units. What is Geoff Gullo's expected profit?

Q12.9 **(CPG Bagels)** CPG Bagels starts the day with a large production run of bagels. Throughout the morning, additional bagels are produced as needed. The last bake is completed at 3 P.M. and the store closes at 8 P.M. It costs approximately $0.20 in materials and labor to make a bagel. The price of a fresh bagel is $0.60. Bagels not sold by the end of the day are sold the next day as "day old" bagels in bags of six, for $0.99 a bag. About two-thirds of the day-old bagels are sold; the remainder are just thrown away. There are many bagel flavors, but for simplicity, concentrate just on the plain bagels. The store manager predicts that demand for plain bagels from 3 P.M. until closing is normally distributed with mean of 54 and standard deviation of 21.

a. How many bagels should the store have at 3 P.M. to maximize the store's expected profit (from sales between 3 P.M. until closing)? (*Hint:* Assume day-old bagels are sold for $0.99/6 = $0.165 each; i.e., don't worry about the fact that day-old bagels are sold in bags of six.)

b. Suppose that the store manager is concerned that stockouts might cause a loss of future business. To explore this idea, the store manager feels that it is appropriate to assign a stockout cost of $5 per bagel that is demanded but not filled. (Customers frequently purchase more than one bagel at a time. This cost is per bagel demanded that is not satisfied rather than per customer that does not receive a complete order.) Given the additional stockout cost, how many bagels should the store have at 3 P.M. to maximize the store's expected profit?

c. Suppose the store manager has 101 bagels at 3 P.M. How many bagels should the store manager expect to have at the end of the day?

Q12.10** **(The Kiosk)** Weekday lunch demand for spicy black bean burritos at the Kiosk, a local snack bar, is approximately Poisson with a mean of 22. The Kiosk charges $4.00 for each burrito, which are all made before the lunch crowd arrives. Virtually all burrito customers also buy a soda that is sold for 60¢. The burritos cost the Kiosk $2.00, while sodas cost the Kiosk 5¢. Kiosk management is very sensitive about the quality of food they serve. Thus, they maintain a strict "No Old Burrito" policy, so any burrito left at the end of the day is disposed of. The distribution function of a Poisson with mean 22 is as follows:

Q	*F(Q)*	*Q*	*F(Q)*	*Q*	*F(Q)*	*Q*	*F(Q)*
1	0.0000	11	0.0076	21	0.4716	31	0.9735
2	0.0000	12	0.0151	22	0.5564	32	0.9831
3	0.0000	13	0.0278	23	0.6374	33	0.9895
4	0.0000	14	0.0477	24	0.7117	34	0.9936
5	0.0000	15	0.0769	25	0.7771	35	0.9962
6	0.0001	16	0.1170	26	0.8324	36	0.9978
7	0.0002	17	0.1690	27	0.8775	37	0.9988
8	0.0006	18	0.2325	28	0.9129	38	0.9993
9	0.0015	19	0.3060	29	0.9398	39	0.9996
10	0.0035	20	0.3869	30	0.9595	40	0.9998

a. Suppose burrito customers buy their snack somewhere else if the Kiosk is out of stock. How many burritos should the Kiosk make for the lunch crowd?

b. Suppose that any customer unable to purchase a burrito settles for a lunch of Pop-Tarts and a soda. Pop-Tarts sell for 75¢ and cost the Kiosk 25¢. (As Pop-Tarts and soda are easily stored, the Kiosk never runs out of these essentials.) Assuming that the Kiosk management is interested in maximizing profits, how many burritos should they prepare?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 13

Assemble-to-Order, Make-to-Order, and Quick Response with Reactive Capacity[1]

A firm facing the newsvendor problem can manage, but not avoid, the possibility of a demand–supply mismatch: order too much and inventory is left over at the end of the season, but order too little and incur the opportunity cost of lost sales. The firm finds itself in this situation because it commits to its entire supply before demand occurs. This mode of operation is often called *make-to-stock* because all items enter finished goods inventory (stock) before they are demanded. In other words, with make-to-stock, the identity of an item's eventual owner is not known when production of the item is initiated.

To reduce the demand–supply mismatches associated with make-to-stock, a firm could attempt to delay at least some production until better demand information is learned. For example, a firm could choose to begin producing an item only when it receives a firm order from a customer. This mode of operation is often called *make-to-order* or *assemble-to-order*. Dell Computer is probably the most well-known and most successful company to have implemented the assemble-to-order model.

Make-to-stock and make-to-order are two extremes in the sense that with one all production begins well before demand is received, whereas with the other production begins only after demand is known. Between any two extremes there also must be an intermediate option. Suppose the lead time to receive an order is short relative to the length of the selling season. A firm then orders some inventory before the selling season starts so that some product is on hand at the beginning of the season. After observing early season sales, the firm then submits a second order that is received well before the end of the season (due to the short lead time). In this situation, the firm should make a conservative initial order and use the second order to strategically respond to initial season sales: Slow-selling products are not replenished midseason, thereby reducing leftover inventory, while fast-selling products are replenished, thereby reducing lost sales.

The capability to place multiple orders during a selling season is an integral part of *Quick Response*. Quick Response is a set of practices designed to reduce the cost of mismatches

[1] The data in this chapter have been modified to protect confidentiality.

between supply and demand. It began in the apparel industry as a response to just-in-time practices in the automobile industry and has since migrated to the grocery industry under the label *Efficient Consumer Response.*

The aspect of Quick Response discussed in this chapter is the use of *reactive capacity,* that is, capacity that allows a firm to place one additional order during the season, which retailers often refer to as a "second buy." As in Chapter 12, we use O'Neill Inc. for our case analysis. Furthermore, we assume throughout this chapter that the normal distribution with mean 3,192 and standard deviation 1,181 is our demand forecast for the Hammer 3/2.

The first part of this chapter evaluates and minimizes the demand–supply mismatch cost to a make-to-stock firm, that is, a firm that has only a single ordering opportunity, as in the newsvendor model. Furthermore, we identify situations in which the cost of demand–supply mismatches is large. Those are the situations in which there is the greatest potential to benefit from Quick Response with reactive capacity or make-to-order production. The second part of this chapter discusses make-to-order relative to make-to-stock. The third part studies reactive capacity: How should we choose an initial order quantity when some reactive capacity is available? And, as with the newsvendor model, how do we evaluate several performance measures? The chapter concludes with a summary and managerial implications.

13.1 Evaluating and Minimizing the Newsvendor's Demand–Supply Mismatch Cost

In this section, the costs associated in the newsvendor model with demand–supply mismatches are identified, then two approaches are outlined for evaluating the expected demand–supply mismatch cost, and finally we show how to minimize those costs. For ease of exposition, we use the shorthand term *mismatch cost* to refer to the "expected demand–supply mismatch cost."

In the newsvendor model, the mismatch cost is divided into two components: the cost of ordering too much and the cost of ordering too little. Ordering too much means there is leftover inventory at the end of the season. Ordering too little means there are lost sales. The cost for each unit of leftover inventory is the overage cost, which we label C_o. The cost for each lost sale is the underage cost, which we label C_u. (See Chapter 12 for the original discussion of these costs.) Therefore, the mismatch cost in the newsvendor model is the sum of the expected overage cost and the expected underage cost:

$$\begin{aligned}\text{Mismatch cost} &= (C_o \times \text{Expected leftover inventory}) \\ &\quad + (C_u \times \text{Expected lost sales})\end{aligned} \tag{13.1}$$

Notice that the mismatch cost includes both a tangible cost (leftover inventory) and an intangible opportunity cost (lost sales). The former has a direct impact on the profit and loss statement, but the latter does not. Nevertheless, the opportunity cost of lost sales should not be ignored.

Not only does equation (13.1) provide us with the definition of the mismatch cost, it also provides us with our first method for evaluating the mismatch cost because we already know how to evaluate the expected leftover inventory and the expected lost sales (from Chapter 12). Let's illustrate this method with O'Neill's Hammer 3/2 wetsuit. The Hammer has a selling price of \$190 and a purchase cost from the TEC Group of \$110. Therefore, the underage cost is \$190 − \$110 = \$80 per lost sale. Leftover inventory is sold at \$90, so the overage cost is \$110 − \$90 = \$20 per wetsuit left at the end of the season. The expected profit-maximizing order quantity is 4,196 units. Using the techniques

TABLE 13.1
Summary of Performance Measures for O'Neill's Hammer 3/2 Wetsuit When the Expected Profit-Maximizing Quantity Is Ordered and the Demand Forecast Is Normally Distributed with Mean 3,192 and Standard Deviation 1,181

Order quantity, Q	= 4,196 units
Expected demand, μ	= 3,192 units
Standard deviation of demand, σ	= 1,181
Expected lost sales	= 130 units
Expected sales	= 3,062 units
Expected leftover inventory	= 1,134 units
Expected revenue	= $683,840
Expected profit	= $222,280

Expected lost sales = 1,181 × L (0.85) = 1,181 × 0.11 = 130
Expected sales = 3,192 − 130 = 3,062
Expected leftover inventory = 4,196 − 3,062 = 1,134
Expected revenue = Price × Expected sales + Salvage value × Expected leftover inventory
= $190 × 3,062 + $90 × 1,134 = $683,840
Expected profit = ($190 − $110) × 3,062 − ($110 − $90) × 1,134 = $222,280

described in Chapter 12, for that order quantity we can evaluate several performance measures, summarized in Table 13.1. Therefore, the mismatch cost for the Hammer 3/2, despite ordering the expected profit-maximizing quantity, is

$$(\$20 \times 1,134) + (\$80 \times 130) = \$33,080$$

Now let's consider a second approach for evaluating the mismatch cost. Imagine O'Neill had the opportunity to purchase a magic crystal ball. Even before O'Neill needs to submit its order to TEC, this crystal ball reveals to O'Neill the exact demand for the entire season. O'Neill would obviously order from TEC the demand quantity observed with this crystal ball. As a result, O'Neill would be in the pleasant situation of avoiding all mismatch costs (there would be no excess inventory and no lost sales) while still providing immediate product availability to its customers. In fact, the only function of the crystal ball is to eliminate all mismatch costs: for example, the crystal ball does not change demand, increase the selling price, or decrease the production cost. Thus, the difference in O'Neill's expected profit with the crystal ball and without it must equal the mismatch cost: The crystal ball increases profit by eliminating mismatch costs, so the profit increase must equal the mismatch cost. Therefore, we can evaluate the mismatch cost by first evaluating the newsvendor's expected profit, then evaluating the expected profit with the crystal ball, and finally taking the difference between those two figures.

We already know how to evaluate the newsvendor's expected profit (again, see Chapter 12). So let's illustrate how to evaluate the expected profit with the crystal ball. If O'Neill gets to observe demand before deciding how much to order from TEC, then there will not be any leftover inventory at the end of the season. Even better, O'Neill will not stock out, so every unit of demand turns into an actual sale. Hence, O'Neill's expected sales with the crystal ball equal expected demand, which is μ. We already know that O'Neill's profit per sale is the gross margin, the retail price minus the production cost, Price − Cost. Therefore O'Neill's expected profit with this crystal ball is expected demand times the profit per unit of demand, which is (Price − Cost) × μ. In fact, O'Neill can never earn a higher expected profit than it does with the crystal ball: There is nothing better than having no leftover inventory and earning the full margin on every unit of potential demand. Hence, let's call that profit the *maximum profit:*

$$\text{Maximum profit} = (\text{Price} - \text{Cost}) \times \mu$$

O'Neill's maximum profit with the Hammer 3/2 is $80 × 3,192 = $255,360. We already know that the newsvendor expected profit is $222,280. So the difference between the

maximum profit (i.e., crystal ball profit) and the newsvendor expected profit is O'Neill's mismatch costs. That figure is \$255,360 − \$222,280 = \$33,080, which matches our calculation with our first method (as it should). To summarize, our second method for evaluating the mismatch cost uses the following equation:

$$\text{Mismatch cost} = \text{Maximum profit} - \text{Expected profit}$$

Incidentally, you can also think of the mismatch cost as the most O'Neill should be willing to pay to purchase the crystal ball; that is, it is the value of perfect demand information.

The second method for calculating the mismatch cost emphasizes that there exists an easily evaluated maximum profit. We might not be able to evaluate expected profit precisely if there is some reactive capacity available to the firm. Nevertheless, we do know that no matter what type of reactive capacity the firm has, that reactive capacity cannot be as good as the crystal ball we just described. Therefore, the expected profit with any form of reactive capacity must be more than the newsvendor's expected profit but less than the maximum profit.

You now may be wondering about how to minimize the mismatch cost and whether that is any different than maximizing the newsvendor's expected profit. The short answer is that these are effectively the same objective, that is, the quantity that maximizes profit also minimizes mismatch costs. One way to see this is to look at the equation above: If expected profit is maximized and the maximum profit does not depend on the order quantity, then the difference between them, which is the mismatch cost, must be minimized.

Now that we know how to evaluate and minimize the mismatch cost, we need to get a sense of its significance. In other words, is \$33,080 a big problem or a little problem? To answer that question, we need to compare it with something else. The maximum profit is one reference point: the demand–supply mismatch cost as a percentage of the maximum profit is \$33,080/\$255,360 = 13 percent. You may prefer expected sales as a point of comparison: the demand–supply mismatch cost per unit of expected sales is \$33,080/3,062 = \$10.8. Alternatively, we can make the comparison with expected revenue, \$683,840, or expected profit, \$222,280: the demand–supply mismatch cost is approximately 4.8 percent of total revenue (\$33,080/\$683,840) and 14.9 percent of expected profit (\$33,080/\$222,280). Companies in the sports apparel industry generally have net profit in the range of 2 to 5 percent of revenue. Therefore, eliminating the mismatch cost from the Hammer 3/2 could potentially double O'Neill's net profit! That is an intriguing possibility.

13.2 When Is the Mismatch Cost High?

No matter which comparison you prefer, the mismatch cost for O'Neill is significant, even if the expected profit-maximizing quantity is ordered. But it is even better to know what causes a large demand–supply mismatch. To answer that question, let's first choose our point of comparison for the mismatch cost. Of the ones discussed at the end of the previous section, only the maximum profit does not depend on the order quantity chosen: unit sales, revenue, and profit all clearly depend on Q. In addition, the maximum profit is representative of the potential for the product: we cannot do better than earn the maximum profit. Therefore, let's evaluate the mismatch cost as a percentage of the maximum profit.

We next need to make an assumption about how much is ordered before the selling season, that is, clearly the mismatch cost depends on the order quantity Q. Let's adopt

the natural assumption that the expected profit-maximizing quantity is ordered, which, as we discussed in the previous section, also happens to minimize the newsvendor's mismatch cost.

If we take the equations for expected lost sales and expected leftover inventory from Chapter 12, plug them into our first mismatch cost equation (13.1), and then do several algebraic manipulations, we arrive at the following observations:

- The expected demand–supply mismatch cost becomes larger as demand variability increases, where demand variability is measured with the coefficient of variation, σ/μ.
- The expected demand–supply mismatch cost becomes larger as the critical ratio, $C_u/(C_o + C_u)$, becomes smaller.

(If you want to see the actual equations and how they are derived, see Appendix D.)

It is intuitive that the mismatch cost should increase as demand variability increases—it is simply harder to get demand to match supply when demand is less predicable. The key insight is how to measure demand variability. The *coefficient of variation* is the correct measure. You may recall in Chapter 8 we discussed the coefficient of variation with respect to the variability of the processing time (CV_p) or the interarrival time to a queue (CV_a). This coefficient of variation, σ/μ, is conceptually identical to those coefficients of variation: it is the ratio of the standard deviation of a random variable (in this case demand) to its mean.

It is worthwhile to illustrate why the coefficient of variation is the appropriate measure of variability in this setting. Suppose you are informed that the standard deviation of demand for an item is 800. Does that tell you enough information to assess the variability of demand? For example, does it allow you to evaluate the probability actual demand will be less than 75 percent of your forecast? In fact, it does not. Consider two cases, in the first the forecast is for 1,000 units and in the second the forecast is for 10,000 units. Demand is less than 75 percent of the 1,000-unit forecast if demand is less than 750 units. What is the probability that occurs? First, normalize the value 750:

$$Z = \frac{Q - \mu}{\sigma} = \frac{750 - 1{,}000}{800} = -0.31$$

Now use the Standard Normal Distribution Function Table to find the probability demand is less than 750: $\Phi(-0.31) = 0.3783$. With the forecast of 10,000, the comparable event has demand that is less than 7,500 units. Repeating the same process yields $z = (7{,}500 - 10{,}000)/800 = -3.1$ and $\Phi(-3.1) = 0.0009$. Therefore, with a standard deviation of 800, there is about a 38 percent chance demand is less than 75 percent of the first forecast but much less than a 1 percent chance demand is less than 75 percent of the second forecast. In other words, the standard deviation alone does not capture how much variability there is in demand. Notice that the coefficient of variation with the first product is 0.8 (800/1,000), whereas it is much lower with the second product, 0.08 (800/10,000).

For the Hammer 3/2, the coefficient of variation is 1,181/3,192 = 0.37. While there is no generally accepted standard for what is a "low," "medium," or "high" coefficient of variation, we offer the following guideline: Demand variability is rather low if the coefficient of variation is less than 0.25, medium if it is in the range 0.25 to 0.75, and high with anything above 0.75. A coefficient of variation above 1.5 is extremely high, and anything above 3 would imply that the demand forecast is essentially meaningless.

Table 13.2 provides data to allow you to judge for yourself what is a "low," "medium," and "high" coefficient of variation.

TABLE 13.2
Forecast Accuracy Relative to the Coefficient of Variation When Demand Is Normally Distributed

Coefficient of Variation	Probability Demand Is Less Than 75% of the Forecast	Probability Demand Is within 25% of the Forecast
0.10	0.6%	98.8%
0.25	15.9	68.3
0.50	30.9	38.3
0.75	36.9	26.1
1.00	40.1	19.7
1.50	43.4	13.2
2.00	45.0	9.9
3.00	46.7	6.6

Recall from Chapters 8 and 9 that the coefficient of variation with an exponential distribution is always one. Therefore, if two processes have exponential distributions, they always have the same amount of variability. The same is not true with the normal distribution because with the normal distribution the standard deviation is adjustable relative to the mean.

Our second observation above relates mismatch costs to the critical ratio. In particular, products with low critical ratios and high demand variability have high mismatch costs and products with high critical ratios and low demand variability have low mismatch costs. Table 13.3 displays data on the mismatch cost for various coefficients of variation and critical ratios.

As we have already mentioned, it is intuitive that the mismatch cost should increase as demand variability increases. The intuition with respect to the critical ratio takes some more thought. A very high critical ratio means there is a large profit margin relative to the loss on each unit of excess inventory. Greeting cards are good examples of products that might have very large critical ratios: the gross margin on each greeting card is large while the production cost is low. With a very large critical ratio, the optimal order quantity is quite large, so there are very few lost sales. There is also a substantial amount of leftover inventory, but the cost of each unit left over in inventory is not large at all, so the total cost of leftover inventory is relatively small. Therefore, the total mismatch cost is small. Now consider a product with a low critical ratio, that is, the per-unit cost of excess inventory is much higher than the cost of each lost sale. Perishable items often fall into this category as well as items that face obsolescence. Given that excess inventory is expensive, the optimal order quantity is quite low, possibly lower than expected demand. As a result, excess inventory is not a problem, but lost sales are a big problem, resulting in a high mismatch cost.

TABLE 13.3
The Mismatch Cost (as a Percentage of the Maximum Profit) When Demand Is Normally Distributed and the Newsvendor Expected Profit-Maximizing Quantity Is Ordered

	Critical Ratio					
Coefficient of Variation	0.4	0.5	0.6	0.7	0.8	0.9
0.10	10%	8%	6%	5%	3%	2%
0.25	24%	20%	16%	12%	9%	5%
0.40	39%	32%	26%	20%	14%	8%
0.55	53%	44%	35%	27%	19%	11%
0.70	68%	56%	45%	35%	24%	14%
0.85	82%	68%	55%	42%	30%	17%
1.00	97%	80%	64%	50%	35%	19%

13.3 Reducing Mismatch Costs with Make-to-Order

When supply is chosen before demand is observed (make-to-stock), there invariably is either too much or too little supply. A purely hypothetical solution to the problem is to find a crystal ball that reveals demand before it occurs. A more realistic solution is to initiate production of each unit only after demand is observed for that unit, which is often called make-to-order or assemble-to-order. This section discusses the pros and cons of make-to-order with respect to its ability to reduce mismatch costs.

In theory, make-to-order can eliminate the entire mismatch cost associated with make-to-stock (i.e., newsvendor). With make-to-order, there is no leftover inventory because production only begins after a firm order is received from a customer. Thus, make-to-order saves on expensive markdown and disposal expenses. Furthermore, there are no lost sales with make-to-order because each customer order is eventually produced. Therefore, products with a high mismatch cost (low critical ratios, high demand variability) would benefit considerably from a switch to make-to-order from make-to-stock.

But there are several reasons to be wary of make-to-order. For one, even with make-to-order, there generally is a need to carry component inventory. Although components may be less risky than finished goods, there still is a chance of having too many or too few of them. Next, make-to-order is never able to satisfy customer demands immediately; that is, customers must wait to have their order filled. If the wait is short, then demand with make-to-order can be nearly as high as with make-to-stock. But there is also some threshold beyond which customers do not wait. That threshold level depends on the product: customers are generally less willing to wait for diapers than they are for custom sofas.

It is helpful to think of queuing theory (Chapters 8 and 9) to understand what determines the waiting time with make-to-order. No matter the number of servers, a key characteristic of a queuing system is that customer service begins only after a customer arrives to the system, just as production does not begin with make-to-order until a customer commits to an order. Another important feature of a queuing system is that customers must wait to be processed if all servers are busy, just as a customer must wait with make-to-order if the production process is working on the backlog of orders from previous customers.

To provide a reference point for this discussion, suppose O'Neill establishes a make-to-order assembly line for wetsuits. O'Neill could keep in inventory the necessary raw materials to fabricate wetsuits in a wide array of colors, styles, and quality levels. Wetsuits would then be produced as orders are received from customers. The assembly line has a maximum production rate, which would correspond to the service rate in a queue. Given that demand is random, the interarrival times between customer orders also would be random, just as in a queuing system.

A key insight from queuing is that a customer's expected waiting time depends nonlinearly (a curve, not a straight line) on the system's utilization (the ratio of the flow rate to capacity): As the utilization approaches 100 percent, the waiting time approaches infinity. (See Figure 8.21.) As a result, if O'Neill wishes to have a reasonably short waiting time for customers, then O'Neill must be willing to operate with less than 100 percent utilization, maybe even considerably less than 100 percent. Less than 100 percent utilization implies idle capacity; for example, if the utilization is 90 percent, then 10 percent of the time the assembly line is idle. Therefore, even with make-to-order production, O'Neill experiences demand–supply mismatch costs. Those costs are divided into two types: idle capacity and lost sales from customers who are unwilling to wait to receive their product. When comparing make-to-stock with make-to-order, you could say that make-to-order replaces the cost of leftover inventory with the cost of idle capacity. Whether or not make-to-order is preferable depends on the relative importance of those two costs.

While a customer's expected waiting time may be significant, customers are ultimately concerned with their total waiting time, which includes the processing time. With make-to-order, the processing time has two components: the time in production and the time from production to actual delivery. Hence, successful implementation of make-to-order generally requires fast and easy assembly of the final product. Next, keeping the delivery time to an acceptable level either requires paying for fast shipping (e.g., air shipments) or moving production close to customers (to reduce the distance the product needs to travel). Fast shipping increases the cost of *every* unit produced, and local production (e.g., North America instead of Asia) may increase labor costs. See Chapter 19 for more discussion.

Although make-to-order is not ideal for all products, Dell discovered that make-to-order is particularly well suited for personal computers for several reasons: Inventory is very expensive to hold because of obsolescence and falling component prices; labor is a small portion of the cost of a PC, in part because the modular design of PCs allows for fast and easy assembly; customers are primarily concerned with price and customization and less concerned with how long they must wait for delivery (i.e., they are patient) and unique design features (i.e., it is hard to differentiate one PC from another with respect to design); there is a large pool of educated customers who are willing to purchase without physically seeing the product (i.e., the phone/Internet channels work); and the cost to transport a PC is reasonable (relative to its total value). The same logic suggests that make-to-order is more challenging in the automobile industry. For example, assembling a vehicle is challenging, customization is less important to consumers, consumers do not like to wait to receive their new vehicle (at least in the United States), and moving vehicles around is costly (relative to their value). Indeed, Toyota once announced that it planned to produce a custom-ordered vehicle in only five days (Simison 1999). However, the company quietly backed away from the project.

As already mentioned, make-to-order is not ideal for all products. Koss Corp., a headphone maker, is an example of a company that discovered that make-to-order is not always a magic bullet (Ramstad 1999). The company experimented with make-to-order and discovered it was unable to provide timely deliveries to its customers (retailers) during its peak season. In other words, demand was variable, but Koss's capacity was not sufficiently flexible. Because it began to lose business due to its slow response time, Koss switched back to make-to-stock so that it would build up inventory before its peak demand period. For Koss, holding inventory was cheaper than losing sales to impatient customers. To summarize, make-to-order eliminates some of the demand–supply mismatches associated with make-to-stock, but make-to-order has its own demand–supply mismatch issues. For example, make-to-order eliminates leftover inventory but it still carries component inventory. More importantly, to ensure acceptable customer waiting times, make-to-order requires some idle capacity, thereby potentially increasing labor and delivery costs.

13.4 Quick Response with Reactive Capacity

O'Neill may very well conclude that make-to-order production is not viable either in Asia (due to added shipping expenses) or in North America (due to added labor costs). If pure make-to-order is out of the question, then O'Neill should consider some intermediate solution between make-to-stock (the newsvendor) and make-to-order (a queue). With the newsvendor model, O'Neill commits to its entire supply before *any* demand occurs; whereas with make-to-order, O'Neill commits to supply only after *all* demand occurs. The intermediate solution is to commit to some supply before demand but then maintain the option to produce

additional supply after some demand is observed. The capacity associated with that later supply is called *reactive capacity* because it allows O'Neill to react to the demand information it learns before committing to the second order. The ability to make multiple replenishments (even if just one replenishment) is a central goal in Quick Response.

Suppose O'Neill approaches TEC with the request that TEC reduce its lead time. O'Neill's motivation behind this request is to try to create the opportunity for a replenishment during the selling season. Recall that the Spring season spans six months, starting in February and ending in July. (See Figure 12.2.) It has been O'Neill's experience that a hot product in the first two months of the season (i.e., a product selling above forecast) almost always turns out to be a hot product in the rest of the season. As a result, O'Neill could surely benefit from the opportunity to replenish the hot products midseason. For example, suppose TEC offered a one-month lead time for a midseason order. Then O'Neill could submit to TEC a second order at the end of the second month (March) and receive that replenishment before the end of the third month, thereby allowing that inventory to serve demand in the second half of the season. Figure 13.1 provides a time line in this new situation.

While it is clear that O'Neill could benefit from the second order, offering a second order with a one-month lead time can be costly to TEC. For example, TEC might need to reserve some capacity to respond to O'Neill's order. If O'Neill's second order is not as large as TEC anticipated, then some of that reserved capacity might be lost. Or O'Neill's order might be larger than anticipated, forcing TEC to scramble for extra capacity, at TEC's expense. In addition, the one-month lead time may force the use of faster shipping, which again could increase costs. The issue is whether the cost increases associated with the second order justify the mismatch cost savings for O'Neill. To address this issue, let's suppose that TEC agrees to satisfy O'Neill's second order but insists on a 20 percent premium for those units to cover TEC's anticipated additional expenses. Given this new opportunity, how should O'Neill adjust its initial order quantity and how much are mismatch costs reduced?

Choosing order quantities with two ordering opportunities is significantly more complex than choosing a single order quantity (i.e., the newsvendor problem). For instance, in addition to our forecast for the entire season's demand, now we need to worry about developing a forecast for demand in the second half of the season given what we observe in the first two months of the season. Furthermore, we do not know what will be our initial sales when we submit our first order, so that order must anticipate all possible outcomes for initial sales and then the appropriate response in the second order for all of those outcomes. In addition, we may stock out within the first half of the season if our first order is not large enough.

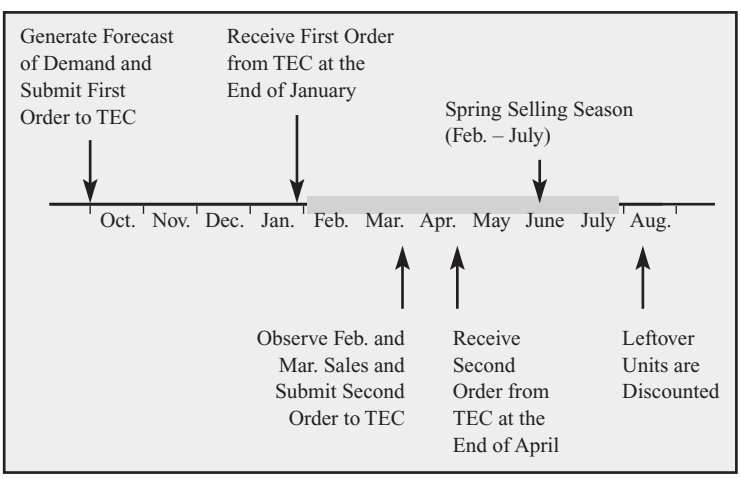


FIGURE 13.1
Time Line of Events for O'Neill's Hammer 3/2 Wetsuit with Unlimited, but Expensive, Reactive Capacity

Finally, even after observing initial sales, some uncertainty remains regarding demand in the second half of the season.

Even though we now face a complex problem, we should not let the complexity overwhelm us. A good strategy when faced with a complex problem is to make it less complex, that is, make some simplifying assumptions that allow for analytical tractability while retaining the key qualitative features of the complex problem. With that strategy in mind, let's assume (1) we do not run out of inventory before the second order arrives and (2) after we observe initial sales we are able to perfectly predict sales in the remaining portion of the season. Assumption 1 is not bad as long as the first order is reasonably large, that is, large enough to cover demand in the first half of the season with a high probability. Assumption 2 is not bad if initial sales are a very good predictor of subsequent sales, which has been empirically observed in many industries.

Our simplifying assumptions are enough to allow us to evaluate the optimal initial order quantity and then to evaluate expected profit. Let's again consider O'Neill's initial order for the Hammer 3/2. It turns out that O'Neill still faces the "too much–too little" problem associated with the newsvendor problem even though O'Neill has the opportunity to make a second order. To explain, note that if the initial order quantity is too large, then there will be leftover inventory at the end of the season. The second order does not help at all with the risk of excess inventory, so the "too much" problem remains.

We also still face the "too little" issue with our initial order, but it takes a different form than in our original newsvendor problem. Recall, with the original newsvendor problem, ordering too little leads to lost sales. But the second order prevents lost sales: After we observe initial sales, we are able to predict total demand for the remainder of the season. If that total demand exceeds our initial order, we merely choose a second order quantity to ensure that all demand is satisfied. This works because of our simplifying assumptions: Lost sales do not occur before the second order arrives, there is no quantity limit on the second order, and initial sales allow us to predict total demand for the season.

Although the second order opportunity eliminates lost sales, it does not mean we should not bother with an initial order. Remember that units ordered during the season are more expensive than units ordered before the season. Therefore, the penalty for ordering too little in the first order is that we may be required to purchase additional units in the second order at a higher cost.

Given that the initial order still faces the "too little–too much" problem, we can actually use the newsvendor model to find the order quantity that maximizes expected profit. The overage cost, C_o, per unit of excess inventory is the same as in the original model; that is, the overage cost is the loss on each unit of excess inventory. Recall that for the Hammer 3/2 Cost = 110 and Salvage value = 90. So $C_o = 20$.

The underage cost, C_u, per unit of demand that exceeds our initial order quantity is the additional premium we must pay to TEC for units in the second order. That premium is 20 percent, which is $20\% \times 110 = 22$. In other words, if demand exceeds our initial order quantity, then the penalty for ordering too little is the extra amount we must pay TEC for each of those units (i.e., we could have avoided that premium by increasing the initial order). Even though we must pay this premium to TEC, we are still better off having the second ordering opportunity: Paying TEC an extra $22 for each unit of demand that exceeds our initial order quantity is better than losing the $80 margin on each of those units if we did not have the second order. So $C_u = 22$.

We are now ready to calculate our optimal initial order quantity. (See Exhibit 12.3 for an outline of this process.) First, evaluate the critical ratio:

$$\frac{C_u}{C_o + C_u} = \frac{22}{20 + 22} = 0.5238$$

Next find the z value in the Standard Normal Distribution Function Table that corresponds to the critical ratio 0.5238: $\Phi(0.05) = 0.5199$ and $\Phi(0.06) = 0.5239$, so let's choose the higher z value, $z = 0.06$. Now convert the z value into an order quantity for the actual demand distribution with $\mu = 3{,}192$ and $\sigma = 1{,}181$:

$$Q = \mu + z \times \sigma = 3{,}192 + 0.06 \times 1{,}181 = 3{,}263$$

Therefore, O'Neill should order 3,263 Hammer 3/2s in the first order to maximize expected profit when a second order is possible. Notice that O'Neill should still order a considerable amount in its initial order so as to avoid paying TEC the 20 percent premium on too many units. However, O'Neill's initial order of 3,263 units is considerably less than its optimal order of 4,196 units when the second order is not possible.

Even though O'Neill must pay a premium with the second order, O'Neill's expected profit should increase by this opportunity. (The second order does not prevent O'Neill from ordering 4,196 units in the initial order, so O'Neill cannot be worse off.) Let's evaluate what that expected profit is for any initial order quantity Q. Our maximum profit has not changed. The best we can do is earn the maximum gross margin on every unit of demand,

$$\text{Maximum profit} = (\text{Price} - \text{Cost}) \times \mu = (190 - 110) \times 3{,}192 = 255{,}360$$

The expected profit is the maximum profit minus the mismatch costs:

$$\begin{aligned}\text{Expected profit} = {} & \text{Maximum profit} - (C_o \times \text{Expected leftover inventory}) \\ & -(C_u \times \text{Expected second order quantity})\end{aligned}$$

The first mismatch cost is the cost of leftover inventory and the second is the additional premium that O'Neill must pay TEC for all of the units ordered in the second order. We already know how to evaluate expected leftover inventory for any initial order quantity. (See Exhibit 12.5 for a summary.) We now need to figure out the expected second order quantity.

If we order Q units in the first order, then we make a second order only if demand exceeds Q. In fact, our second order equals the difference between demand and Q, which would have been our lost sales if we did not have a second order. This is also known as the loss function. Therefore,

$$\text{Expected second order quantity} = \text{Newsvendor's expected lost sales}$$

We already know how to evaluate the newsvendor's expected lost sales. (See Exhibit 12.4 for a summary.) First look up $L(z)$ in the Standard Normal Loss Function Table for the z value that corresponds to our order quantity, $z = 0.06$. We find in that table $L(0.06) = 0.3697$. Next, finish the calculation:

$$\text{Expected lost sales} = \sigma \times L(z) = 1{,}181 \times 0.3697 = 437$$

Recall that

$$\text{Expected sales} = \mu - \text{Expected lost sales} = 3{,}192 - 437 = 2{,}755$$

where expected sales is the quantity the newsvendor would sell with an order quantity of 3,263. We want to evaluate expected sales for the newsvendor so that we can evaluate the last piece we need:

$$\text{Expected leftover inventory} = Q - \text{Expected sales} = 3{,}263 - 2{,}755 = 508$$

We are now ready to evaluate expected profit for the Hammer 3/2 if there is a second order:

$$\begin{aligned}\text{Expected profit} &= \text{Maximum profit} - (C_o \times \text{Expected leftover inventory}) \\ &\quad -(C_u \times \text{Expected second order quantity}) \\ &= \$255{,}360 - (\$20 \times 508) - (\$22 \times 437) \\ &= \$235{,}586\end{aligned}$$

Recall that O'Neill's expected profit with just one ordering opportunity is $222,280. Therefore, the second order increases profit by ($235,586 − $222,280)/$222,280 = 6.0 percent even though TEC charges a 20 percent premium for units in the second order. We also can think in terms of how much the second order reduces the mismatch cost. Recall that the mismatch cost with only one order is $33,080. Now the mismatch cost is $255,360 − $235,586 = $19,744, which is a 40 percent reduction in the mismatch cost (1 − $19,774/$33,080). In addition, O'Neill's in-stock probability increases from about 80 percent to essentially 100 percent and the number of leftover units at the end of the season that require markdowns to sell is cut in half (from 1,134 to 508). Therefore, even though reactive capacity in the form of a midseason replenishment does not eliminate all mismatch costs, it provides a feasible strategy for significantly reducing mismatch costs.

13.5 Summary

With the newsvendor's make-to-stock system, the firm commits to its entire supply before any updated demand information is learned. As a result, there are demand–supply mismatch costs that manifest themselves in the form of leftover inventory or lost sales. This chapter identifies situations in which the mismatch cost is high and considers several improvements to the newsvendor situation to reduce those mismatch costs.

Mismatch costs are high (as a percentage of a product's maximum profit) when a product has a low critical ratio and/or a high coefficient of variation. A low critical ratio implies that the cost of leftover inventory is high relative to the cost of a lost sale. Perishable products or products that face obsolescence generally have low critical ratios. The coefficient of variation is the ratio of the standard deviation of demand to expected demand. It is high for products that are hard to forecast. Examples include new products, fashionable products, and specialty products with small markets. The important lesson here is that actions that lower the critical ratio or increase the coefficient of variation also increase demand–supply mismatch costs.

Make-to-order is an extreme solution to the newsvendor situation. With make-to-order, the firm begins producing an item only after the firm has an order from a customer. In other words, production begins only when the ultimate owner of an item becomes known. A key advantage with make-to-order is that leftover inventory is eliminated. However, a make-to-order system is not immune to the problems of demand–supply mismatches because it behaves like a queuing system. As a result, customers must wait to be satisfied and the length of their waiting time is sensitive to the amount of idle capacity.

The intermediate solution between make-to-order and make-to-stock has the firm commit to some production before any demand information is learned, but the firm also has the capability to react to early demand information via a second order, which is called reactive capacity. Reactive capacity can substantially reduce (but not eliminate) the newsvendor's mismatch cost. Still, this approach may be attractive because it does not suffer from all of the challenges faced by make-to-order.

Table 13.4 provides a summary of the key notation and equations presented in this chapter.

TABLE 13.4
A Summary of the Key Notation and Equations in Chapter 13

Q = Order quantity	
C_u = Underage cost	C_o = Overage cost
μ = Expected demand	σ = Standard deviation of demand

Mismatch cost = (C_o × Expected leftover inventory) + (C_u × Expected lost sales)
= Maximum profit − Expected profit

Maximum profit = (Price − Cost) × μ

Coefficient of variation = Standard deviation/Expected demand

13.6 Further Reading

More responsive, more flexible, more reactive operations have been the goal over the last 20 years in most industries, in large part due to the success of Dell Inc. in the personal computer business. For an insightful review of Dell's strategy, see Magretta (1998). See McWilliams and White (1999) for an interview with Michael Dell on his views on how the auto industry should change with respect to its sales and production strategy.

For a comprehensive treatment of Quick Response in the apparel industry, see Abernathy, Dunlop, Hammond, and Weil (1999), Vitzthum (1998) describes how Zara, a Spanish fashion retailer, is able to produce "fashion on demand."

Fisher (1997) discusses the pros and cons of flexible supply chains and Zipkin (2001) does the same for mass customization. Karmarkar (1989) discusses the pros and cons of push versus pull production systems.

See Fisher and Raman (1996) or Fisher, Rajaram, and Raman (2001) for technical algorithms to optimize order quantities when early sales information and reactive capacity are available.

13.7 Practice Problems

Q13.1* **(Teddy Bower)** Teddy Bower sources a parka from an Asian supplier for $10 each and sells them to customers for $22 each. Leftover parkas at the end of the season have no salvage value. (Recall Q12.6.) The demand forecast is normally distributed with mean 2,100 and standard deviation 1,200. Now suppose Teddy Bower found a reliable vendor in the United States that can produce parkas very quickly but at a higher price than Teddy Bower's Asian supplier. Hence, in addition to parkas from Asia, Teddy Bower can buy an unlimited quantity of additional parkas from this American vendor at $15 each after demand is known.

a. Suppose Teddy Bower orders 1,500 parkas from the Asian supplier. What is the probability that Teddy Bower will order from the American supplier once demand is known?
b. Again assume that Teddy Bower orders 1,500 parkas from the Asian supplier. What is the American supplier's expected demand; that is, how many parkas should the American supplier expect that Teddy Bower will order?
c. Given the opportunity to order from the American supplier at $15 per parka, what order quantity from its Asian supplier now maximizes Teddy Bower's expected profit?
d. Given the order quantity evaluated in part c, what is Teddy Bower's expected profit?
e. If Teddy Bower didn't order any parkas from the Asian supplier, then what would Teddy Bower's expected profit be?

Q13.2* **(Flextrola)** Flextrola, Inc., an electronics system integrator, is developing a new product. As mentioned in Q11.4, Solectrics can produce a key component for this product. Solectrics sells this component to Flextrola for $72 per unit and Flextrola must submit its order well in advance of the selling season. Flextrola's demand forecast is a normal distribution with mean of 1,000 and standard deviation of 600. Flextrola sells each unit, after integrating some software, for $131. Leftover units at the end of the season are sold for $50.

(* indicates that the solution is at the end of the book)

Xandova Electronics (XE for short) approached Flextrola with the possibility of also supplying Flextrola with this component. XE's main value proposition is that they offer 100 percent in-stock and one-day delivery on all of Flextrola's orders, no matter when the orders are submitted. Flextrola promises its customers a one-week lead time, so the one-day lead time from XE would allow Flextrola to operate with make-to-order production. (The software integration that Flextrola performs can be done within one day.) XE's price is $83.50 per unit.

a. Suppose Flextrola were to procure exclusively from XE. What would be Flextrola's expected profit?

b. Suppose Flextrola plans to procure from both Solectrics and XE; that is, Flextrola will order some amount from Solectrics before the season and then use XE during the selling season to fill demands that exceed that order quantity. How many units should Flextrola order from Solectrics to maximize expected profit?

c. Concerned about the potential loss of business, Solectrics is willing to renegotiate their offer. Solectrics now offers Flextrola an "options contract": Before the season starts, Flextrola purchases Q options and pays Solectrics $25 per option. During the selling season, Flextrola can exercise up to the Q purchased options with a one-day lead time—that is, Solectrics delivers on each exercised option within one day—and the exercise price is $50 per unit. If Flextrola wishes additional units beyond the options purchased, Solectrics will deliver units at XE's price, $83.50. For example, suppose Flextrola purchases 1,500 options but then needs 1,600 units. Flextrola exercises the 1,500 options at $50 each and then orders an additional 100 units at $83.50 each. How many options should Flextrola purchase from Solectrics?

d. Continuing with part c, given the number of options purchased, what is Flextrola's expected profit?

Q13.3* **(Wildcat Cellular)** Marisol is new to town and is in the market for cellular phone service. She has settled on Wildcat Cellular, which will give her a free phone if she signs a one-year contract. Wildcat offers several calling plans. One plan that she is considering is called "Pick Your Minutes." Under this plan, she would specify a quantity of minutes, say x, per month that she would buy at 5¢ per minute. Thus, her upfront cost would be 0.05x$. If her usage is less than this quantity x in a given month, she loses the minutes. If her usage in a month exceeds this quantity x, she would have to pay 40¢ for each extra minute (that is, each minute used beyond x). For example, if she contracts for $x = 120$ minutes per month and her actual usage is 40 minutes, her total bill is $\$120 \times 0.05 = \$6.00$. However, if actual usage is 130 minutes, her total bill would be $\$120 \times 0.05 + (130 - 120) \times 0.40 = \$10.00$. The same rates apply whether the call is local or long distance. Once she signs the contract, she cannot change the number of minutes specified for a year. Marisol estimates that her monthly needs are best approximated by the normal distribution, with a mean of 250 minutes and a standard deviation of 24 minutes.

a. If Marisol chooses the "Pick Your Minutes" plan described above, how many minutes should she contract for?

b. Instead, Marisol chooses to contract for 240 minutes. Under this contract, how much (in dollars) would she expect to pay at 40 cents per minute?

c. A friend advises Marisol to contract for 280 minutes to ensure limited surcharge payments (i.e., the 40-cents-per-minute payments). Under this contract, how many minutes would she expect to waste (i.e., unused minutes per month)?

d. If Marisol contracts for 260 minutes, what would be her approximate expected monthly cell phone bill?

e. Marisol has decided that she indeed does not like surcharge fees (the 40-cents-per-minute fee for her usage in excess of her monthly contracted minutes). How many minutes should she contract for if she wants only a 5 percent chance of incurring any surcharge fee?

(* indicates that the solution is at the end of the book)

f. Wildcat Cellular offers another plan called "No Minimum" that also has a $5.00 fixed fee per month but requires no commitment in terms of the number of minutes per month. Instead, the user is billed 7¢ per minute for her actual usage. Thus, if her actual usage is 40 minutes in a month, her bill would be $5.00 + 40 × 0.07 = $7.80. Marisol is trying to decide between the "Pick Your Minutes" plan described above and the "No Minimum" plan. Which should she choose?

Q13.4** **(Sarah's Wedding)** Sarah is planning her wedding. She and her fiancé have signed a contract with a caterer that calls for them to tell the caterer the number of guests that will attend the reception a week before the actual event. This "final number" will determine how much they have to pay the caterer; they must pay $60 per guest that they commit to. If, for example, they tell the caterer that they expect 90 guests, they must pay $5,400 (= 90 × $60) even if only, say, 84 guests show up. The contract calls for a higher rate of $85 per extra guest for the number of guests beyond what the couple commits to. Thus, if Sarah and her fiancé commit to 90 guests but 92 show up, they must pay $5,570 (the original $5,400 plus 2 × $85).

The problem Sarah faces is that she still does not know the exact number of guests to expect. Despite asking that friends and family members reply to their invitations a month ago, some uncertainty remains: her brother may—or may not—bring his new girlfriend; her fiancé's college roommate may—or may not—be able to take a vacation from work; and so forth. Sarah has determined that the expected number of guests (i.e., the mean number) is 100, but the actual number could be anywhere from 84 to 116:

Q	$f(Q)$	$F(Q)$	$L(Q)$	Q	$f(Q)$	$F(Q)$	$L(Q)$
84	0.0303	0.0303	16.00	101	0.0303	0.5455	3.64
85	0.0303	0.0606	15.03	102	0.0303	0.5758	3.18
86	0.0303	0.0909	14.09	103	0.0303	0.6061	2.76
87	0.0303	0.1212	13.18	104	0.0303	0.6364	2.36
88	0.0303	0.1515	12.30	105	0.0303	0.6667	2.00
89	0.0303	0.1818	11.45	106	0.0303	0.6970	1.67
90	0.0303	0.2121	10.64	107	0.0303	0.7273	1.36
91	0.0303	0.2424	9.85	108	0.0303	0.7576	1.09
92	0.0303	0.2727	9.09	109	0.0303	0.7879	0.85
93	0.0303	0.3030	8.36	110	0.0303	0.8182	0.64
94	0.0303	0.3333	7.67	111	0.0303	0.8485	0.45
95	0.0303	0.3636	7.00	112	0.0303	0.8788	0.30
96	0.0303	0.3939	6.36	113	0.0303	0.9091	0.18
97	0.0303	0.4242	5.76	114	0.0303	0.9394	0.09
98	0.0303	0.4545	5.18	115	0.0303	0.9697	0.03
99	0.0303	0.4848	4.64	116	0.0303	1.0000	0.00
100	0.0303	0.5152	4.12				

Q = Number of guests that show up to the wedding
$f(Q)$ = Density function = Prob{Q guests show up}
$F(Q)$ = Distribution function = Prob{Q or fewer guests show up}
$L(Q)$ = Loss function = Expected number of guests above Q

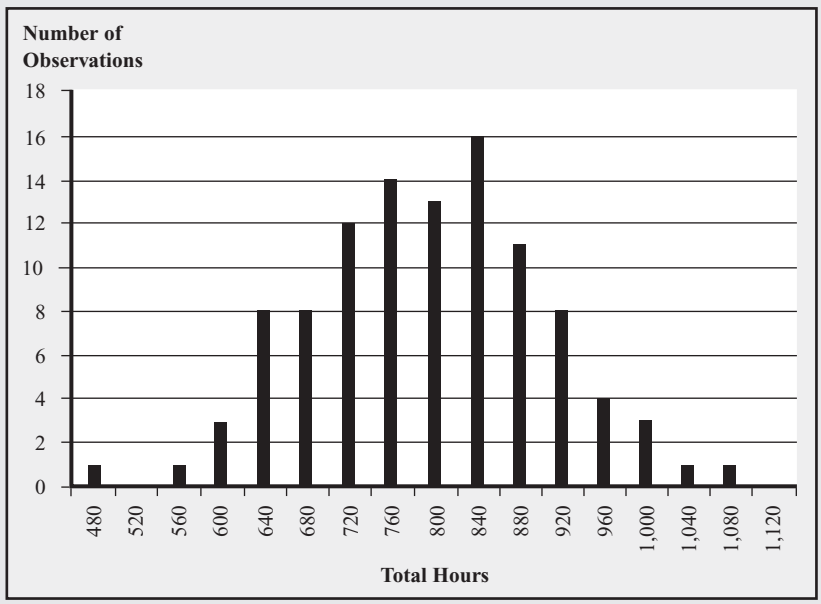
a. How many guests should Sarah commit to with the caterer?

b. Suppose Sarah commits to 105 guests. What is Sarah's expected bill?

c. Suppose that the caterer is willing to alter the contract so that if fewer than the number of guests they commit to show up, they will get a partial refund. In particular, they only have to pay $45 for each "no-show." For example, if they commit to 90 but only 84 show, they will have to pay 84 × $60 + 6 × $45 = $5,310. Now how many guests should she commit to?

d. The caterer offers Sarah another option. She could pay $70 per guest, no matter how many guests show up; that is, she wouldn't have to commit to any number before the wedding. Should Sarah prefer this option or the original option ($60 per committed guest and $85 each guest beyond the commitment)?

Q13.5 **(Lucky Smokes)** Lucky Smokes currently operates a warehouse that serves the Virginia market. Some trucks arrive at the warehouse filled with goods to be stored in the warehouse. Other trucks arrive at the warehouse empty to be loaded with goods. Based on the number of trucks that arrive at the warehouse in a week, the firm is able to accurately estimate the total number of labor hours that are required to finish all of the loading and unloading. The following histogram plots these estimates for each week over the past two



years. (There are a total of 104 weeks recorded in the graph.) For example, there were three weeks in this period that required 600 total labor hours and only one week that recorded 1,080 hours of required labor.

The mean of the data is 793 and the standard deviation is 111. Labor is the primary variable cost in the operation of a warehouse. The Virginia warehouse employed 20 workers, who were guaranteed at least 40 hours of pay per week. Thus, in weeks with less than 800 hours of required labor, the workers either went home early on some days or were idle. On weeks with more than 800 hours of required labor, the extra hours were obtained with overtime. Workers were paid time and a half for each hour of overtime.

You have been placed in charge of a new warehouse scheduled to serve the North Carolina market. Marketing suggests that the volume for this warehouse should be comparable to the Virginia warehouse. Assume that you must pay each worker for at least 40 hours of work per week and time and a half for each hour of overtime. Assume there is no limit on overtime for a given week. Further, assume you approximate your workload requirement with a normal distribution.

a. If you hire 22 workers, how many weeks a year should you expect to use overtime?

b. If you hire 18 workers, how many weeks a year will your workers be underutilized?

c. If you are interested in minimizing your labor cost, how many workers should you hire (again, assuming your workload forecast is normally distributed)?

d. You are now concerned the normal distribution might not be appropriate. For example, you can't hire 20.5 workers. What is the optimal number of workers to hire if you use the empirical distribution function constructed with the data in the above histogram?

Q13.6 **(Shillings)** You are traveling abroad and have only American dollars with you. You are currently in the capital but you will soon be heading out to a small town for an extended stay. In the town, no one takes credit cards and they only accept the domestic currency (shillings). In the capital, you can convert dollars to shillings at a rate of two shillings per dollar. In the town, you learn that one dollar only buys 1.6 shillings. Upon your return to the capital at the end of your trip, you can convert shillings back to dollars at a rate of 2.5 shillings per dollar. You estimate that your expenditures in the town will be normally distributed with mean of 400 shillings and standard deviation of 100 shillings.

a. How many dollars should you convert to shillings before leaving the capital?
b. After some thought, you feel that it might be embarrassing if you run out of shillings and need to ask to convert additional dollars, so you really do not want to run out of shillings. How many dollars should you convert to shillings if you want to ensure there is no more than a 1 in 200 chance you will run out of shillings?

Q13.7 **(TEC)** Consider the relationship between TEC and O'Neill with unlimited, but expensive, reactive capacity. Recall that TEC is willing to give O'Neill a midseason replenishment (see Figure 13.1) but charges O'Neill a 20 percent premium above the regular wholesale price of $110 for those units. Suppose TEC's gross margin is 25 percent of its selling price for units produced in the first production run. However, TEC estimates that its production cost per unit for the second production run (any units produced during the season after receiving O'Neill's second order) is twice as large as units produced for the initial order. Wetsuits produced that O'Neill does not order need to be salvaged at the end of the season. With O'Neill's permission, TEC estimates it can earn $30 per suit by selling the extra suits in Asian markets.

a. What is TEC's expected profit with the traditional arrangement (i.e., a single order by O'Neill well in advance of the selling season)? Recall that O'Neill's optimal newsvendor quantity is 4,101 units.
b. What is TEC's expected profit if it offers the reactive capacity to O'Neill and TEC's first production run equals O'Neill's first production order? Assume the demand forecast is normally distributed with mean 3,192 and standard deviation 1,181. Recall, O'Neill's optimal first order is 3,263 and O'Neill's expected second order is 437 units.
c. What is TEC's optimal first production quantity if its CEO authorizes its production manager to choose a quantity that is greater than O'Neill's first order?
d. Given the order chosen in part c, what is TEC's expected profit? (*Warning:* This is a hard question.)

Q13.8 **(Office Supply Company)** Office Supply Company (OSC) has a spare parts warehouse in Alaska to support its office equipment maintenance needs. Once every six months, a major replenishment shipment is received. If the inventory of any given part runs out before the next replenishment, then emergency air shipments are used to resupply the part as needed. Orders are placed on January 15 and June 15, and orders are received on February 15 and July 15, respectively.

OSC must determine replenishment quantities for its spare parts. As an example, historical data show that total demand for part 1AA-66 over a six-month interval is Poisson with mean 6.5. The cost of inventorying the unneeded part for six months is $5 (which includes both physical and financial holding costs and is charged based on inventory at the end of the six-month period). The variable production cost for 1AA-66 is $37 per part. The cost of a regular, semiannual shipment is $32 per part, and the cost of an emergency shipment is $50 per part.

It is January 15 and there are currently three 1AA-66 parts in inventory. How many parts should arrive on February 15?

Q13.9* **(Steve Smith)** Steve Smith is a car sales agent at a Ford dealership. He earns a salary and benefits, but a large portion of his income comes from commissions: $350 per vehicle sold for the first five vehicles in a month and $400 per vehicle after that. Steve's historical sales can be well described with a Poisson distribution with mean 5.5; that is, on average, Steve sells 5.5 vehicles per month. On average, how much does Steve earn in commissions per month?

(* indicates that the solution is at the end of the book)

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 14

Service Levels and Lead Times in Supply Chains: The Order-up-to Inventory Model[1]

Many products are sold over a long time horizon with numerous replenishment opportunities. To draw upon a well-known example, consider the Campbell Soup Company's flagship product, chicken noodle soup. It has a long shelf life and future demand is assured. Hence, if in a particular month Campbell Soup has more chicken noodle soup than it needs, it does not have to dispose of its excess inventory. Instead, Campbell needs only wait for its pile of inventory to draw down to a reasonable level. And if Campbell finds itself with less inventory than it desires, its soup factory cooks up another batch. Because obsolescence is not a major concern and Campbell is not limited to a single production run, the newsvendor model (Chapters 12 and 13) is not the right inventory tool for this setting. The right tool for this job is the *order-up-to model.*

Although multiple replenishments are feasible, the order-up-to model still faces the "too little–too much" challenge associated with matching supply and demand. Because soup production takes time (i.e., there is a lead time to complete production), Campbell cannot wait until its inventory draws down to zero to begin production. (You would never let your vehicle's fuel tank go empty before you begin driving to a refueling station!) Hence, production of a batch should begin while there is a sufficient amount of inventory to buffer against uncertain demand while we wait for the batch to finish. Since buffer inventory is not free, the objective with the order-up-to model is to strike a balance between running too lean (which leads to undesirable stockouts, i.e., poor service) and running too fat (which leads to inventory holding costs).

Instead of soup, this chapter applies the order-up-to model to the inventory management of a technologically more sophisticated product: a pacemaker manufactured by Medtronic Inc. We begin with a description of Medtronic's supply chain for pacemakers and then detail the order-up-to model. Next, we consider how to use the model to hit target service

[1] Data in this chapter have been modified to protect confidentiality.

levels, discuss what service targets are appropriate, and explore techniques for controlling how frequently we order. We conclude with general managerial insights.

14.1 Medtronic's Supply Chain

Medtronic is a designer and manufacturer of medical technology. They are well known for their line of cardiac rhythm products, and, in particular, pacemakers, but their product line extends into numerous other areas: products for the treatment of cardiovascular diseases and surgery, diabetes, neurological diseases, spinal surgery, and eye/nose/throat diseases.

Inventory in Medtronic's supply chain is held at three levels: manufacturing facilities, distribution centers (DCs), and field locations. The manufacturing facilities are located throughout the world, and they do not hold much finished goods inventory. In the United States there is a single distribution center, located in Mounds View, Minnesota, responsible for the distribution of cardiac rhythm products. That DC ships to approximately 500 sales representatives, each with his or her own defined territory. All of the Medtronic DCs are responsible for providing very high availability of inventory to the sales representatives they serve in the field, where availability is measured with the in-stock probability.

The majority of finished goods inventory is held in the field by the sales representatives. In fact, field inventory is divided into two categories: consignment inventory and trunk inventory. Consignment inventory is inventory owned by Medtronic at a customer's location, usually a closet in a hospital. Trunk inventory is literally inventory in the trunk of a sales representative's vehicle. A sales representative has easy access to both of these kinds of field inventory, so they can essentially be considered a single pool of inventory.

Let's now focus on a particular DC, a particular sales representative, and a particular product. The DC is the one located in Mounds View, Minnesota. The sales representative is Susan Magnotto and her territory includes the major medical facilities in Madison, Wisconsin. Finally, the product is the InSync ICD Model 7272 pacemaker, which is displayed in Figure 14.1.

A pacemaker is demanded when it is implanted in a patient via surgery. Even though a surgeon can anticipate the need for a pacemaker for a particular patient, a surgeon may not know the appropriate model for a patient until the actual surgery. For this reason, and for the need to maintain a good relationship with each physician, Susan attends each surgery and always carries the various models that might be needed. Susan can replenish her inventory after an implant by calling an order in to Medtronic's Customer Service, which then sends the request to the Mounds View DC. If the model she requests is available in inventory at the DC, then it is sent to her via an overnight carrier. The time between when Susan orders a unit and when she receives the unit is generally one day, and rarely more than two days.

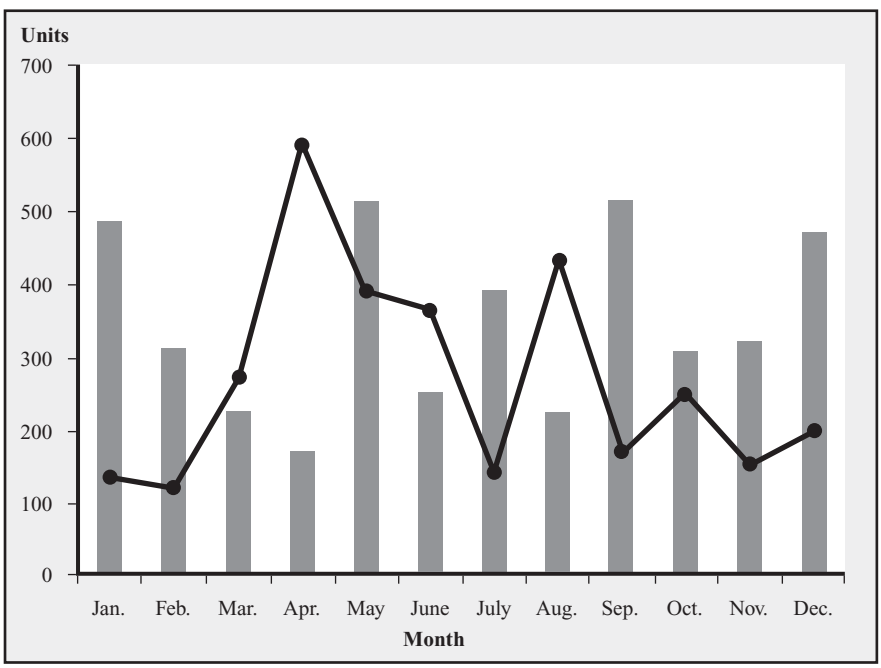
The Mounds View DC requests replenishments from the production facilities on a weekly basis. With the InSync pacemaker, there is currently a three-week lead time to receive each order.

For the InSync pacemaker, Figure 14.2 provides one year's data on monthly shipments and end-of-month inventory at the Mounds View DC. Figure 14.3 provides data on monthly implants (i.e., demand) and inventory for the InSync pacemaker in Susan's territory over the same year. As can be seen from the figures, there is a considerable amount

FIGURE 14.1
Medtronic's InSync ICD Pacemaker

of variation in the number of units demanded at the DC and in particular in Susan's territory. Interestingly, it appears that there is more demand in the summer months in Susan's territory, but the aggregate shipments through the DC do not indicate the same pattern. Therefore, it is reasonable to conclude that the "pattern" observed in Susan's demand data is not real: Just like a splotch of ink might look like something on a piece of paper, random events sometimes appear to form a pattern.

FIGURE 14.2
Monthly Shipments (bar) and End-of-Month Inventory (line) for the InSync Pacemaker at the Mounds View Distribution Center



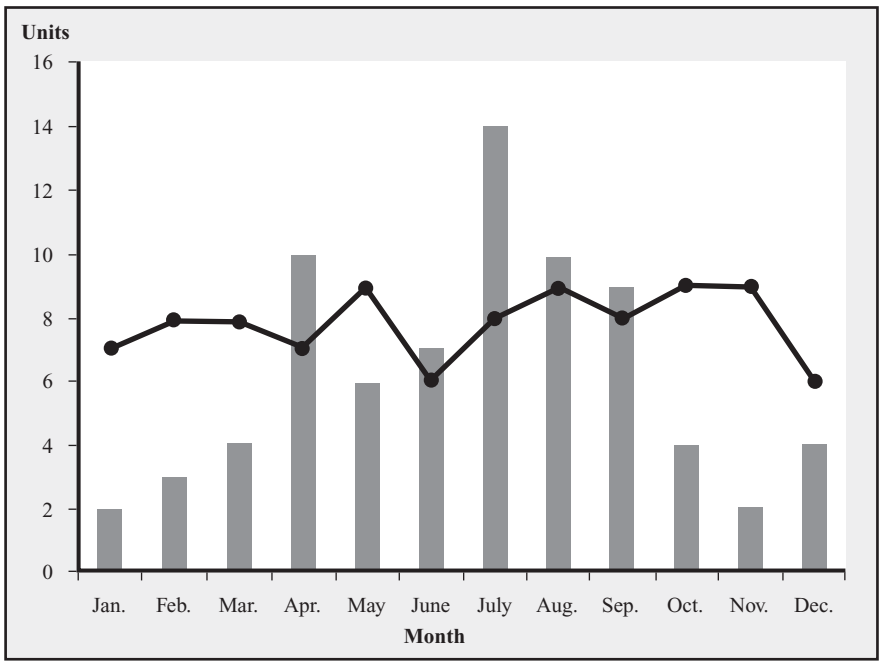


FIGURE 14.3
Monthly Implants (bar) and End-of-Month Inventory (line) for the InSync Pacemaker in Susan's Territory

As a sales representative, Susan's primary responsibility is to ensure that Medtronic's products are the choice products of physicians in her territory. To encourage active sales effort, a considerable portion of her yearly income is derived from bonuses to achieve aggressive sales thresholds.

If the decision on inventory investment were left up to Susan, she would err on the side of extra inventory. There are a number of reasons why she would like to hold a considerable amount of inventory:

- Due to the sales incentive system, Susan never wants to miss a sale due to a lack of inventory. Because patients and surgeons do not tolerate waiting for back-ordered inventory, if Susan does not have the right product available, then the sale is almost surely lost to a competitor.
- Medtronic's products are generally quite small, so it is possible to hold a considerable amount of inventory in a relatively small space (e.g., the trunk of a vehicle).
- Medtronic's products have a relatively long shelf life, so spoilage is not a major concern. (However, spoilage can be a concern if a rep fails to stick to a "first-in-first-out" regime, thereby allowing a unit to remain in inventory for a disproportionately long time. Given that spoilage is not a significant issue if first-in-first-out is implemented, we'll not consider this issue further in this discussion.)
- While Susan knows that she can be replenished relatively quickly from the DC (assuming the DC has inventory available), she is not always able to find the time to place an order immediately after an implant. An inventory buffer thereby allows her some flexibility with timing her replenishment requests.
- Although the production facilities are supposed to ensure that the DCs never stock out of product, sometimes a product can become unavailable for several weeks, if not several months. For example, the *production yield* might not be as high as initially planned or

a supplier of a key component might be capacity-constrained. Whatever the cause, having a few extra units of inventory helps protect Susan against these shortages.

To ensure that each sales representative holds a reasonable amount of inventory, each sales representative is given a *par level* for each product. The par level specifies the maximum number of units the sales representative can have on-order plus on-hand at any given time. Therefore, once a sales representative's inventory equals her par level, she cannot order an additional unit until one is implanted. The par levels are set quarterly based on previous sales and anticipated demand. If a sales representative feels a higher par level is warranted, he or she can request an adjustment. Even though Medtronic does not wish to give the sales representative full reign over inventory, due to Medtronic's large gross margins, neither does Medtronic want to operate too lean.

An issue for Medtronic is whether its supply chain is supporting its aggressive growth objectives. This chapter first considers the management of field inventory. As of now, the sales representatives are responsible for managing their own inventory (within the limits of set par levels), but maybe a computer-based system should be considered that would choose stocking levels and automatically replenish inventory. This system would relieve Susan Magnotto and other representatives from the task of managing inventory so that they can concentrate on selling product. While that is attractive to Susan, a reduction in product availability is nonnegotiable. After exploring the management of field inventory, attention is turned to the management of the Mounds View distribution center inventory. It is essential that the DC provide excellent availability to the field representatives without holding excessive inventory.

14.2 The Order-up-to Model Design and Implementation

The order-up-to model is designed to manage inventory for a product that has the opportunity for many replenishments over a long time horizon. This section describes the assumptions of the model and how it is implemented in practice. The subsequent sections consider the evaluation of numerous performance measures, how historical data can be used to choose a distribution to represent demand, and how to calibrate the model to achieve one of several possible objectives.

We are working with a single product that is sold over a long period of time. Opportunities to order replenishment inventory occur at regular intervals. The time between two ordering opportunities is called a *period,* and all of the periods are of the same duration. While one day seems like a natural period length for the InSync pacemaker in the field (e.g., in Susan's territory), one week is a more natural period length for the Mounds View DC. In other settings, the appropriate period length could be an hour, a month, or any other interval. See Section 14.8 for additional discussion on the appropriate period length. For the sake of consistency, let's also assume that orders are submitted at the same point in time within the period, say, at the beginning of the period.

Random demand occurs during each period. As with the newsvendor model, among the most critical inputs to the order-up-to model are the parameters of the demand distribution, which is the focus of Section 14.4. However, it is worth mentioning that the model assumes the same demand distribution represents demand in every period. This does not mean that actual demand is the same in every period; it just means that each period's demand is the outcome of a single distribution. The model can be extended to accommodate more complex demand structures, but, as we will see, our simpler structure is adequate for our task.

Receiving a replenishment is the third event within each period. We assume that replenishments are only received at the beginning of a period, before any demand occurs in the period. Hence, if a shipment arrives during a period, then it is available to satisfy demand during that period.

Replenishment orders are received after a fixed amount of time called the *lead time,* which is represented with the variable l. The lead time is measured in periods; if one day is a period, then the lead time to receive an order should be measured in days. Hence, not only should the period length be chosen so that it matches the frequency at which orders can be made and replenishments can be received, it also should be chosen so that the replenishment lead time can be measured in an integer (0, 1, 2, . . .) number of periods.

There is no limit to the quantity that can be ordered within a period, and no matter the order quantity, the order is always received in the lead time number of periods. Therefore, supply in this model is not capacity-constrained, but delivery of an order does take some time.

Inventory left over at the end of a period is carried over to the next period; there is no obsolescence, theft, or spoilage of inventory.

To summarize, at the start of each period, a replenishment order can be submitted and a replenishment can be received, then random demand occurs. There is no limit imposed on the quantity of any order, but an order is received only after l periods. For example, if the period length is one day and $l = 1$, then a Monday morning order is received Tuesday morning. Each period has the same duration and the same sequence of events occurs in each period (order, receive, demand). Figure 14.4 displays the sequence of events over a sample of three periods when the lead time to receive orders is one period, $l = 1$.

Now let's define several terms we use to describe our inventory system and then we show how the order-up-to level is used to choose an order quantity.

On-order inventory is relatively intuitive: The on-order inventory is the number of units that we ordered in previous periods that we have not yet received. Our on-order inventory should never be negative, but it can be zero.

On-hand inventory is also straightforward: It is the number of units of inventory we have on-hand, immediately available to serve demand.

Back-order is the number of units on back order, that is, the total amount of demand that has occurred but has not been satisfied. To get the mathematics of the order-up-to model to work precisely, it is necessary to assume that *all* demand is eventually filled, that is, if demand occurs and no units are available in current inventory, then that demand is back-ordered and filled as soon as inventory becomes available. In other words, the order-up-to model assumes there are no lost sales. In some settings, this is not a problem: complete back-ordering is commonplace in the management of inventory between two firms within a supply chain. However, as with the InSync pacemaker in the field, when end consumers generate demand (instead of a firm), the back-order assumption is probably

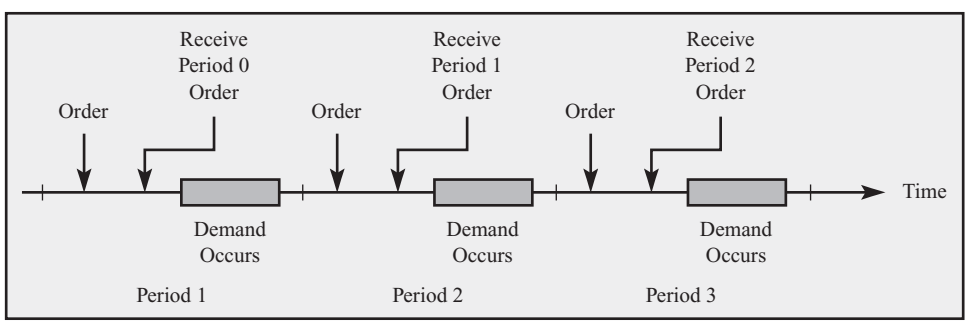


FIGURE 14.4
Sample Sequence of Events in the Order-up-to Model with a One-Period Lead Time, $l = 1$, to Receive Orders

violated (at least to some extent). Nevertheless, if the order-up-to level is chosen so that back orders are rare, then the order-up-to model is a reasonable approximation. Hence, we use it for the InSync pacemaker to manage both the DC inventory as well as Susan's field inventory.

The next measure combines on-hand inventory with the back order:

$$\text{Inventory level} = \text{On-hand inventory} - \text{Back order}$$

Unlike the on-hand inventory and the back order, which are never negative, the inventory level can be negative. It is negative when we have units back-ordered. For example, if the inventory level is -3, then there are three units of demand waiting to be filled.

The following measure combines all of the previous measures:

$$\begin{aligned}\text{Inventory position} &= \text{On-order inventory} + \text{On-hand inventory} - \text{Back order} \\ &= \text{On-order inventory} + \text{Inventory level}\end{aligned}$$

The *order-up-to level* is the maximum inventory position we are willing to have. Let's denote the order-up-to level with the variable S. For example, if $S = 2$, then we are allowed an inventory position up to two units, but no more. Our order-up-to level is essentially equivalent to the par level Medtronic currently uses. It has also been referred to as the *base stock level.* (The order-up-to model is sometimes called the *base stock model.*)

The implementation of our order-up-to policy is relatively straightforward: If we observe at the beginning of any period that our inventory position is less than the order-up-to level S, then we order enough inventory to raise our inventory position to S; that is, in each period, we order the difference between S and the inventory position:

$$\text{Each period's order quantity} = S - \text{Inventory position}$$

Because the inventory position includes our on-order inventory, after we submit the order, our inventory position immediately increases to S.

To illustrate an ordering decision, suppose we observe at the beginning of a period that our inventory level is -4 (four units are back-ordered), our on-order inventory is one, and our chosen order-up-to level is $S = 3$. In this situation, we need to order six units: our inventory position is $1 - 4 = -3$ and our order quantity should be S minus the inventory position, $3 - (-3) = 6$.

If we find ourselves in a period with an inventory position that is greater than S, then we should not order anything. Eventually our inventory position will drop below S. After that time, we will begin ordering and our inventory position will never again be greater than S as long as we do not change S (because we only order to raise our inventory position to S, never more).

Notice that our inventory position drops below S only when demand occurs. Suppose $S = 3$ and we observe that our inventory position is one at the beginning of the period. If we followed our order-up-to policy in the previous period, then we must have had an inventory position of three after our order in the previous period. The only way that we could then observe an inventory position of one in this period is if two units of demand occurred in the previous period. Thus, we will order two units in this period (to raise our inventory position back to $S = 3$). Hence,

The order quantity in each period exactly equals the demand in the previous period in the order-up-to inventory model.

Due to this observation, an order-up-to policy is sometimes called a *one-for-one ordering policy:* each unit of demand triggers an order for one replenishment unit.

The order-up-to model is an example of a system that operates on the pull principle of production/inventory control. The key feature of a *pull system* is that production-replenishment of a unit is only initiated when a demand of another unit occurs. Therefore, in a pull system, inventory is pulled through the system only by the occurrence of demand. In contrast, with a *push system,* production-replenishment occurs in anticipation of demand. The newsvendor model is a push system. A kanban system, which is a critical component of any just-in-time system, operates with pull. (See Chapter 11.) Pull systems impose the discipline to prevent the excessive buildup of inventory, but they do not anticipate shifts in future demand. Thus, pull systems are most effective when average demand remains steady, as we have assumed in our order-up-to model.

14.3 The End-of-Period Inventory Level

The inventory level (on-hand inventory minus the back order) is an important metric in the order-up-to model: If the inventory level is high, then we incur holding costs on on-hand inventory, but if the inventory level is low, then we may not be providing adequate availability to our customers. Hence, we need to know how to control the inventory level via our decision variable, the order-up-to level. The following result suggests there actually is a relatively simple relationship between them:

The inventory level measured at the end of a period equals the order-up-to level S *minus demand over* l + *1 periods.*

If that result is (magically) intuitive to you, or if you are willing to believe it on faith, then you can now skip ahead to the next section. For the rest of us, the remainder of this section explains and derives that result.

We'll derive our result with the help of a seemingly unrelated example. Suppose at a neighborhood picnic you have a large pot with 30 cups of soup in it. Over the course of the picnic, you add 20 additional cups of soup to the pot and a total of 40 cups are served. How many cups of soup are in the pot at the end of the picnic? Not too hard: start with 30, add 20, and then subtract 40, so you are left with 10 cups of soup in the pot. Does the answer change if you first subtract 40 cups and then add 20 cups? The answer is no as long as people are patient. To explain, if we subtract 40 cups from the original 30 cups, then we will have -10 cups, that is, there will be people waiting in line to receive soup. Once the 20 cups are added, those people in line are served and 10 cups remain. The sequence of adding and subtracting does not matter precisely because everyone is willing to wait in line, that is, there are no lost sales of soup. In other words, the sequence of adding and subtracting does not matter, only the total amount added and the total amount subtracted matter.

Does the answer change in our soup example if the 20 cups are added one cup at a time or in random quantities (e.g., sometimes half a cup, sometime a whole cup, sometimes more than a cup)? Again, the answer is no: the increments by which the soup is added or subtracted do not matter, only the total amount added or subtracted.

Keep the soup example in mind, but let's switch to another example. Suppose a firm uses the order-up-to model, its order-up-to level is $S = 3$, and the lead time is two days, $l = 2$. What is the inventory level at the end of any given day? This seems like a rather hard question to answer, but let's tackle it anyway. To provide a concrete reference, randomly choose a period, say period 10. Let IL be the inventory level at the start of period 10. We use a variable for the inventory level because we really do not know the exact inventory level. It turns out, as we will see, that we do not need to know the exact inventory level.

After we submit our order in period 10, we will have a total of $3 - IL$ units on order. When we implement the order-up-to model, we must order so that our inventory level

(IL) plus our on-order inventory ($3 - IL$) equals our order-up-to level ($3 = IL + 3 - IL$). Some of the on-order inventory may have been ordered in period 10, some of it in period 9. No matter when the on-order inventory was ordered, it will *all* be received by the end of period 12 because the lead time is two periods. For example, the period 10 order is received in period 12, so all of the previously ordered inventory should have been received by period 12 as well.

Now recall the soup example. Think of IL as the amount of soup you start with. How much is added to the "pot of inventory" over periods 10 to 12? That is the amount that was on order in period 10, that is, $3 - IL$. So the pot starts with IL and then $3 - IL$ is added over periods 10 to 12. How much is subtracted from the pot of inventory over periods 10 to 12? Demand is what causes subtraction from the pot of inventory. So it is demand over periods 10 to 12 that is subtracted from inventory; that is, demand over the $l + 1$ periods (10 to 12 are three periods). So how much is in the pot of inventory at the end of period 12? The answer is simple: just as in the soup example, it is how much we start with (IL), plus the amount we add ($3 - IL$), minus the amount we subtract (demand over periods 10 to 12):

$$\begin{aligned}\text{Inventory level at the end of period 12} &= IL + 3 - IL - \text{Demand in periods 10 to 12}\\ &= 3 - \text{Demand in periods 10 to 12}\end{aligned}$$

In other words, our inventory level at the end of a period is the order-up-to level (in this case 3) minus demand over $l + 1$ periods (in this case, periods 10 to 12). Hence, we have derived our result.

Just as in the soup example, it does not matter the sequence by which inventory is added or subtracted; all that matters is the total amount that is added ($3 - IL$) and the total amount that is subtracted (total demand over periods 10 to 12). (This is why the back-order assumption is needed.) Nor do the increments by which inventory is added or subtracted matter. In other words, we can add and subtract at constant rates, or we could add and subtract at random rates; either way, it is only the totals that matter.

You still may be a bit confused about why it is demand over $l + 1$ periods that is relevant rather than demand over just l periods. Recall that we are interested in the inventory level at the *end* of the period, but we make our ordering decision at the *start* of a period. The time from when an order is placed at the start of a period to the end of the period in which the order arrives is actually $l + 1$ periods' worth of demand.

Now you might wonder why we initiated our analysis at the start of a period, in this case period 10. Why not begin by measuring the inventory position at some other time during a period? The reason is that the inventory position measured at the start of a period is always equal to the order-up-to level, but we cannot be sure about what the inventory position will be at any other point within a period (because of random demand). Hence, we anchor our analysis on something we know for sure, which is that the inventory position equals S at the start of every period when an order-up-to policy is implemented.

To summarize, in the order-up-to model, the inventory level at the end of a period equals the order-up-to level S minus demand over $l + 1$ periods. Therefore, while we need to know the distribution of demand for a single period, we also need to know the distribution of demand over $l + 1$ periods.

14.4 Choosing Demand Distributions

Every inventory management system must choose a demand distribution to represent demand. In our case, we need a demand distribution for the Mounds View DC and Susan Magnotto's territory. Furthermore, as discussed in the previous section, we need a demand

distribution for one period of demand and a demand distribution for $l + 1$ periods of demand. As we will see, the normal distribution works for DC demand, but the Poisson distribution is better for demand in Susan's territory.

The graph in Figure 14.2 indicates that Mounds View's demand is variable, but it appears to have a stable mean throughout the year. This is a good sign: as we already mentioned, the order-up-to model assumes average demand is the same across periods. Average demand across the sample is 349 and the standard deviation is 122.38. Seven months of the year have demand less than the mean, so the demand realizations appear to be relatively symmetric about the mean. Finally, there do not appear to be any extreme outliers in the data: the maximum is 1.35 standard deviations from the mean and the minimum is 1.46 standard deviations from the mean. Overall, the normal distribution with a mean of 349 and a standard deviation of 122.38 is a reasonable choice to represent the DC's monthly demand.

However, because the DC orders on a weekly basis and measures its lead time in terms of weeks, the period length for our order-up-to model applied to the DC should be one week. Therefore, we need to pick a distribution to represent weekly demand; that is, we have to chop our monthly demand distribution into a weekly demand distribution. If we are willing to make the assumption that one week's demand is independent of another week's demand, and if we assume that there are 4.33 weeks per month (52 weeks per year/12 months), then we can convert the mean and standard deviation for our monthly demand distribution into a mean and standard deviation for weekly demand:

$$\text{Expected weekly demand} = \frac{\text{Expected monthly demand}}{4.33}$$

$$\text{Standard deviation of weekly demand} = \frac{\text{Standard deviation of monthly demand}}{\sqrt{4.33}}$$

Exhibit 14.1 summarizes the process of converting demand distributions from one period length to another.

In the case of the Mounds View DC, expected weekly demand is $349/4.33 = 80.6$ and the standard deviation of weekly demand is $122.38/\sqrt{4.33} = 58.81$. So we will use a normal distribution with mean 80.6 and standard deviation 58.81 to represent weekly demand at the Mounds View DC.

We also need demand for the InSync pacemaker over $l + 1$ periods, which in this case is demand over $3 + 1 = 4$ weeks. Again using Exhibit 14.1, demand over four weeks has mean $4 \times 80.6 = 322.4$ and standard deviation $\sqrt{4} \times 58.81 = 117.6$.

Now consider demand for the InSync pacemaker in Susan's territory. From the data in Figure 14.3, total demand over the year is 75 units, which translates into average demand of 6.25 (75/12) units per month, 1.44 units per week (75/52), and 0.29 (1.44/5) unit per day, assuming a five-day week.

Our estimate of 0.29 unit per day for expected demand implicitly assumes expected demand on any given day of the year is the same as for any other day of the year. In other words, there is no seasonality in demand across the year, within a month, or within a week. There probably is not too much promotion-related volatility in demand (buy one pacemaker, get one free), nor is there much volatility due to gift giving (what more could a dad want than a new pacemaker under the Christmas tree). There probably is not much variation within the week (the same number of implants on average on Friday as on Monday) or within the month. However, those conjectures could be tested with more refined data. Furthermore, from the data in Figure 14.2, it appears demand is stable throughout the year and

Exhibit 14.1

HOW TO CONVERT A DEMAND DISTRIBUTION FROM ONE PERIOD LENGTH TO ANOTHER

If you wish to divide a demand distribution from a long period length (e.g., a month) into n short periods (e.g., a week), then

$$\text{Expected demand in the short period} = \frac{\text{Expected demand in the long period}}{n}$$

$$\text{Standard deviation of demand in the short period} = \frac{\text{Standard deviation of demand in the long period}}{\sqrt{n}}$$

If you wish to combine demand distributions from n short periods (e.g., a week) into one long period (e.g, a three-week period, $n = 3$), then

$$\text{Expected demand in the long period} = n \times \text{Expected demand in the short period}$$

$$\text{Standard deviation of demand in the long period} = \sqrt{n} \times \text{Standard deviation of demand in the short period}$$

The above equations assume the same demand distribution represents demand in each period and demands across periods are independent of each other.

there are no upward or downward trends in the data. Hence, our assumption of a constant expected daily demand is reasonable.

Using Exhibit 14.1, if average demand over one day is 0.29 unit, then expected demand over $l + 1$ days must be $2 \times 0.29 = 0.58$.

Unlike the normal distribution, which is defined by two parameters (its mean and its standard deviation), the Poisson distribution is defined by only a single parameter, its mean. For the InSync pacemaker, it is natural to choose the mean equal to the observed mean demand rate: 0.29 for demand over one period and 0.58 for demand over two periods. Even though the Poisson distribution does not allow you to choose any standard deviation while holding the mean fixed, the Poisson distribution does have a standard deviation:

$$\text{Standard deviation of a Poisson distribution} = \sqrt{\text{Mean of the distribution}}$$

For example, with a mean of 0.29, the standard deviation is $\sqrt{0.29} = 0.539$. Table 14.1 provides the distribution and density functions for the chosen Poisson distributions.

TABLE 14.1
The Distribution and Density Functions for Two Poisson Distributions.
In Excel, $F(S)$ is evaluated with the function POISSON(*S, Expected demand,* 1) and $f(S)$ is evaluated with the function POISSON(*S, Expected demand,* 0).

Mean Demand = 0.29			Mean Demand = 0.58		
S	$F(S)$	$f(S)$	S	$F(S)$	$f(S)$
0	0.74826	0.74826	0	0.55990	0.55990
1	0.96526	0.21700	1	0.88464	0.32474
2	0.99672	0.03146	2	0.97881	0.09417
3	0.99977	0.00304	3	0.99702	0.01821
4	0.99999	0.00022	4	0.99966	0.00264
5	1.00000	0.00001	5	0.99997	0.00031

$F(S)$ = Prob{Demand is less than or equal to S}
$f(S)$ = Prob{Demand is exactly equal to S}

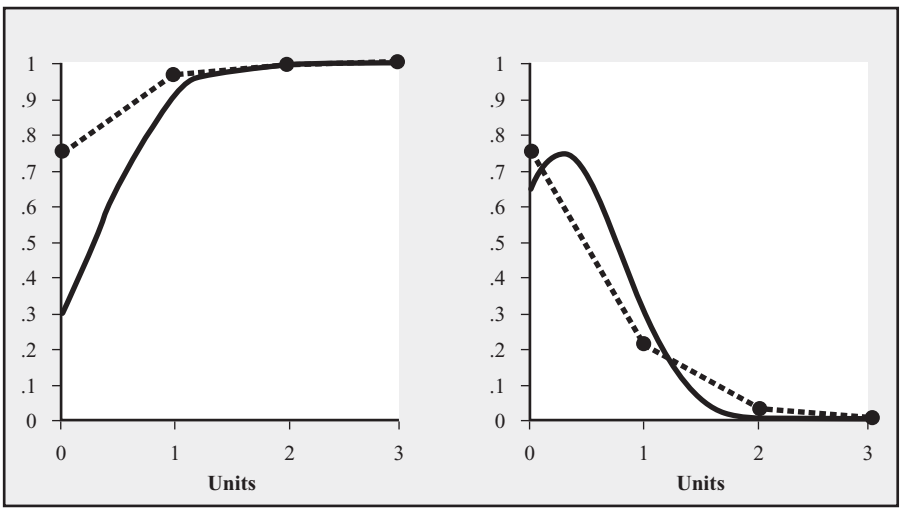


FIGURE 14.5
The Distribution (left graph) and Density Functions (right graph) of a Poisson Distribution with a Mean of 0.29 (bullets and dashed lines) and a Normal Distribution with a Mean of 0.29 and a Standard Deviation of 0.539 (solid line)

Because it can be hard to visualize a distribution from a table, Figure 14.5 displays the graphs of the distribution and density functions of the Poisson distribution with mean 0.29. For comparison, the comparable functions for the normal distribution are also included. (The dashed lines with the Poisson distribution are only for visual effect; that is, those functions exist only for integer values.)

The graphs in Figure 14.5 highlight that the Poisson and normal distributions are different in two key respects: (1) the Poisson distribution is discrete (it has integer outcomes), whereas the normal distribution is continuous, and (2) the distribution and density functions for those two distributions have different shapes. The fractional quantity issue is not a major concern if demand is 500 units (or probably even 80 units), but it is a concern when average demand is only 0.29 unit. Ideally, we want a discrete demand distribution like the Poisson.

Yet another argument can be made in support of the Poisson distribution as our model for demand in Susan's territory. Recall that with the queuing models (Chapters 8 and 9) we use the exponential distribution to describe the time between customer arrivals, which is appropriate if customers arrive independently of each other; that is, the arrival time of one customer does not provide information concerning the arrival time of another customer. This is particularly likely if the arrival rate of customers is quite slow, as it is with the InSync pacemaker. So it is likely that the interarrival time of InSync pacemaker demand has an exponential distribution. And here is the connection to the Poisson distribution: If the interarrival times are exponentially distributed, then the number of arrivals in any fixed interval of time has a Poisson distribution. For example, if the interarrival times between InSync pacemaker demand in Susan's territory are exponentially distributed with a mean of 3.45 days, then the average number of arrivals (demand) per day has a Poisson distribution with a mean of $1/3.45 = 0.29$ unit.

If we had daily demand data, we would be able to confirm whether or not our chosen Poisson distribution is a good fit to the data. Nevertheless, absent those data, we have probably made the best educated guess.

To summarize, we shall use a normal demand distribution with mean 80.6 and standard deviation 58.81 to represent weekly demand for the InSync pacemaker at the Mounds View DC and a normal demand distribution with mean 322.4 and standard deviation 117.6 to represent demand over $l + 1 = 4$ weeks. We will use a Poisson distribution with mean 0.29 to represent daily demand in Susan Magnotto's territory and a Poisson distribution with mean 0.58 to represent demand over $l + 1 = 2$ days.

14.5 Performance Measures

This section considers the evaluation of several performance measures with the order-up-to method. We consider these measures at two locations in the supply chain: Susan Magnotto's territory and the Mounds View distribution center.

Recall we use a Poisson distribution with mean 0.29 to represent daily demand in Susan's territory and a Poisson distribution with mean 0.58 to represent demand over $l + 1 = 2$ days. We shall evaluate the performance measures assuming Susan uses $S = 3$ as her order-up-to level. The Mounds View weekly demand is normally distributed with mean 80.6 and standard deviation 58.81 and over $l + 1 = 4$ weeks it is normally distributed with mean $\mu = 322.4$ and standard deviation $\sigma = 117.6$. We evaluate the performance measures assuming the order-up-to level $S = 625$ is implemented at Mounds View.

Figure 14.6 summarizes the necessary inputs to evaluate each performance measure.

In-Stock and Stockout Probability

A *stockout* occurs when demand arrives and there is no inventory available to satisfy that demand immediately. A stockout is not the same as being *out of stock,* which is the condition of having no inventory on hand. With our definition of a stockout, we must be out of stock *and* a demand must occur. Thus, if we are out of stock and no demand occurs, then a stockout never happened. We are *in stock* in a period if all demand was satisfied in that period. With this definition, if we start a period with five units and demand is five units, then we are in stock in that period even though we end the period without inventory.

The *in-stock probability* is the probability we are in stock in a period, and the *stockout probability* is the probability a stockout occurs. We used these same definitions in the newsvendor model, Chapter 12. As in the newsvendor model, an alternative measure is the fill rate, which is the probability a customer will be able to purchase an item. See Appendix D for the procedure to evaluate the fill rate in the order-up-to model.

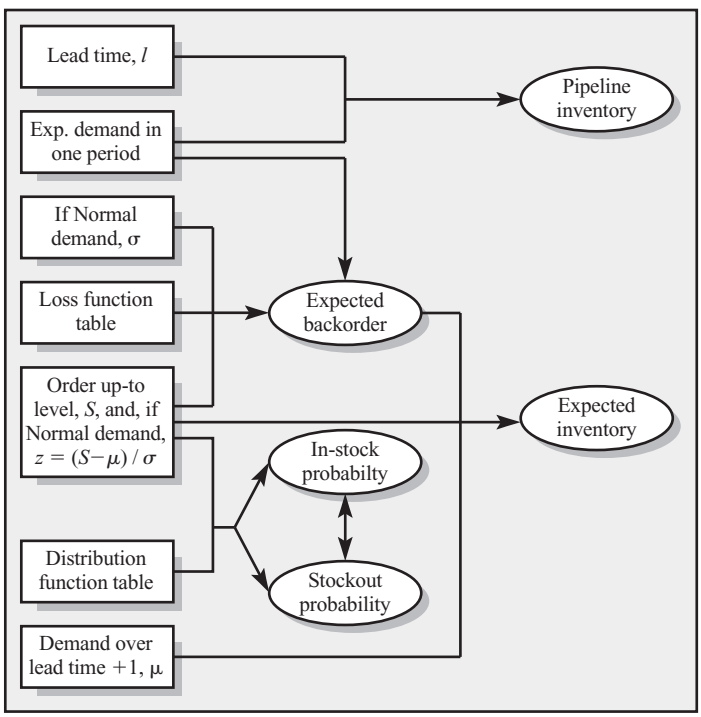


FIGURE 14.6
The Relationship between Inputs (boxes) and Performance Measures (ovals) in the Order-up-to Model
μ = Expected demand over $l + 1$ periods and σ = Standard deviation of demand over $l + 1$ periods.

A stockout causes a back order. Hence, a stockout occurs in a period if one or more units are back-ordered at the end of the period. If there are back orders at the end of the period, then the inventory level at the end of the period is negative. The main result from Section 14.3 is that the inventory level is related to the order-up-to level and demand over $l + 1$ periods in the following way:

$$\text{Inventory level at the end of the period} = S - \text{Demand over } l + 1 \text{ periods}$$

Therefore, the inventory level at the end of the period is negative if demand over $l + 1$ periods exceeds the order-up-to level. Therefore,

$$\begin{aligned}\text{Stockout probability} &= \text{Prob}\{\text{Demand over } l + 1 \text{ periods} > S\} \\ &= 1 - \text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq S\}\end{aligned} \qquad \textbf{(14.1)}$$

Equation (14.1) is actually an approximation of the stockout probability, but it happens to be an excellent approximation if the chosen service level is high (i.e., if stockouts are rare). See Appendix D for why equation (14.1) is an approximation and for the exact, but more complicated, stockout probability equation.

Because either all demand is satisfied immediately from inventory or not, we know that the

$$\text{In-stock probability} = 1 - \text{Stockout probability}$$

Combining the above equation with equation (14.1), we get

$$\begin{aligned}\text{In-stock probability} &= 1 - \text{Stockout probability} \\ &= \text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq S\}\end{aligned}$$

The above probability equations do not depend on which distribution has been chosen to represent demand, but the process for evaluating those probabilities does depend on the particular demand distribution.

When the demand distribution is given in the form of a table, as with the Poisson distribution, then we can obtain the in-stock probability directly from the table. Looking at Table 14.1, for Susan's territory with an order-up-to level $S = 3$,

$$\begin{aligned}\text{In-stock probability} &= \text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq 3\} \\ &= 99.702\% \\ \text{Stockout probability} &= 1 - \text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq 3\} \\ &= 1 - 0.99702 \\ &= 0.298\%\end{aligned}$$

For the Mounds View distribution center, we need to work with the normal distribution. Recall that with the normal distribution you first do the analysis as if demand is a standard normal distribution and then you convert those outcomes into the answers for the actual normal distribution.

Note that the process for evaluating the in-stock and stockout probabilities in the order-up-to model, which is summarized in Exhibit 14.2, is identical to the one described in Table 12.4 for the newsvendor model except the order quantity Q is replaced with the order-up-to level S. However, it is critical to use the demand forecast for $l + 1$ periods, not the demand forecast for a single period (unless the lead time happens to be 0).

First, we normalize the order-up-to level, which is $S = 625$, using the parameters for demand over $l + 1$ periods:

$$z = \frac{S - \mu}{\sigma} = \frac{625 - 322.4}{117.6} = 2.57$$

Exhibit 14.2

IN-STOCK PROBABILITY AND STOCKOUT PROBABILITY EVALUATION IN THE ORDER-UP-TO MODEL

If the demand over $l + 1$ periods is a normal distribution with mean μ and standard deviation σ, then follow steps A through D (see Exhibit 14.1 for the process of evaluating μ and σ if you have demand over a single period):

A. Evaluate the z-statistic for the order-up-to level: $z = \dfrac{S - \mu}{\sigma}$.

B. Use the z-statistic to look up in the Standard Normal Distribution Function Table the probability the standard normal demand is z or lower, $\Phi(z)$.

C. In-stock probability $= \Phi(z)$ and Stockout probability $= 1 - \Phi(z)$.

D. In Excel, In-stock probability = Normsdist(z) and Stockout probability = 1 − Normsdist(z).

If the demand over $l + 1$ periods is a discrete distribution function, then In-stock probability $= F(S)$ and Stockout probability $= 1 - F(S)$, where $F(S)$ is the probability demand over $l + 1$ periods is S or lower.

Next, we look up $\Phi(z)$ (the probability the outcome of a standard normal is less than or equal to z) in the Standard Normal Distribution Function Table in Appendix B: $\Phi(2.57) = 0.9949$. Therefore, with $S = 625$, the in-stock probability for the DC is 99.49 percent. The stockout probability is $1 - \Phi(z) = 0.0051$, or 0.51 percent.

Expected Back Order

The *expected back order* is the expected number of back orders at the end of any period. We need the expected back order to evaluate the expected on-hand inventory, which is of direct interest to any manager.

Recall from Section 14.3 that the inventory level at the end of the period is S minus demand over $l + 1$ periods. Hence, if demand over $l + 1$ periods is greater than S, then there will be back orders. The number of back orders equals the difference between demand over $l + 1$ periods and S. Therefore, in the order-up-to model, the expected back order equals the loss function of demand over $l + 1$ periods evaluated at the threshold S. *Note:* This is analogous to the expected lost sales in the newsvendor model. In the order-up-to model, the number of units back-ordered equals the difference between random demand over $l + 1$ periods and S; in the newsvendor model, the expected lost sales are the difference between random demand and Q. So all we need to evaluate the expected back order is the loss function of demand over $l + 1$ periods.

Let's begin with the expected back order in Susan's territory. Recall that with a discrete distribution function table, we need to have a column that has the loss function $L(S)$. Table 14.2 displays the loss function we need. (Appendix C describes how to use the data

TABLE 14.2
Distribution and Loss Function for Two Poisson Distributions

Mean Demand = 0.29			Mean Demand = 0.58		
S	$F(S)$	$L(S)$	S	$F(S)$	$L(S)$
0	0.74826	0.29000	0	0.55990	0.58000
1	0.96526	0.03826	1	0.88464	0.13990
2	0.99672	0.00352	2	0.97881	0.02454
3	0.99977	0.00025	3	0.99702	0.00335
4	0.99999	0.00001	4	0.99966	0.00037
5	1.00000	0.00000	5	0.99997	0.00004

$F(S)$ = Prob{Demand is less than or equal to S}
$L(S)$ = Loss function = Expected back order = Expected amount demand exceeds S

Exhibit 14.3

EXPECTED BACK ORDER EVALUATION FOR THE ORDER-UP-TO MODEL

If the demand over $l + 1$ periods is a normal distribution with mean μ and standard deviation σ, then follow steps A through D (see Exhibit 14.1 for the process of evaluating μ and σ if you have demand over a single period):

A. Evaluate the z-statistic for the order-up-to level S: $z = \dfrac{S - \mu}{\sigma}$.

B. Use the z-statistic to look up in the Standard Normal Loss Function Table the expected loss with the standard normal distribution, $L(z)$.

C. Expected back order $= \sigma \times L(z)$.

D. With Excel, expected back order can be evaluated with the following equation:

$$\text{Expected back order} = \sigma^* (\text{Normdist}(z, 0, 1, 0) - z^*(1 - \text{Normsdist}(z)))$$

If the demand forecast for $l + 1$ periods is a discrete distribution function table, then expected back order equals $L(S)$, where $L(S)$ is the loss function. If the table does not include the loss function, then see Appendix C for a procedure to evaluate it.

in Table 14.1 to evaluate $L(S)$.) Appendix B has the loss function table for other Poisson distributions. With $S = 3$ and mean demand over $l + 1$ periods equal to 0.58, we see that $L(3) = 0.00335$. Therefore, the expected back order in Susan's territory is 0.00335 unit if she operates with $S = 3$.

With the Mounds View DC, we follow the process of evaluating expected lost sales with a normal distribution. (See Exhibit 12.4.) First, find the z-statistic that corresponds to the order-up-to level:

$$z = \frac{S - \mu}{\sigma} = \frac{625 - 322.4}{117.6} = 2.57$$

Note again that we are using the mean and standard deviation of the normal distribution that represents demand over $l + 1$ periods. Now look up in the Standard Normal Distribution Loss Function Table the loss function with the standard normal distribution and a z-statistic of 2.57: $L(2.57) = 0.0016$. Next, convert that expected loss with the standard normal distribution into the expected back order:

$$\text{Expected back order} = \sigma \times L(z) = 117.6 \times 0.0016 = 0.19$$

Exhibit 14.3 summarizes the process.

Expected On-Hand Inventory

Expected on-hand inventory, or just *expected inventory* for short, is the expected number of units of inventory at the end of a period. We choose to measure inventory at the end of the period because that is when inventory is at its lowest point in the period.

Recall that the inventory level at the end of a period is equal to the order-up-to level S minus demand over $l + 1$ periods. Hence, inventory at the end of a period is the difference between S and demand over $l + 1$ periods: if $S = 5$ and demand over $l + 1$ periods is three, then there are two units left in inventory. In other words, expected inventory is the expected amount by which S exceeds demand over $l + 1$ periods. Referring to the insights

Exhibit 14.4

EVALUATION OF EXPECTED ON-HAND INVENTORY, AND PIPELINE/EXPECTED ON-ORDER INVENTORY IN THE ORDER-UP-TO MODEL

For the expected on-hand inventory:

A. Evaluate the expected back order (see Exhibit 14.3).
B. Expected on-hand inventory = S − Expected demand over $l + 1$ periods + Expected back order.

See Exhibit 14.1 for how to evaluate *expected demand over* $l + 1$ periods.

For the pipeline inventory (which is also known as expected on-order inventory):

Expected on-order inventory = Expected demand in one period × Lead time

from the newsvendor model, if we think of S in terms of the order quantity and demand over $l + 1$ periods in terms of "sales," then inventory is analogous to "leftover inventory." Recall that in the newsvendor model

$$\begin{aligned}\text{Expected leftover inventory} &= Q - \text{Expected sales} \\ &= Q - \mu + \text{Expected lost sales}\end{aligned}$$

As a result, in the order-up-to model

$$\text{Expected inventory} = S - \text{Expected demand over } l + 1 \text{ periods} + \text{Expected back order}$$

In Susan's territory with $S = 3$, the expected inventory is $3 - 0.58 + 0.00335 = 2.42$. At the Mounds View DC with $S = 625$, the expected inventory is $625 - 322.4 + 0.19 = 302.8$. Exhibit 14.4 summarizes the process.

Pipeline Inventory/Expected On-Order Inventory

Pipeline inventory, which also will be called *expected on-order inventory,* is the average amount of inventory on order at any given time. It is relevant because Medtronic owns the inventory between the Mounds View distribution center and Susan Magnotto's territory. To evaluate pipeline inventory, we refer to Little's Law, described in Chapter 2,

$$\text{Inventory} = \text{Flow rate} \times \text{Flow time}$$

Now let's translate the terms in the Little's Law equation into the comparable terms in this setting: inventory is the number of units on order; flow rate is the expected demand in one period (the expected order in a period equals expected demand in one period, so on-order inventory is being created at a rate equal to expected demand in one period); and flow time is the lead time, since every unit spends l periods on order. Therefore,

$$\text{Expected on-order inventory} = \text{Expected demand in one period} \times \text{Lead time}$$

In the case of the InSync pacemaker, Susan's territory has $0.29 \times 1 = 0.29$ unit on order on average and the Mounds View DC has $80.6 \times 3 = 241.8$ units on order. Exhibit 14.4 summarizes the process.

The expected on-order inventory is based on demand over l periods of time, and not $l + 1$ periods of time. Furthermore, the above equation for the expected on-order inventory holds for any demand distribution because Little's Law depends only on average rates, and not on the variability of those rates.

14.6 Choosing an Order-up-to Level to Meet a Service Target

This section discusses the actual choice of InSync order-up-to levels for Susan Magnotto's territory and the Mounds View DC. To refer to a previously mentioned analogy, the order-up-to level is somewhat like the point in the fuel gauge of your car at which you decide to head to a refueling station. The more you are willing to let the dial fall below the "E," the higher the chance you will run out of fuel. However, while increasing that trigger point in the fuel gauge makes you feel safer, it also increases the average amount of fuel you drive around with. With that trade-off in mind, this section considers choosing an order-up-to level to minimize inventory while achieving an in-stock probability no lower than an in-stock target level. This objective is equivalent to minimizing inventory while yielding a stockout probability no greater than one minus the in-stock target level.

Given Medtronic's large gross margin, let's say we want the in-stock probability to be at least 99.9 percent for the InSync pacemaker in Susan's territory as well as at the Mounds View DC. With a 99.9 percent in-stock probability, a stockout should occur no more than 1 in 1,000 days on average. Section 14.7 discusses whether we have chosen a reasonable target.

From Section 14.5 we know that the in-stock probability is the probability demand over $l + 1$ periods is S or lower. Hence, when demand is modeled with a discrete distribution function, we find the appropriate order-up-to level by looking directly into that table. From Table 14.2, we see that in Susan's territory, $S = 0$ clearly does not meet our objective with an in-stock probability of about 56 percent, that is, $F(0) = 0.5599$. Neither is $S = 3$ sufficient because it has an in-stock probability of about 99.7 percent. However, with $S = 4$ our target is met: the in-stock probability is 99.97 percent. In fact, $S = 4$ exceeds our target by a considerable amount: that translates into one stockout every $1/0.00034 = 2{,}941$ days, or one stockout every 11.31 years, if we assume 260 days per year.

With the Mounds View DC, we must work with the normal distribution. We first find the order-up-to level that meets our in-stock probability service requirement with the standard normal distribution and then convert that standard normal order-up-to level to the order-up-to level that corresponds to the actual demand distribution. In the Standard Normal Distribution Function Table, we see that $\Phi(3.08) = 0.9990$, so an order-up-to level of 3.08 would generate our desired in-stock probability if demand over $l + 1$ periods followed a standard normal. It remains to convert that z-statistic into an order-up-to level: $S = \mu + z \times \sigma$. Remember that the mean and standard deviation should be from the normal distribution of demand over $l + 1$ periods. Therefore,

$$S = 322.4 + 3.08 \times 117.62 = 685$$

See Exhibit 14.5 for a summary of the process to choose an order-up-to level to achieve a target in-stock probability.

14.7 Choosing an Appropriate Service Level

So far in our discussion, we have chosen high service levels because we suspect that a high service level is appropriate. This section puts more rigor behind our hunch. For the sake of brevity, we'll explicitly consider only the management of field inventory. At the end of the section, we briefly discuss the management of distribution center inventory.

The appropriate service level minimizes the cost of holding inventory plus the cost of poor service. The holding cost of inventory is usually expressed as a *holding cost rate,* which is the cost of holding one unit in inventory for one year, expressed as a percentage of the item's cost. For example, if a firm assigns its holding cost rate to be 20 percent, then it

Exhibit 14.5

HOW TO CHOOSE AN ORDER-UP-TO LEVEL *S* TO ACHIEVE AN IN-STOCK PROBABILITY TARGET IN THE ORDER-UP-TO MODEL

If the demand over $l + 1$ periods is a normal distribution with mean μ and standard deviation σ, then follow steps A and B (see Exhibit 14.1 for the process of evaluating μ and σ if you have demand over a single period):

A. In the Standard Normal Distribution Function Table, find the probability that corresponds to the target in-stock probability. Then find the *z*-statistic that corresponds to that probability. If the target in-stock probability falls between two entries in the table, choose the entry with the larger *z*-statistic.

   In Excel the appropriate *z*-statistic can be found with the following equation:

$$z = \text{Normsinv(Target in-stock probability)}$$

B. Convert the *z*-statistic chosen in part A to an order-up-to level: $S = \mu + z \times \sigma$. Recall that you are using the mean and standard deviation of demand over $l + 1$ periods.

If the demand forecast for $l + 1$ periods is a discrete distribution function table, then find the S in the table such that $F(S)$ equals the target in-stock probability, where $F(S)$ is the probability demand is less than or equal to S over $l + 1$ periods. If the target in-stock probability falls between two entries in the table, choose the larger S.

believes the cost of holding a unit in inventory for one year equals 20 percent of the item's cost. The holding cost includes the opportunity cost of capital, the cost of spoilage, obsolescence, insurance, storage, and so forth, all variable costs associated with holding inventory. Because Medtronic is a growing company, with a high internal opportunity cost of capital, let's say their holding cost rate is 35 percent for field inventory. We'll use the variable h to represent the holding cost. See Chapter 2 for additional discussion on the holding cost rate.

If we assume the InSync pacemaker has a 75 percent gross margin, then the cost of an InSync pacemaker is $(1 - 0.75) \times \text{Price} = 0.25 \times \text{Price}$, where Price is the selling price.[2] Therefore, the annual holding cost is $0.35 \times 0.25 \times \text{Price} = 0.0875 \times \text{Price}$ and the daily holding cost, assuming 260 days per year, is $0.875 \times \text{Price}/260 = 0.000337 \times \text{Price}$.

The cost of poor service requires some thought. We first need to decide how we will measure poor service and then decide on a cost for poor service. In the order-up-to model, a natural measure of poor service is the occurrence of a back order. Therefore, we say that we incur a cost for each unit back-ordered and we'll let the variable b represent that cost. We'll also refer to the variable b as the *back-order penalty cost.* Now we must decide on an appropriate value for b. A natural focal point with field inventory (i.e., inventory for serving final customers) is to assume each back order causes a lost sale and the cost of a lost sale equals the product's gross margin. However, if you believe there are substantial long-run implications of a lost sale (e.g., the customer will switch his or her future business to a competitor), then maybe the cost of a lost sale is even higher than the gross margin. On the other hand, if customers are somewhat patient, that is, a back order does not automatically lead to a lost sale, then maybe the cost of a back order is lower than the gross margin. In the case of Medtronic, the former story is more likely. Let's suppose each back order leads to a lost sale and, to be conservative, the cost of a back order is just the gross margin; that is, $b = 0.75 \times \text{Price}$.

Now let's minimize Medtronic's holding and back-order costs. The holding cost in a period is h times the number of units in inventory (which we measure at the end of the

[2] Medtronic's gross margin across all products, as reported on their income statement, is approximately 80 percent. Because there are competing products, we assume the actual gross margin of the InSync is slightly lower than this average.

period). The back-order cost in a period is b times the number of units back-ordered.[3] As a result, we face the "too little–too much" challenge: Choose S too high and incur excessive inventory holding costs; but if S is too low, then we incur excessive back-order costs. We can actually use the newsvendor logic to strike the correct balance.

Our overage cost is $C_o = h$: the consequence of setting S too high is inventory and the cost per unit of inventory per period is h. Our underage cost is $C_u = b$: back orders are the consequence of setting S too low and the cost per back order is b. In the newsvendor model, we chose an order quantity Q such that the critical ratio equals the probability demand is Q or lower, which is the same as the probability that a stockout does not occur. In the order-up-to model, the probability a stockout does not occur in a period is

$$\text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq S\}$$

Hence, the order-up-to level that minimizes costs in a period satisfies the following newsvendor equation:

$$\text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq S\} = \frac{C_u}{C_o + C_u} = \frac{b}{h + b} \qquad \textbf{(14.2)}$$

For Medtronic, the critical ratio is

$$\frac{b}{h + b} = \frac{(0.75 \times \text{Price})}{(0.00037 \times \text{Price}) + (0.75 \times \text{Price})} = 0.9996$$

Notice the following with respect to equation (14.2):

- We do not need to know the product's actual price, Price, because it cancels out of both the numerator and the denominator of the critical ratio.
- It is important that we use the holding cost per unit per period to evaluate the critical ratio because the order-up-to level determines the expected inventory in a period. In other words, h should be the holding cost for a single unit for a single period.

Now we are ready to justify our service level based on costs. Recall that

$$\text{In-stock probability} = \text{Prob}\{\text{Demand over } l + 1 \text{ periods} \leq S\}$$

If we combine the above equation with equation (14.2), then the in-stock probability that is consistent with cost minimization is

$$\text{In-stock probability} = \text{Critical ratio} = \frac{b}{h + b} \qquad \textbf{(14.3)}$$

In other words, the appropriate in-stock probability equals the critical ratio. Recall that we chose 99.9 percent as our target in-stock probability. Even though that might seem high, our calculations above suggest that an in-stock probability of up to 99.96 percent is consistent with cost minimization.

[3] If you have been reading carefully, you might realize that this is not entirely correct. The back-order cost in a period is b times the number of demands *in that period* that are back-ordered, that is, we do not incur the cost b per unit that became back-ordered in a previous period and still is on back order. However, with a high in-stock probability, it should be the case that units are rarely back-ordered, and if they are back-ordered, then they are back-ordered for no more than one period. Hence, with a high in-stock probability, assuming the back-order cost is b times the number of units back-ordered is an excellent approximation.

TABLE 14.3
The Optimal Target In-Stock Probability for Various Gross Margins
The annual holding cost rate is 35 percent, the back order penalty cost equals the gross margin, and inventory is reviewed daily.

Gross Margin	Optimal Target In-Stock Probability	Gross Margin	Optimal Target In-Stock Probability
1%	88.24%	35%	99.75%
2	93.81	57	99.90
3	95.83	73	99.95
4	96.87	77	99.96
6	97.93	82	99.97
12	99.02	87	99.98
21	99.50	93	99.99

Holding inventory is not cheap for Medtronic (35 percent holding cost rate), but due to Medtronic's large gross margins, the underage cost ($0.75 \times$ Price) is still about 2,200 times greater than the overage cost ($0.000337 \times$ Price)! With such a lopsided allocation of costs, it is no surprise that the appropriate in-stock probability is so high.

Table 14.3 indicates for various gross margins the optimal target in-stock probability. We can see that an obscene gross margin is needed (93 percent) to justify a 99.99 percent in-stock probability, but a modest gross margin (12 percent) is needed to justify a 99 percent in-stock probability.

Now consider the appropriate service level at the distribution center. While the opportunity cost of capital remains the same whether it is tied up in inventory in the field or at the distribution center, all other inventory holding costs are likely to be lower at the distribution center (e.g., physical space, theft, spoilage, insurance, etc.). But even with a lower holding cost, the appropriate service level at the distribution center is unlikely to be as high as it is in the field because the distribution center's back-order cost should be lower. Why? A back order in the field is likely to lead to a lost sale, but a back order at the distribution center does not necessarily lead to a lost sale. Each field representative has a buffer of inventory and that buffer might prevent a lost sale as long as the back order at the distribution center does not persist for too long. This is not to suggest that the appropriate in-stock probability at the distribution center is low. Rather, it suggests that the appropriate in-stock probability might not be 99.9 percent.[4]

The main insight from this section is that the optimal target in-stock probability in the order-up-to model is likely to be quite high (99 percent and above), even with a relatively modest gross margin and high annual holding cost rate. However, that result depends on two key assumptions: back orders lead to lost sales and inventory does not become obsolete. The latter assumption highlights a connection and a useful contrast between the order-up-to model and the newsvendor model. In the newsvendor model, obsolescence is the primary concern; that is, demand is not expected to continue into the future, so leftover inventory is expensive. As a result, optimal service levels in the newsvendor model are rarely as high as in the order-up-to model. Furthermore, the appropriate model to employ depends on where a *product* is in its *life cycle.* Up to and including the mature stage of a product's life cycle, the order-up-to model is more appropriate. As a product's end of life approaches, the newsvendor model is needed. Some products have very long life cycles—for example, chicken noodle soup—so the newsvendor model is never needed. Others have very short life cycles—for example, O'Neill's Hammer 3/2—so a firm is relegated to the newsvendor model almost immediately. It is the products with an intermediate life

[4] Evaluation of the appropriate in-stock probability for the distribution center is beyond the scope of this discussion. However, simulation can be a useful tool to begin to understand the true back-order cost at the distribution center. Via simulation it is possible to estimate the likelihood that a back order at the distribution center causes a lost sale in the field.

cycle (one to two years)—for example, the InSync pacemaker—that can be very tricky to manage. A firm should start thinking in terms of the order-up-to model and then switch to the newsvendor model shortly before the product dies. Many firms botch this "end-of-life" transition: by holding on to high service levels too long, they find themselves with far too much inventory when the product becomes obsolete.

14.8 Controlling Ordering Costs

In our analysis of Medtronic's supply chain, the focus has been on the service level (the in-stock probability) and the expected amount of inventory on hand at the end of each period. Although we have not addressed the issue of *order frequency* (i.e., how many shipments are made each year to the DC or to each sales territory), there are other settings for which it is important to control the order frequency. For example, most online book shoppers realize, due to how online retailers charge for shipping, that five separate orders with one book in each order is generally more expensive than one book order containing the same five books. In other words, when there is a significant cost incurred with each order that is independent of the amount ordered (i.e., a fixed cost), it is necessary to be smart about how often orders are made. The focus of this section is on how we can account for fixed ordering costs in the order-up-to model.

As we have already seen, in the order-up-to model, the order quantity in a period equals the demand in the previous period. Hence, an order is submitted in a period whenever demand in the previous period is not zero. Therefore, the probability we submit an order in a period is $1 - \text{Prob}\{\text{Demand in one period} = 0\}$ and the frequency at which we submit orders is

$$\frac{1 - \text{Prob}\{\text{Demand in one period} = 0\}}{\text{Length of period}}$$

For example, if there is a 90 percent probability we order in a period and a period is two weeks, then our order frequency is 0.9/2 weeks = 0.45 order per week. If demand occurs frequently, so the probability of zero demand is very small no matter the length of the period, then it follows that we can reduce our ordering frequency by increasing the length of our period; that is, we are likely to submit nearly twice as many orders with a one-week period than with a two-week period. But increasing the length of the period is costly from the perspective of inventory holding costs. We illustrate that point via an example.

Suppose all orders are received precisely eight weeks after they are submitted to a supplier, weekly demand is normally distributed with mean 100 and standard deviation 75, the target in-stock probability is 99.25 percent, and demands across weeks are independent. We can choose a period length of one, two, four, or eight weeks. If the period is one week, then the lead time is eight periods, whereas if the period length is four weeks, then the lead time is two periods. Using the methods developed in the previous sections, we can determine the end-of-period average inventory for each period length. Those results are summarized in Table 14.4. The table reveals that our end-of-period inventory is indeed higher as we lengthen the period. But that is not really a fair comparison across our different options.

As we have already stated, the average order quantity equals average demand in the previous period. Thus, our average order quantity with a period length of one week is 100 units, whereas our average order quantity with an eight-week period is 800 units. Figure 14.7 plots the average inventory level over time for our four options; on average, inventory increases at the start of the period by the average order quantity and then decreases at the rate of 100 units per week, that is, average inventory follows a "saw-toothed" pattern. (Due to randomness in demand, the actual inventory pattern varies around those patterns, but those saw-toothed

TABLE 14.4
Analysis of Ending Inventory for Different Period Lengths
In each case, the delivery time is eight weeks and demand is normally distributed and independent across weeks.

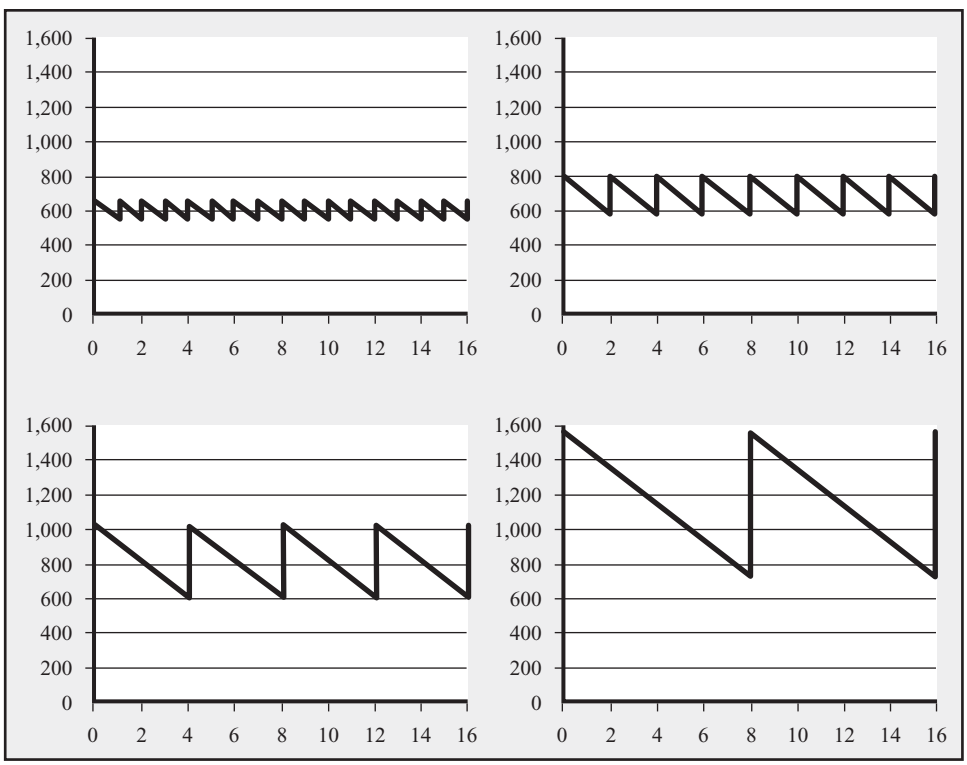
	Period Length (in weeks)			
	1	**2**	**4**	**8**
One period expected demand	100	200	400	800
One period standard deviation	75.0	106.1	150.0	212.1
Lead time (in periods)	8	4	2	1
Target in-stock probability	99.25%	99.25%	99.25%	99.25%
z	2.43	2.43	2.43	2.43
S	1,447	1,576	1,831	2,329
Average back order	0.56	0.59	0.65	0.75
Average ending inventory	548	577	632	730

patterns capture the average behavior of inventory.) The average inventory over time is the average end-of-period inventory plus half of the average order quantity, which for our four options is 598, 677, 832, and 1,130 respectively. Hence, longer periods mean less-frequent ordering but more inventory.

Incidentally, you may recall that the graphs in Figure 14.7 resemble Figure 2.11 in Chapter 2. Back in Chapter 2 we used the term *cycle inventory* to refer to the inventory held due to lumpy ordering. In this case, the average cycle inventory would be half of the average order quantity: with four-week periods, the average cycle inventory is 400/2 = 200 units. The average end-of-period inventory is often referred to as *safety inventory* because that is the inventory that is needed to buffer demand variability. The average inventory over time is then safety inventory plus cycle inventory.

To balance the cost of more inventory with the benefit of fewer orders, we need information about holding and ordering costs. Let's say this item costs $50, annual holding costs are

FIGURE 14.7
Average Inventory Pattern over Time for Four Different Period Lengths
Upper left, one week; upper right, two weeks; lower left, four weeks; and lower right, eight weeks.



25 percent, and we incur a fixed cost of \$275 per shipment (e.g., we could be talking about a truck delivery). If the period length is one week, then the average inventory is 598 units, which has value $598 \times \$50 = \$29{,}900$ and costs us $25\% \times \$29{,}900 = \$7{,}475$ per year. With mean demand of 100 and a standard deviation of 75, the z-statistic for 0 is $(0 - 100)/75 = -1.33$. Hence, the probability we order in any given week is $1 - \Phi(-1.33) = 0.91$.[5] With 52 weeks per year, we can expect to make $0.91 \times 52 = 47.32$ orders per year for a total ordering cost of $47.32 \times \$275 = \$13{,}013$. Total cost is then $\$7{,}475 + \$13{,}013 = \$20{,}488$. Repeating those calculations for the remaining three period-length options reveals their annual costs to be \$15,398, \$13,975, and \$15,913. Figure 14.8 plots those costs as well as the inventory holding and ordering costs of the four options.

Figure 14.8 reveals that our best option is to set the period length to four weeks (which implies the lead time is then two periods). A shorter period length results in too many orders so the extra ordering costs dominate the reduced holding costs. A longer period suffers from too much inventory.

Although this analysis has been done in the context of the order-up-to model, it may very well remind you of another model, the *Economic Order Quantity (EOQ)* model discussed in Chapter 7. Recall that in the EOQ model there is a fixed cost per order/batch K and a holding cost per unit per unit of time h and demand occurs at a constant flow rate R; in this case, $R = 100$ per week or $R = 5{,}200$ per year. The key difference between our model and the EOQ model is that here we have random demand whereas the EOQ model assumes demand occurs at a constant rate. Nevertheless, it is interesting to evaluate the EOQ model in this setting. We already know that the fixed ordering cost is K \$275. The holding cost per unit per year is $25\% \times \$50 = \$12.5$. So the EOQ quantity (see Chapter 7) is

$$Q = \sqrt{\frac{2 \times K \times R}{h}} = \sqrt{\frac{2 \times 275 \times 5200}{12.5}} = 478$$

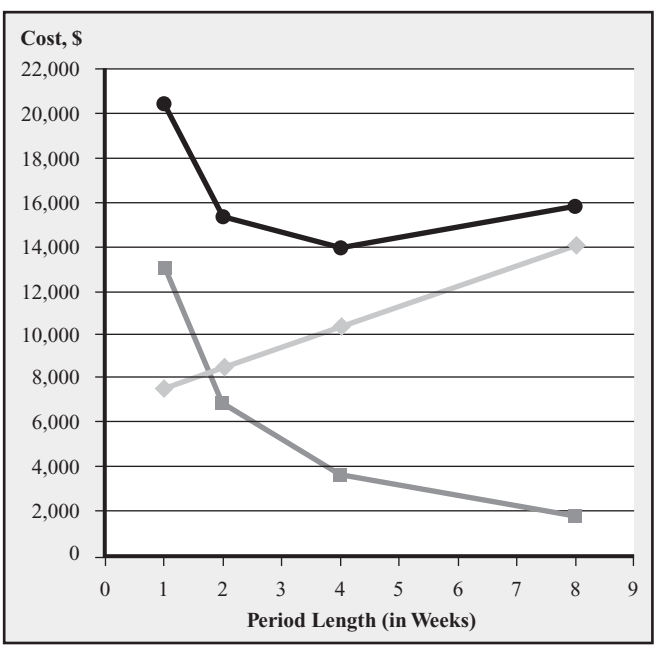


FIGURE 14.8
Annual Ordering Costs (squares), Inventory Costs (diamonds), and Total Costs (circles) for Periods of Length One, Two, Four, and Eight Weeks

[5] We actually just evaluated the probability that demand is *less* than or equal to zero because the normal distribution allows for negative demand. We are implicitly assuming that all negative realizations of demand are really zero demand outcomes.

(Note that we need to use the yearly flow rate because the holding cost is per unit per year.) Hence, the EOQ model suggests that each order should be for 478 units, which implies submitting an order every 478/100 = 4.78 weeks. (This follows from Little's Law.) Hence, even though the order-up-to and the EOQ models are different, the EOQ model's recommendation is quite similar (order every 4.78 weeks versus order every 4 weeks). Although we have only demonstrated this for one example, it can be shown that the EOQ model generally gives a very good recommendation for the period length (note that the EOQ actually recommends an order quantity that can then be converted to a period length).

One limitation of our order-up-to model is that the lead time must equal an integer number of periods. In our example, because the delivery time is eight weeks, this allows us to choose period lengths of one, two, four, or eight, but we cannot choose a period length of 3 or 5 or 4.78 weeks (because with a period length of 3 weeks the lead time is 2.67 periods, i.e., deliveries would be received two-thirds of the way into a period instead of at the beginning of the period). If the delivery time were three weeks, then we would be even more restricted in our period length options. Fortunately, the order-up-to model can be extended to handle situations in which the lead time is a fraction of the period length. But that extension is beyond the scope of this text, and, rest assured, the qualitative insights from our model carry over to that more complex setting.

So we have shown that we can adjust our period length in the order-up-to model to control our ordering costs. Furthermore, the average order quantity with the optimal period length will approximately equal the EOQ quantity. (Hence, the EOQ formula gives us an easy way to check if our period length is reasonable.) One advantage of this approach is that we submit orders on a regular schedule. This is a useful feature if we need to coordinate the orders across multiple items. For example, since we incur a fixed cost per truck shipment, we generally deliver many different products on each truck because no single product's demand is large enough to fill a truck (imagine sending a tractor trailer load of spices to a grocery store). In that situation, it is quite useful to order items at the same time so that the truck can be loaded quickly and we can ensure a reasonably full shipment (given that there is a fixed cost per shipment, it makes sense to utilize the cargo capacity as much as possible). Therefore, we need only ensure that the order times of different products align.

Instead of using fixed order intervals, as in the order-up-to model, we could control ordering costs by imposing a minimum order quantity. For example, we could wait for Q units of demand to occur and then order exactly Q units. With such a policy, we would order on average every Q/R units of time, but due to the randomness in demand, the time between orders would vary. Not surprisingly, the EOQ quantity provides an excellent recommendation for that minimum order quantity, but we omit the analytical details as they are beyond the scope of this text. The important insight from this discussion is that it is possible to control ordering costs by restricting ourselves to a periodic schedule of orders (as in the order-up-to model) or we could restrict ourselves to a minimum order quantity. With the first option, there is little variability in the timing of orders, which facilitates the coordination of orders across multiple items, but the order quantities are variable (which may increase handling costs). With the second option, the order quantities are not variable (we always order Q), but the timing of those orders varies.

14.9 Managerial Insights

This section discusses general managerial insights from the order-up-to model.

One of the key lessons from the queuing and newsvendor chapters is that variability in demand is costly. (Recall that the mismatch cost in the newsvendor model is increasing with the coefficient of variation, which is the ratio of the standard deviation of demand to expected

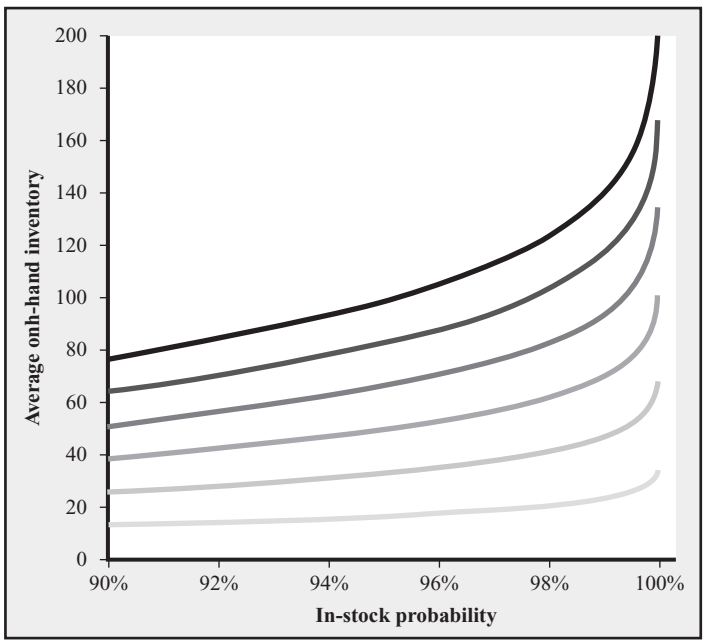


FIGURE 14.9
The Trade-off between Inventory and In-Stock with Normally Distributed Demand and Mean 100 over $l + 1$ Periods
The curves differ in the standard deviation of demand over $l + 1$ periods: 60, 50, 40, 30, 20,10 from top to bottom.

demand.) That result continues to hold in the order-up-to model. Figure 14.9 illustrates the result graphically. The figure presents the trade-off curve between the in-stock probability and expected inventory: as the desired in-stock probability increases, so does the required amount of inventory. Furthermore, we see that for any given in-stock probability, the expected inventory increases in the standard deviation of demand over $l + 1$ periods: increased variability means more inventory is needed on average to achieve a fixed service level.

In addition to the variability in demand, the expected inventory in the order-up-to model is sensitive to the lead time, as illustrated by Figure 14.10: as the lead time is reduced, so is the required inventory for any service target.

While expected inventory depends on the variability of demand and the lead time, the expected on-order inventory, or pipeline inventory, depends only on the lead time. Therefore, while reducing the uncertainty in demand reduces expected inventory, pipeline

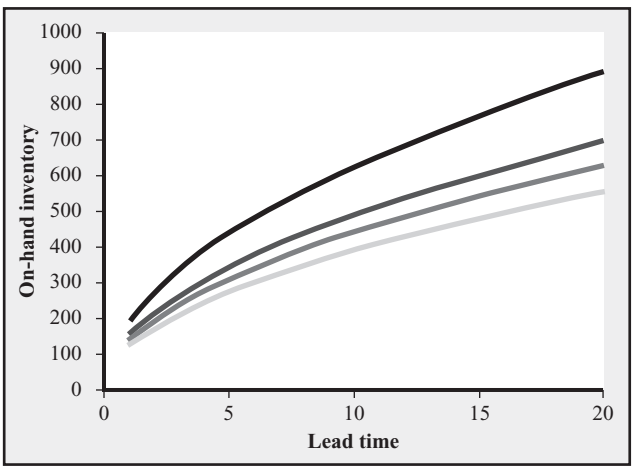


FIGURE 14.10
The Impact of Lead Time on Expected Inventory for Four In-Stock Targets
In-Stock targets are 99.9, 99.5, 99.0, and 98 percent, top curve to bottom curve, respectively. Demand in one period is normally distributed with mean 100 and standard deviation 60.

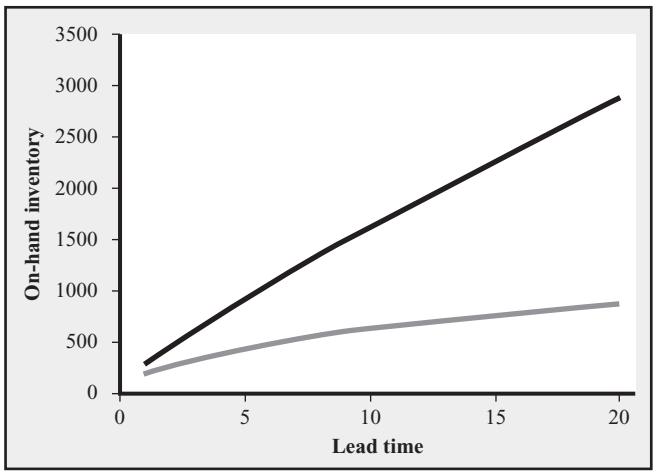


FIGURE 14.11
Expected Inventory (circles) and Total Inventory (squares), Which Is Expected Inventory Plus Pipeline Inventory, with a 99.9 Percent In-Stock Requirement
Demand in one period is normally distributed with mean 100 and standard deviation 60.

inventory can only be reduced with a faster lead time. (Actually, reducing demand also reduces pipeline inventory, but that is rarely an attractive option, and reducing demand does not even reduce pipeline inventory when it is measured relative to the demand rate, e.g., with inventory turns or days of demand.) Furthermore, the amount of pipeline inventory can be considerable, especially for long lead times, as demonstrated in Figure 14.11, where the distance between the two curves is the pipeline inventory, which is clearly growing as the lead time increases.

14.10 Summary

This chapter illustrates the application of the order-up-to model to one product, the InSync pacemaker, at two different levels in Medtronic's supply chain: the Mounds View distribution center and Susan Magnotto's Madison, Wisconsin, territory. The order-up-to model periodically reviews (weekly at Mounds View, daily for Susan) the inventory position at a location and submits an order, which is received after a fixed lead time, to raise the inventory position to an order-up-to level. The order-up-to level is chosen, based on the demand distribution, to minimize inventory while maintaining a service standard such as an in-stock probability.

The analysis of the order-up-to model reveals that raising the desired service level increases the required inventory investment and the amount of inventory needed increases nonlinearly as the target service level increases. In other words, as high service levels are desired, proportionally more inventory is needed.

There are two other key factors that determine the amount of inventory that is needed: the variability of demand, measured by the coefficient of variation, and the length of the lead time. Just as we saw in the newsvendor model, an increase in the coefficient of variation leads to an increase in the amount of inventory needed for any fixed service level.

The length of the lead time is critical for two reasons. First, a reduction in the lead time reduces the amount of inventory needed at any location. Second, and maybe even more importantly, a reduction in the lead time reduces the amount of inventory in transit between locations, that is, the pipeline inventory. In fact, reducing the lead time is the only way to reduce the pipeline inventory: While reducing the variability of demand reduces the expected inventory at a location, it has no effect on pipeline inventory because of Little's Law.

Table 14.5 provides a summary of the key notation and equations presented in this chapter.

TABLE 14.5
Summary of Key Notation and Equations in Chapter 14

l = Lead time

S = Order-up-to level

Inventory level = On-hand inventory − Back order

Inventory position = On-order inventory + Inventory level

In-stock probability = 1 − Stockout probability
= Prob{Demand over l + 1 periods $\leq S$}

Expected back order:

If demand over l + 1 periods is normally distributed with mean μ and standard deviation σ:
Expected back order = $\sigma \times L(z)$, where $z = (S - \mu)/\sigma$

In Excel:

Expected back order = σ*(Normdist(z,0,1,0) − z*(1 − Normsdist(z)))

If demand over l + 1 periods is a discrete distribution function table, then
Expected back order = $L(S)$

Expected inventory = S − Expected demand over l + 1 periods + Expected back order

Expected on-order inventory = Expected demand in one period × Lead time

14.11 Further Reading

The order-up-to model is just one of many possible inventory policies that could be implemented in practice. For example, there are policies that account for stochastic lead times, lost sales, and/or batch ordering (ordering in integer multiples of a fixed batch quantity). However, no matter what extensions are included, the key insights remain: Inventory increases as demand variability increases or as the lead time increases.

See Zipkin (2000) for an extensive treatment of the theory of inventory management. For less technical, but still sophisticated, treatments, see Nahmias (2005) or Silver, Pyke, and Peterson (1998). Those texts cover the additional polices we discussed in the chapter (for example, a minimum order quantity with a fixed lead time and stochastic demand). In addition, they discuss the issue of the appropriate service level for upstream stages in a supply chain.

See Simchi-Levi, Kaminsky, and Simchi-Levi (2003) and Chopra and Meindl (2004) for managerial discussions of supply-chain management.

14.12 Practice Problems

Q14.1* **(Furniture Store)** You are the store manager at a large furniture store. One of your products is a study desk. Weekly demand for the desk is normally distributed with mean 40 and standard deviation 20. The lead time from the assembly plant to your store is two weeks and you order inventory replenishments weekly. You use the order-up-to model to control inventory.

a. Suppose your order-up-to level is $S = 220$. You are about to place an order and note that your inventory level is 100 and you have 85 desks on order. How many desks will you order?

b. Suppose your order-up-to level is $S = 220$. You are about to place an order and note that your inventory level is 160 and you have 65 desks on order. How many desks will you order?

c. What is the optimal order-up-to level if you want to target a 98 percent in-stock probability?

d. Suppose your order-up-to level is $S = 120$. What is your expected on-hand inventory?

e. Suppose your order-up-to level is $S = 120$. Your internal cost of capital is 15 percent and each desk costs $200. What is your total cost of capital for the year for inventory in the store?

(* indicates that the solution is at the end of the book)

Q14.2* **(Campus Bookstore)** A campus bookstore sells the Palm m505 handheld for $399. The wholesale price is $250 per unit. The store estimates that weekly demand averages 0.5 unit and has a Poisson distribution. The bookstore's annual inventory holding cost is 20 percent of the cost of inventory. Assume orders are made weekly and the lead time to receive an order from the distributor is four weeks.

a. What base stock level minimizes inventory while achieving a 99 percent in-stock probability?

b. Suppose the base stock level is $S = 4$. What is the average pipeline inventory?

c. Suppose the base stock level is $S = 5$. What is the average inventory held at the end of the week in the store?

d. Suppose the base stock level is $S = 6$. What is the probability a stockout occurs during a week (i.e., some customer is back-ordered)?

e. Suppose the base stock level is $S = 6$. What is the probability the store is out of stock (i.e., has no inventory) at the end of a week?

f. Suppose the base stock level is $S = 6$. What is the probability the store has one or more units of inventory at the end of a week?

The bookstore is concerned that it is incurring excessive ordering costs by ordering weekly. For parts g and h, suppose the bookstore now submits orders every two weeks. The demand forecast remains the same and the lead time is still four weeks.

g. What base stock level yields at least a 99 percent in-stock probability while minimizing inventory?

h. What is the average pipeline stock?

Q14.3* **(Quick Print)** Quick Print Inc. uses plain and three-hole-punched paper for copying needs. Demand for each paper type is highly variable. Weekly demand for the plain paper is estimated to be normally distributed with mean 100 and standard deviation 65 (measured in boxes). Each week, a replenishment order is placed to the paper factory and the order arrives five weeks later. All copying orders that cannot be satisfied immediately due to the lack of paper are back-ordered. The inventory holding cost is about $1 per box per year.

a. Suppose that Quick Print decides to establish an order-up-to level of 700 for plain paper. At the start of this week, there are 523 boxes in inventory and 180 boxes on order. How much will Quick Print order this week?

b. What is Quick Print's optimal order-up-to level for plain paper if Quick Print operates with a 99 percent in-stock probability?

Q14.4* **(Main Line Auto Distributor)** Main Line Auto Distributor is an auto parts supplier to local garage shops. None of its customers have the space or capital to store all of the possible parts they might need so they order parts from Main Line several times a day. To provide fast service, Main Line uses three pickup trucks to make its own deliveries. Each Friday evening, Main Line orders additional inventory from its supplier. The supplier delivers early Monday morning. Delivery costs are significant, so Main Line only orders on Fridays. Consider part A153QR, or part A for short. Part A costs Main Line $175 and Main Line sells it to garages for $200. If a garage orders part A and Main Line is out of stock, then the garage finds the part from some other distributor. Main Line has its own capital and space constraints and estimates that each unit of part A costs $0.50 to hold in inventory per week. (Assume you incur the $0.50 cost for units left in inventory at the end of the week, not $0.50 for your average inventory during the week or $0.50 for your inventory at the start of the week.) Average weekly demand for this part follows a Poisson distribution with mean 1.5 units. Suppose it is Friday evening and Main Line currently doesn't have any part A's in stock. The distribution and loss functions for a Poisson distribution with mean 1.5 can be found in Appendix B.

a. How many part A's should Main Line order from the supplier?

(* indicates that the solution is at the end of the book)

b. Suppose Main Line orders three units. What is the probability Main Line is able to satisfy all demand during the week?

c. Suppose Main Line orders four units. What is the probability Main Line is ***not*** able to satisfy all demand during the week?

d. If Main Line seeks to hit a target in-stock probability of 99.5 percent, then how many units should Main Line order?

e. Suppose Main Line orders five units. What is Main Line's expected holding cost for the upcoming week?

Q14.5* **(Hotspices.com)** You are the owner of Hotspices.com, an online retailer of hip, exotic, and hard-to-find spices. Consider your inventory of saffron, a spice (generally) worth more by weight than gold. You order saffron from an overseas supplier with a shipping lead time of four weeks and you order weekly. Average quarterly demand is normally distributed with a mean of 415 ounces and a standard deviation of 154 ounces. The holding cost per ounce per week is $0.75. You estimate that your back-order penalty cost is $50 per ounce. Assume there are 4.33 weeks per month.

a. If you wish to minimize inventory holding costs while maintaining a 99.25 percent in-stock probability, then what should your order-up-to level be?

b. If you wish to minimize holding and back-order penalty costs, then what should your order-up-to level be?

c. Now consider your inventory of pepperoncini (Italian hot red peppers). You can order this item daily and your local supplier delivers with a two-day lead time. While not your most popular item, you do have enough demand to sell the five-kilogram bag. Average demand per day has a Poisson distribution with mean 1.0. The holding cost per bag per day is $0.05 and the back-order penalty cost is about $5 per bag. What is your optimal order-up-to level?

Q14.6** **(Blood Bank)** Dr. Jack is in charge of the Springfield Hospital's Blood Center. Blood is collected in the regional Blood Center 200 miles away and delivered to Springfield by airplane. Dr. Jack reviews blood reserves and places orders every Monday morning for delivery the following Monday morning. If demand begins to exceed supply, surgeons postpone nonurgent procedures, in which case blood is back-ordered.

Demand for blood on every given week is normal with mean 100 pints and standard deviation 34 pints. Demand is independent across weeks.

a. On Monday morning, Dr. Jack reviews his reserves and observes 200 pints in on-hand inventory, no back orders, and 73 pints in pipeline inventory. Suppose his order-up-to level is 285. How many pints will he order? Choose the closest answer.

b. Dr. Jack targets a 99 percent in-stock probability. What order-up-to level should he choose? Choose the closest answer.

c. Dr. Jack is planning to implement a computer system that will allow daily ordering (seven days per week) and the lead time to receive orders will be one day. What will be the average order quantity?

Q14.7** **(Schmears Shirts)** Schmears Inc. is a catalog retailer of men's shirts. Daily demand for a particular SKU (style and size) is Poisson with mean 1.5. It takes three days for a replenishment order to arrive from Schmears' supplier and orders are placed daily. Schmears uses the order-up-to model to manage its inventory of this shirt.

a. Suppose Schmears uses an order-up-to level of 9. What is the average number of shirts on order?

b. Now suppose Schmears uses an order-up-to level of 8. What is the probability during any given day that Schmears does not have sufficient inventory to meet the demand from all customers?

c. Suppose Schmears wants to ensure that 90 percent of customer demand is satisfied immediately from stock. What order-up-to level should they use?

(* indicates that the solution is at the end of the book)

d. Schmears is considering a switch from a "service-based" stocking policy to a "cost-minimization" stocking policy. They estimate their holding cost per shirt per day is \$0.01. Forty-five percent of customers order more than one item at a time, so they estimate their stockout cost on this shirt is \$6 per shirt. What order-up-to level minimizes the sum of their holding and back-order costs?

Q14.8 **(ACold)** ACold Inc. is a frozen food distributor with 10 warehouses across the country. Iven Tory, one of the warehouse managers, wants to make sure that the inventory policies used by the warehouse are minimizing inventory while still maintaining quick delivery to ACold's customers. Since the warehouse carries hundreds of different products, Iven decided to study one. He picked Caruso's Frozen Pizza. Demand for CFPs averages 400 per day with a standard deviation of 200. Weekly demand (five days) averages 2,000 units with a standard deviation of 555. Since ACold orders at least one truck from General Foods each day (General Foods owns Caruso's Pizza), ACold can essentially order any quantity of CFP it wants each day. In fact, ACold's computer system is designed to implement a base stock policy for each product. Iven notes that any order for CFPs arrives four days after the order. Further, it costs ACold \$0.01 per day to keep a CFP in inventory, while a back order is estimated to cost ACold \$0.45.

a. What base stock level should Iven choose for CFPs if his goal is to minimize holding and back-order costs?

b. Suppose the base stock level 2,800 is chosen. What is the average amount of inventory on order?

c. Suppose the base stock level 2,800 is chosen. What is the annual holding cost? (Assume 260 days per year.)

d. What base stock level minimizes inventory while maintaining a 97 percent in-stock probability?

Q14.9 **(Cyber Chemicals)** Cyber Chemicals uses liquid nitrogen on a regular basis. Average daily demand is 178 gallons with a standard deviation of 45. Due to a substantial ordering cost, which is estimated to be \$58 per order (no matter the quantity in the order), Cyber currently orders from its supplier on a weekly basis. Cyber also incurs holding costs on its inventory. Cyber recognizes that its inventory is lowest at the end of the week but prefers a more realistic estimate of its average inventory. In particular, Cyber estimates its average inventory to be its average end-of-week inventory plus half of its average order quantity. The holding cost Cyber incurs on that average inventory is \$0.08 per gallon per week. Cyber's supplier delivers in less than a day. Assume 52 weeks per year, five days per week.

a. Cyber wishes to maintain a 99.9 percent in-stock probability. If it does so, what is Cyber's annual inventory holding cost?

b. What is Cyber's annual ordering cost?

c. Should Cyber consider ordering every two weeks?

Q14.10 **(Southern Fresh)** Shelf space in the grocery business is a valuable asset. Every good supermarket spends a significant amount of effort attempting to determine the optimal shelf space allocation across products. Many factors are relevant to this decision: the profitability of each product, the size of each product, the demand characteristics of each product, and so forth. Consider Hot Bull corn chips, a local favorite. Average daily demand for this product is 55, with a standard deviation of 30. Bags of Hot Bull can be stacked 20 deep per facing. (A facing is the width on a shelf required to display one item of a product.) Deliveries from Southern Fresh's central warehouse occur two days after a store manager submits an order. (Actually, in most stores, orders are generated by a centralized computer system that is linked to its point-of-sales data. But even these orders are received two days after they are transmitted.)

a. How many facings are needed to achieve a 98.75 percent in-stock probability?

b. Suppose Southern Fresh allocates 11 facings to Hot Bull corn chips. On average, how many bags of Hot Bull are on the shelf at the end of the day?

c. Although Southern Fresh does not want to incur the cost of holding inventory, it does want to leave customers with the impression that it is well stocked. Hence, Southern Fresh employees continually roam the aisles of the store to adjust the presentation of the product. In particular, they shift product around so that there is an item in each facing whenever possible. Suppose Southern Fresh allocates 11 facings to Hot Bull corn chips. What is the probability that at the end of the day there will be an empty facing, that is, a facing without any product?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 15

Risk-Pooling Strategies to Reduce and Hedge Uncertainty[1]

Uncertainty is the bane of operations. No matter in what form—for example, uncertain demand, uncertain supply, or uncertain quality—operational performance never benefits from the presence of uncertainty. Previous chapters have discussed models for coping with uncertainty (e.g., queuing, newsvendor, and order-up-to) and have emphasized the need to quantify uncertainty. Some strategies for reducing and hedging uncertainty have already been suggested: combine servers in a queuing system (Chapter 9); reduce uncertainty by collecting data to ensure that the best demand forecast is always implemented (Chapter 12); establish make-to-order production and invest in reactive capacity to better respond to demand (Chapter 13).

This chapter explores several additional strategies based on the concept of risk pooling. The idea behind risk pooling is to redesign the supply chain, the production process, or the product to either reduce the uncertainty the firm faces or hedge uncertainty so that the firm is in a better position to mitigate the consequence of uncertainty. Several types of risk pooling are presented (location pooling, virtual pooling, product pooling, lead time pooling, and capacity pooling), but these are just different names to describe the same basic phenomenon. With each strategy, we work through a practical example to illustrate its effectiveness and to highlight the situations in which the strategy is most appropriate.

15.1 Location Pooling

The newsvendor and the order-up-to inventory models are tools for deciding how much inventory to put at a single location to serve demand. An equally important decision, and one that we have ignored so far, is in how many different locations should the firm store inventory to serve demand. To explain, consider the Medtronic supply chain discussed in Chapter 14. In that supply chain, each sales representative in the field manages a cache of inventory to serve the rep's territory and there is a single distribution center to serve the entire U.S. market. Should there be one stockpile of inventory per sales representative or should the demands from multiple territories be served from a single location? Should

[1] Data in this chapter have been disguised to protect confidentiality.

there be a single distribution center or should the U.S. market demand be divided among multiple distribution centers? We explore those questions in this section.

Pooling Medtronic's Field Inventory

Let's begin with where to locate Medtronic's field inventory. Instead of the current system in which each sales representative manages his or her own inventory, maybe the representatives in adjacent territories could share inventory. For example, Medtronic could rent a small space in a centrally located and easily accessible location (e.g., a back room in a strip mall off the interchange of two major highways) and two to five representatives could pool their inventory at that location. Sharing inventory means that each representative would only carry inventory needed for immediate use; that is, each representative's trunk and consignment inventory would be moved to this shared location. Control of the pooled inventory would be guided by an automatic replenishment system based on the order-up-to model. What impact would this new strategy have on inventory performance?

Recall that average daily demand for Medtronic's InSync pacemaker in Susan Magnotto's Madison, Wisconsin, territory is represented with a Poisson distribution with mean 0.29 unit per day. For the sake of argument, let's suppose there are several other territories adjacent to Susan's, each with a single sales representative and each with average daily demand of 0.29 unit for the InSync pacemaker. Instead of each representative carrying his or her own inventory, now they share a common pool of inventory. We refer to the combined territories in this new system as the *pooled territory* and the inventory there as the *pooled inventory.* In contrast, we refer to the territories in the current system as the *individual territories* and the inventory in one of those territories as the *individual inventory.* We refer to the strategy of combining the inventory from multiple territories/locations into a single location as *location pooling.* We have already evaluated the expected inventory with the current individual territory system, so now we need to evaluate the performance of the system with pooled territories, that is, the impact of location pooling.

The order-up-to model is used to manage the inventory at the pooled territory. The same aggressive target in-stock probability is used for the pooled territory as is used at the individual territories, 99.9 percent. Furthermore, the lead time to replenish the pooled territory is also one day. (There is no reason to believe the lead time to the pooled territory should be different than to the individual territories.)

As discussed in Chapter 14, if the Poisson distribution represents demand at two different territories, then their combined demand has a Poisson distribution with a mean that equals the sum of their means. (See Exhibit 14.1.) For example, suppose Susan shares inventory with two nearby sales representatives and they all have mean demand for the InSync pacemaker of 0.29 unit per day. Then total demand across the three territories is Poisson with mean $3 \times 0.29 = 0.87$ unit per day. We then can apply the order-up-to model to that pooled territory assuming a lead time of one day and a mean demand of 0.87 unit.

Table 15.1 presents data on the impact of pooling the sales representatives' territories. To achieve the 99.9 percent in-stock probability for three sales representatives requires $S = 7$, where S is the order-up-to level. If Susan's inventory is not combined with another representative's, then (as we evaluated in Chapter 14) $S = 4$ is needed to hit the target in-stock probability. The expected inventory at the pooled location is 5.3 units, in contrast to 3.4 units for each individual location. However, the total inventory for three individual locations is $3 \times 3.4 = 10.2$ units. Hence, pooling three locations reduces expected inventory by about 48 percent [$(10.2 - 5.3)/10.2$], without any degradation in service!

TABLE 15.1
The Impact on InSync Pacemaker Inventory from Pooling Sales Representatives' Territories
Demand at each territory is Poisson with average daily demand of 0.29 unit, the target in-stock probability is 99.9 percent, and the lead time is one day.

			Expected Inventory		Pipeline Inventory	
Number of Territories Pooled	Pooled Territory's Expected Demand per Day (a)	*S*	Units (b)	Days-of-Demand (b/a)	Units (c)	Days-of-Demand (c/a)
1	0.29	4	3.4	11.7	0.29	1.0
2	0.58	6	4.8	8.3	0.58	1.0
3	0.87	7	5.3	6.1	0.87	1.0
4	1.16	8	5.7	4.9	1.16	1.0
5	1.45	9	6.1	4.2	1.45	1.0
6	1.74	10	6.5	3.7	1.74	1.0
7	2.03	12	7.9	3.9	2.03	1.0
8	2.32	13	8.4	3.6	2.32	1.0

There is another approach to make the comparison between pooled territories and individual territories: Evaluate each inventory quantity relative to the demand it serves, that is, calculate expected inventory measured in days-of-demand rather than units:

$$\text{Expected inventory in days-of-demand} = \frac{\text{Expected inventory in units}}{\text{Expected daily demand}}$$

Table 15.1 also provides that measure of expected inventory. We see that inventory at each individual territory equals $3.4/0.29 = 11.7$ days-of-demand whereas inventory at three pooled territories equals only $5.3/0.87 = 6.1$ days-of-demand. Using our days-of-demand measure, we see that pooling three territories results in a 48 percent $[(11.7 - 6.1)/11.7]$ reduction in inventory investment. We obtain the same inventory reduction (48 percent) because the two measures of inventory, units and days-of-demand, only differ by a constant factor (the expected daily demand). Hence, we can work with either measure.

While pooling two or three territories has a dramatic impact on inventory, Table 15.1 indicates that there are decreasing marginal returns to pooling territories; that is, each new territory added to the pool brings a smaller reduction in inventory than the previous territory added to the pool. For example, adding two more territories to a pool of six (to make a total of eight combined territories) has very little impact on the inventory investment (3.6 days-of-demand versus 3.7 days-of-demand), whereas adding two more territories to a pool of one (to make a total of three combined territories) has a dramatic impact in inventory (6.1 days-of-demand versus 11.7 days-of-demand). This is good news: the majority of the benefit of pooling territories comes from the first couple of territories combined, so there is little value in trying to combine many territories together.

Although location pooling generally reduces inventory, a careful observer of the data in Table 15.1 would discover that this is not always so: adding the seventh location to the pool slightly increases inventory (3.9 days-of-demand versus 3.7 days-of-demand). This is due to the restriction that the order-up-to level must be an integer (0, 1, 2, . . .) quantity. As a result, the in-stock probability might be even higher than the target: the in-stock probability with six pooled territories is 99.90 percent, whereas it is 99.97 percent with seven pooled territories. Overall, this issue does not invalidate the general trend that location pooling reduces inventory.

This discussion obviously leads to the question of why does location pooling reduce the required inventory investment? We'll find a good answer by looking at how demand variability changes as locations are added to the pooled location. And, as we have already discussed, the coefficient of variation (the ratio of the standard deviation to the mean) is our choice for measuring demand variability.

Recall that the standard deviation of a Poisson distribution equals the square root of its mean. Therefore,

$$\text{Coefficient of variation of a Poisson distribution} = \frac{\text{Standard deviation}}{\text{Mean}} = \frac{\sqrt{\text{Mean}}}{\text{Mean}} = \frac{1}{\sqrt{\text{Mean}}} \qquad \textbf{(15.1)}$$

As the mean of a Poisson distribution increases, its coefficient of variation decreases, that is, the Poisson distribution becomes less variable. Less variable demand leads to less inventory for any given service level. Hence, combining locations with Poisson demand reduces the required inventory investment because a higher demand rate implies less variable demand. However, because the coefficient of variation decreases with the square root of the mean, it decreases at a decreasing rate. In other words, each incremental increase in the mean has a proportionally smaller impact on the coefficient of variation, and, hence, on the expected inventory investment.

Figure 15.1 displays the relationship between inventory and the coefficient of variation for the data in Table 15.1. Notice that the decreasing pattern in inventory closely mimics the decreasing pattern in the coefficient of variation.

In addition to the total expected inventory in the field, we also are interested in the total pipeline inventory (inventory on order between the distribution center and the field). Table 15.1 provides the pipeline inventory in terms of units and in terms of days-of-demand. While location pooling decreases the expected inventory in days-of-demand, it has absolutely no impact on the pipeline inventory in terms of days-of-demand! Why? Little's

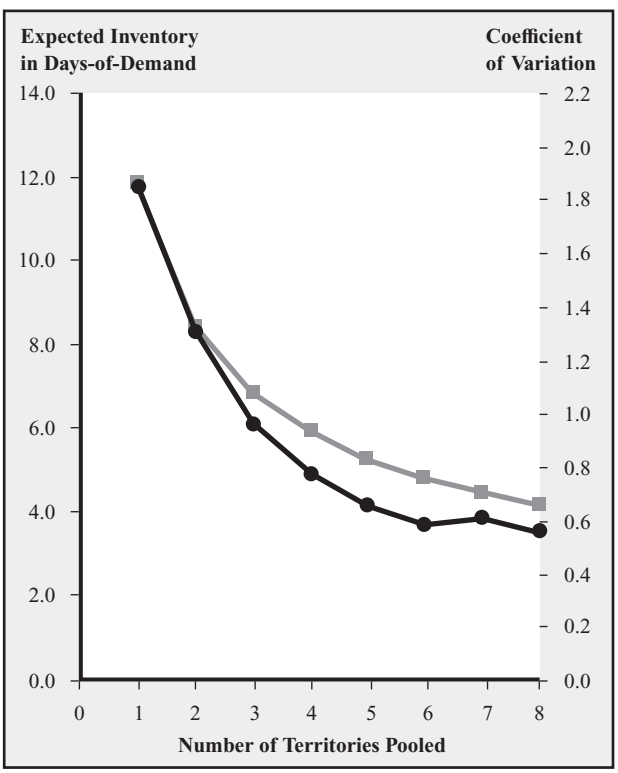


FIGURE 15.1
The Relationship between Expected Inventory (circles) and the Coefficient of Variation (squares) as Territories Are Pooled
Demand in each territory is Poisson with mean 0.29 unit per day, the target in-stock probability is 99.9 percent, and the lead time is one day.

TABLE 15.2
Using Location Pooling to Raise the In-Stock Probability While Maintaining the Same Inventory Investment
Demand at each territory is Poisson with average daily demand of 0.29 unit, and the lead time is one day.

Number of Territories Pooled	Pooled Territory's Expected Demand per Day	*S*	Expected Inventory		In-Stock Probability
			Units	Days-of-Demand	
1	0.29	4	3.4	11.7	99.96615%
2	0.58	8	6.8	11.7	99.99963
3	0.87	12	10.3	11.8	100.00000

Law governs pipeline inventory, and Little's Law depends on averages, not variability. Hence, because pooling territories reduces the variability of demand, it reduces expected inventory in the field, but it has no impact on the pipeline inventory. As we mentioned before, the only way to reduce pipeline inventory is to get a faster lead time.

While we can exploit location pooling to reduce inventory while maintaining a service level, we also can use location pooling to increase our service level. For example, we could choose an order-up-to level in the pooled territory that generates the same inventory investment as the individual territories (measured in days-of-demand) and see how much higher our in-stock could be. Table 15.2 presents those data for pooling up to three territories; beyond three territories we can raise the in-stock to essentially 100 percent with the same inventory investment as the individual territories.

Because the in-stock probability target with individual territories is so high (99.9 percent), it probably makes better sense to use location pooling to reduce the inventory investment rather than to increase the service level. However, in other settings it may be more desirable to increase the service level, especially if the target service level is deemed to be too low.

Figure 15.2 provides another perspective on this issue. It displays the inventory–service trade-off curves with four different degrees of location pooling: individual territories, two territories pooled, four territories pooled, and eight territories pooled. As displayed in the figure, pooling territories shifts the inventory–service trade-off curve down and to the right. Hence, location pooling gives us many options: we can choose to (1) maintain the same service with less inventory, (2) maintain the same inventory with a higher service, or (3) reduce inventory and increase service simultaneously (i.e., "we can have our cake and eat it too"). We saw a similar effect when pooling servers in a queuing environment. There you can use pooling to reduce waiting time without having to staff extra workers, or you can reduce workers while maintaining the same responsiveness, or a combination of both. Furthermore, we should note that these results are not specific to the order-up-to model or Poisson demand; they are quite general and we use this model and demand only to illustrate our point.

Although our analysis highlights the potential dramatic benefit of location pooling, this does not imply that Medtronic should pool territories without further thought. There will be an explicit storage cost for the space to house the pooled inventory, whereas the current system does not have a storage cost for trunk and consignment inventory. However, location pooling might reduce theft and spoilage costs because inventory is stored in fewer locations. Furthermore, location pooling probably would reduce shipping costs because the number of items per delivery is likely to increase.

The greatest concern with location pooling is the impact on the efficiency of the sales representatives. Even if only a few territories are pooled, it is likely that the pooled location would not be as convenient to each sales representative as their own individual inventory.

The physical separation between user and inventory can be mitigated via *virtual pooling:* Representatives maintain control of their inventory, but inventory information is

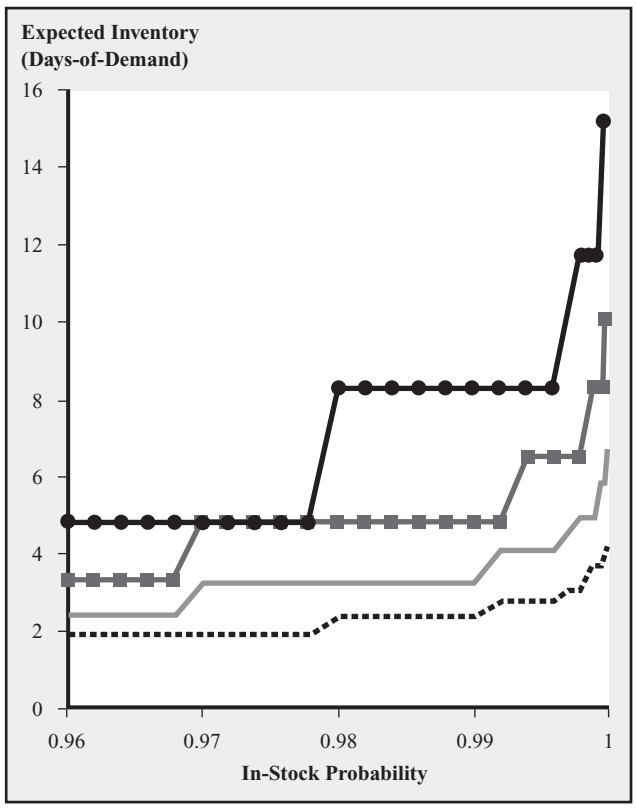


FIGURE 15.2
The Inventory–Service Trade-off Curve for Different Levels of Location Pooling
The curves represent, from highest to lowest, individual territories, two pooled territories, four pooled territories, and eight pooled territories. Demand in each territory is Poisson with mean 0.29 unit per day and the lead time is one day.

shared among all representatives so that each rep can obtain inventory from the central distribution center and any other rep that has excess inventory. Although virtual pooling has its own challenges (e.g., the additional cost of maintaining the necessary information systems, the added expense of transshipping inventory among territories, and the sticky design issue of how to decide when inventory can be taken from one rep to be given to another rep), it can still be better than the current system that has isolated pockets of inventory.

Medtronic's Distribution Center(s)

Now let's turn our attention to the distribution center. For the U.S. market, Medtronic currently operates a single distribution center in Mounds View, Minnesota. Suppose Medtronic were to subdivide the United States into two or more regions, with each region assigned a single distribution center. This idea is location pooling in reverse. Hence, the total inventory investment is likely to increase. Let's see by how much.

Recall that weekly demand of the InSync Pacemaker at the Mounds View DC is normally distributed with mean 80.6 and standard deviation 58.81. There is a three-week lead time and the target in-stock probability is 99.9 percent. Table 15.3 provides data on the expected inventory required given the number of DCs Medtronic operates.

Table 15.3 reveals that it is indeed costly to subdivide the U.S. market among multiple distribution centers: eight DCs require nearly three times more inventory to achieve the same service level as a single DC! (To be precise, it requires 12.8/4.5 = 2.84 times more inventory.)

TABLE 15.3
The Increase in Inventory Investment as More Distribution Centers Are Operated
Assume demand is equally divided among the DCs, demands across DCs are independent, total demand is normally distributed with mean 80.6 and standard deviation 58.8, and the lead time is three weeks in all situations.

	Weekly Demand Parameters at Each DC			Expected Inventory at Each DC	
Number of DCs	Mean	Standard Deviation	Coefficient of Variation	Units	Weeks-of-Demand
1	80.6	58.8	0.73	364	4.5
2	40.3	41.6	1.03	257	6.4
3	26.9	34.0	1.26	210	7.8
4	20.2	29.4	1.46	182	9.0
5	16.1	26.3	1.63	163	10.1
6	13.4	24.0	1.79	148	11.0
7	11.5	22.2	1.93	137	11.9
8	10.1	20.8	2.06	127	12.8

In this situation, the connection between the coefficient of variation and the expected inventory savings from location pooling (or "dissavings" from location disintegration, as in this case) is even stronger than we saw with field inventory, as displayed in Figure 15.3. In fact, expected inventory and the coefficient of variation in this setting are proportional to one another (i.e., their ratio is a constant no matter the number of distribution centers).

Electronic Commerce

No discussion on location pooling is complete without discussing electronic commerce. One of the well-known advantages to the e-commerce model, especially with respect to e-tailers, is the ability to operate with substantially lower inventory. As our analysis suggests, keeping inventory in fewer locations should allow an e-tailer to turn inventory much faster than a comparable brick-and-mortar retailer. However, there are extra costs to position inventory in a warehouse rather than in a neighborhood store: shipping individual

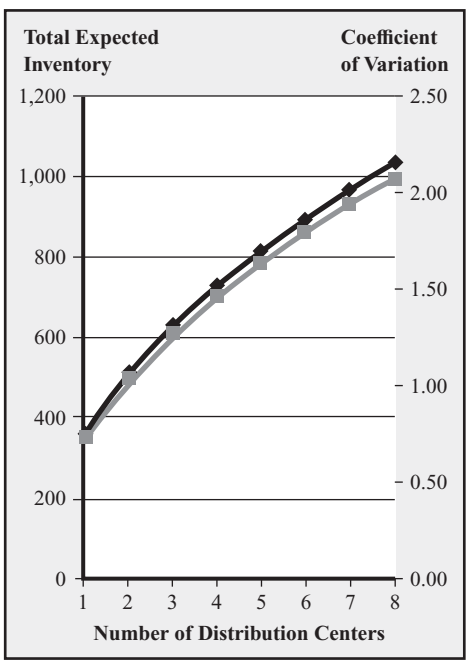


FIGURE 15.3
The Expected Inventory in Units (circles) and the Coefficient of Variation (squares) Depending on the Number of Distribution Centers Medtronic Operates
Demand is assumed to be equally divided and independent across distribution centers. The target in-stock probability is 99.9 percent and the lead time is three weeks in all cases.

items to consumers is far more expensive than shipping in bulk to retail stores and, while physical stores need not be constructed, an e-tailer needs to invest in the technology to create an electronic store (i.e., user interface, logistics management, etc.).

We also saw that there are declining returns to location pooling. Not surprisingly, while many e-tailers, such as Amazon.com, started with a single distribution center, they now operate several distribution centers in the United States. This requires that some products are stored in multiple locations, but it also means that the average customer is located closer to a distribution center, which accelerates the average delivery time and reduces shipping costs.

The ability to offer customers a huge product selection is another advantage of the e-commerce model, possibly the most important advantage. While we have focused on using location pooling to reduce inventory, location pooling also can enable a broad product assortment. Consider an item that sells but is a rather slow seller. Unfortunately for most businesses, the majority of products fall into that category. To include this item in the product assortment requires at least one unit. Placing one unit in hundreds of locations may not be economical, but it may be economical to place a few units in a single location.

To illustrate this point, consider a slow-moving product that could be sold by a retailer with 200 stores. The product would sell at each store at the average rate of 0.01 unit per week. Consequently, the retailer's total demand across all stores is $0.01 \times 200 = 2$ per week. You may think this is ridiculously slow, but in fact there are many products that sell at this pace. For example, Brynjolfsson, Hu, and Smith (2003) estimated that 40 percent of Amazon's sales came from items that sold no more than 1.5 units per week. Returning to our example, suppose this retailer must stock at least one unit in each store (the product must be available at the store). Given each store's sales rate, the retailer will stock only one unit and each item will spend nearly two years ($1/0.01 = 100$ weeks) on the shelf. That sales rate implies a measly 0.5 inventory turn (inventory is turned over once every two years). To finalize this analysis, if inventory cost 20 percent per year to hold (capital cost and, more importantly, the cost of shelf space), then this item will incur $2 \times 20\% = 40$ percent in holding costs. Most retailers do not have anywhere near a 40 percent gross margin, so it is unlikely that this product is profitable—the retailer cannot carry this item profitably because it just doesn't turn fast enough. Now contrast those economics with an e-tailer with one warehouse. If the e-tailer's demand is Poisson with mean two per week, replenishment lead time is two weeks, and the target in-stock is 99 percent, we can use the order-up-to model to determine that the retailer will have on average about six units of inventory. If total yearly demand is about 104 units (52 weeks at 2 per week), then our e-tailer turns inventory $104/6 = 17.3$ times per year. The e-tailer stands a chance to make money stocking this item, whereas the brick-and-mortar retailer does not. To summarize, there are many slow selling products in this world (which can sum up to a lot of sales, as evidenced by Amazon.com), but location pooling may be necessary for a retailer to profitably include them in the assortment.

15.2 Product Pooling

The previous section considered serving demand with fewer inventory locations. A closely related idea is to serve demand with fewer products. To explain, consider O'Neill's Hammer 3/2 wetsuit discussed in Chapters 12 and 13. The Hammer 3/2 we studied is targeted to the market for surfers, but O'Neill sells another Hammer 3/2 that serves the market for recreational divers. The two wetsuits are identical with the exception that the surf Hammer has the "wave" logo (see Figure 12.1) silk screened on the chest, while the dive Hammer has O'Neill's dive logo, displayed in Figure 15.4. O'Neill's current product line has two products to serve demand for a Hammer 3/2 wetsuit, some of it from surfers, the

FIGURE 15.4
O'Neill's Logo for Dive Wetsuits

other portion from divers. An alternative is to combine these products into a single product to serve all Hammer 3/2 wetsuit demand, that is, a *universal design.* The strategy of using a universal design is called *product pooling.* This section focuses on the merits of the product-pooling strategy with a universal design.

Recall that demand for the surf Hammer is normally distributed with mean 3,192 and standard deviation 1,181. For the sake of simplicity, let's assume demand for the dive Hammer is also normally distributed with the same mean and standard deviation. Both wetsuits sell for $190, are purchased from O'Neill's supplier for $110, and are liquidated at the end of the season for $90.

We have already evaluated the optimal order quantity and expected profit for the surf Hammer: ordering 4,196 units earns an expected profit of $222,280 (see Table 13.1). Because the dive Hammer is identical to the surf Hammer, it has the same optimal order quantity and expected profit. Therefore, the total profit from both Hammer wetsuits is $2 \times \$222{,}280 = \$444{,}560$.

Now let's consider what O'Neill should do if it sold a single Hammer wetsuit, which we call the universal Hammer. We need a distribution to represent demand for the universal Hammer and then we need an order quantity. Expected demand for the universal Hammer is $3{,}192 \times 2 = 6{,}384$ units. If demand in the dive market is independent of demand in the surf market, then the standard deviation for the universal Hammer is $1{,}181 \times \sqrt{2} = 1{,}670$ (see Exhibit 14.1). The underage cost for the universal Hammer is still $C_u = 190 - 110 = 80$ and the overage cost is still $C_o = 110 - 90 = 20$. Hence, the critical ratio has not changed:

$$\frac{C_u}{C_o + C_u} = \frac{80}{20 + 80} = 0.8$$

The corresponding z-statistic is still 0.85, and so the optimal order quantity is

$$Q = \mu + \sigma \times z = 6{,}384 + 1{,}670 \times 0.85 = 7{,}804$$

The expected profit with the universal Hammer is

$$\begin{aligned}\text{Expected profit} &= (C_u \times \text{Expected sales}) - (C_o \times \text{Expected leftover inventory}) \\ &= (80 \times 6{,}200) - (20 \times 1{,}604) \\ &= \$463{,}920\end{aligned}$$

Therefore, pooling the surf and dive Hammers together can potentially increase profit by 4.4 percent [(463,920 − 444,560)/444,560]. This profit increase is 1.4 percent of the expected revenue when O'Neill sells two wetsuits. Given that net profit in this industry ranges from 2 percent to 5 percent of revenue, this potential improvement is not trivial.

As with the location pooling examples at Medtronic, the potential benefit O'Neill receives from product pooling occurs because of a reduction in the variability of demand. With two Hammer wetsuits, O'Neill faces a coefficient of variation of about 0.37 with each suit. With a universal Hammer, the coefficient of variation is about 1,670/6,384 = 0.26. Recall from Chapter 13 that the mismatch cost in the newsvendor model is directly proportional to the coefficient of variation, hence the connection between a lower coefficient of variation and higher expected profit.

Given this link between the coefficient of variation and the benefit of product pooling, it is important for us to understand how product pooling influences the coefficient of variation. In this example, as well as the Medtronic examples in the previous two sections, we make a key assumption that the demands we are combining are independent. Recall that independence means that the outcome of one demand provides no information about the outcome of the other demand. There are many settings in which demands are indeed independent. But there are also situations in which demands are not independent.

The link between two random events can be measured by their correlation, which ranges from -1 to 1. Independent random events have zero correlation. Positive correlation means two random events tend to move in lock step; that is, when one is high, the other tends to be high as well, and when one is low, the other tends to be low as well. In contrast, negative correlation means two random events tend to move in opposite directions; that is, when one is high, the other tends to be low, and when one is low, the other tends to be high.

We can illustrate the effect of correlation graphically with two products. Figure 15.5 displays the outcome of 100 random demand realizations for two products in three scenarios. (For example, if the random demands of the two products are five and seven respectively, then a point is plotted at {5,7}.) In the first scenario, the products' demands are negatively correlated, in the second they are independent, and in the third they are positively correlated. In the independent scenario (scenario two), we see that the outcomes form a "cloud" that roughly fits into a circle; that is, the outcome of one demand says nothing about the outcome of the other demand. In the negative correlation scenario (scenario one), the outcome cloud is a downward-sloping ellipse: high demand with one product suggests low demand with the other product. The positive correlation scenario (scenario three) also has an outcome cloud shaped like an ellipse, but now it is upward sloping: high demand with one product suggests high demand with the other product.

Many different demand outcomes lead to the same total demand. For example, in the graphs in Figure 15.5, the total demand is 20 units if the products' demands are {0,20}, {1,19}, . . . , {19,1}, {20,0}. In other words, all of the points along the dashed line in each graph have total demand of 20 units. In general, all points along the same downward-sloping 45° line have the same total demand. Because the outcome ellipse in the negative correlation scenario is downward sloping along a 45° line, the total demands of those outcomes are nearly the same. In contrast, because the outcome ellipse in the positive correlation scenario is *upward* sloping, those outcomes generally sum to different total demands. In other words, we expect to see more variability in the total demand with positive correlation than with negative correlation.

We can now be more precise about the impact of correlation. If we combine two demands with the same mean μ and standard deviation σ, then the pooled demand has the following parameters:

$$\text{Expected pooled demand} = 2 \times \mu$$

$$\text{Standard deviation of pooled demand} = \sqrt{2 \times (1 + \text{Correlation})} \times \sigma$$

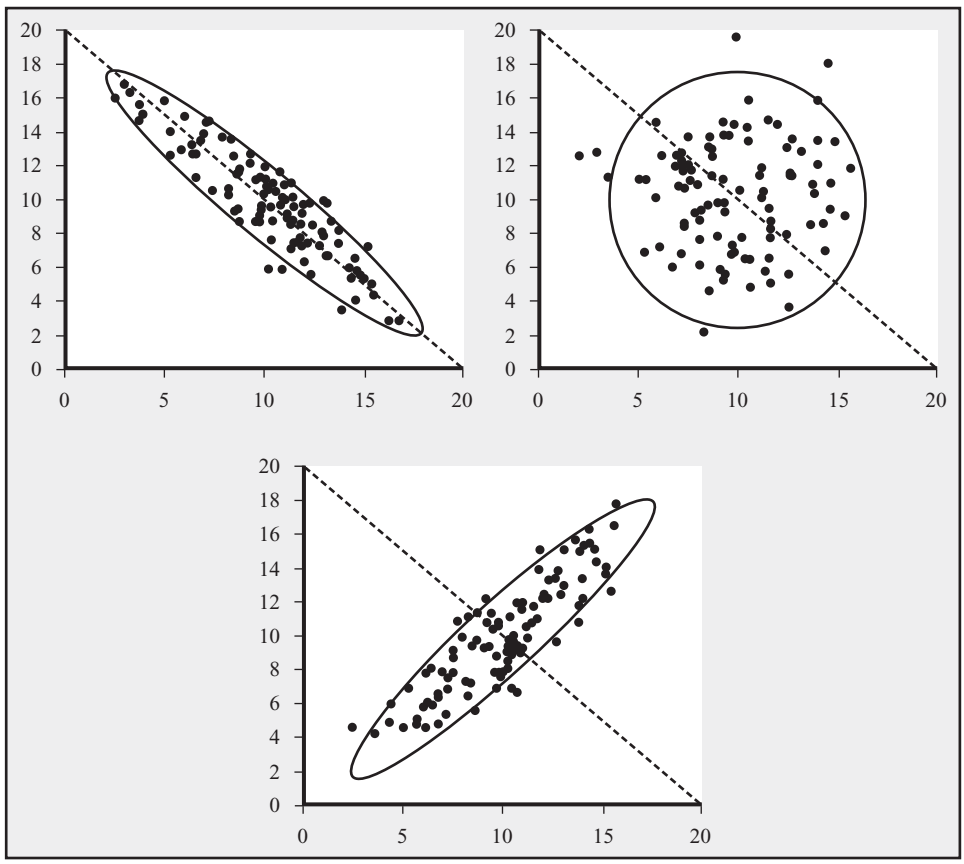


FIGURE 15.5
Random Demand for Two Products
In the graphs, x-axis is product 1 and y-axis is product 2. In scenario 1 (upper-left graph), the correlation is −0.90; in scenario 2 (upper-right graph), the correlation is 0; and in scenario 3 (the lower graph), the correlation is 0.90. In all scenarios, demand is normally distributed for each product with mean 10 and standard deviation 3.

Notice that the correlation has no impact on the expected demand, but it does influence the standard deviation. Furthermore, the above equations are equivalent to the ones we have been using (e.g., Exhibit 14.1) when the correlation is zero, that is, when the two demands are independent.

The coefficient of variation for the pooled demand is then

$$\text{Coefficient of variation of pooled demand} = \sqrt{\frac{1}{2}(1 + \text{Correlation})} \times \left(\frac{\sigma}{\mu}\right)$$

As the correlation increases, the coefficient of variation of pooled demand increases as well, just as the graphs in Figure 15.5 suggest.

Now let's visualize what happens when we choose quantities for both the dive and the surf suits. Figure 15.6 displays the result of our quantity choices for different demand outcomes. For example, if the demand outcome is in the lower-left-hand "square" of the graph, then we have leftover surf and dive suits. The ideal outcome is if demand for each suit happens to equal its order quantity, an outcome labeled with a circle in the graph. The demand–supply mismatch penalty increases as the demand outcome moves further away from that ideal point in any direction.

The comparable graph for the universal Hammer is different, as is shown in Figure 15.7. Now any demand outcome along the downward-sloping 45° line (circles) is an ideal outcome because total demand equals the quantity of universal suits. In other words, the number of ideal demand outcomes with the universal suit has expanded considerably relative

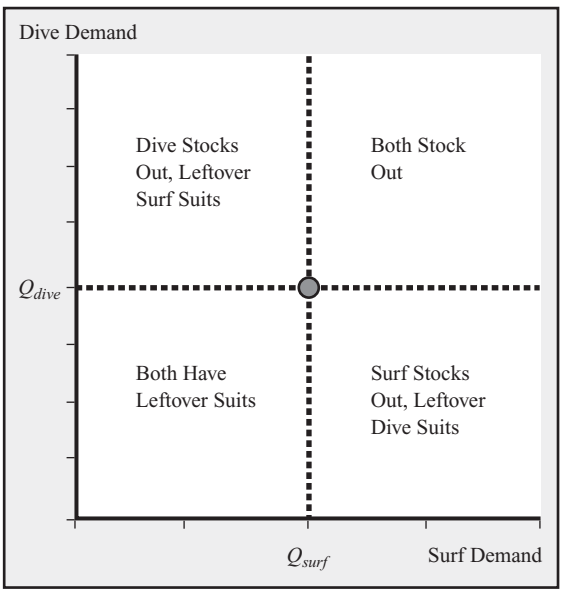


FIGURE 15.6
The Inventory/ Stockout Outcome Given the Order Quantities for Surf and Dive Suits, Q_{surf} and Q_{dive}

to the single ideal demand outcome with two suits. How likely are we to be close to one of those ideal points? Figure 15.7 also superimposes the three "outcome clouds" from Figure 15.5. Clearly, with negative correlation we are more likely to be close to an ideal point (the downward-sloping ellipse) and with positive correlation we are least likely to be near an ideal point.

We can confirm the intuition developed with the graph in Figure 15.7 by actually evaluating O'Neill's optimal order quantity for the universal Hammer 3/2 and its expected profit for the entire range of correlations. We first notice that the optimal order quantity for the Hammer 3/2 is generally *not* the sum of the optimal order quantities of the two suits. For example, O'Neill's total order with two wetsuits is $4{,}196 \times 2 = 8{,}392$ units, but with correlation 0.2 the optimal order for the universal Hammer is 7,929 units and with correlation -0.7 the optimal order is 7,162.

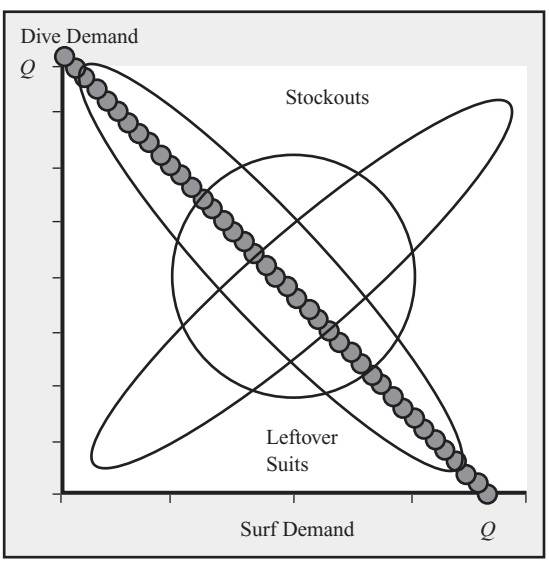


FIGURE 15.7
Outcomes for the Universal Hammer Given Q Units Purchased
Outcomes on the diagonal line with circles are ideal; there is no leftover inventory and no stockouts. Outcomes below and to the left of that line have leftover suits; outcomes to the right and above that line result in stockouts. Ellipses identify likely outcomes under different correlations.

The results with respect to expected profit are displayed in Figure 15.8: We indeed see that the expected profit of the universal Hammer declines as surf and dive demand become more positively correlated.

The extreme ends in Figure 15.8 are interesting. With perfectly positive correlation (i.e., correlation = 1), there is absolutely no benefit from inventory pooling: The expected profit with the universal Hammer is $444,560, and that is also the profit with two Hammer wetsuits! At the other end of the spectrum, correlation = −1, the coefficient of variation of total Hammer demand is 0, and so the maximum profit is achieved, $510,720! In fact, in that situation, the optimal order quantity for universal suits is just 6,384 units, which also happens to be the expected demand for universal suits. (This makes sense; we only earn the maximum profit if we sell on average the expected demand and we never have leftover inventory.)

While we have been discussing the impact of demand correlation on the efficacy of product pooling, this issue applies even with location pooling. If the demands at two locations are negatively correlated, then location pooling is even more effective than if the demands were merely independent. And if demands are positively correlated across locations, then location pooling is less effective than we evaluated, given our assumption of independence.

We also should discuss the conditions that we can expect when demand has a particular type of correlation. Positive correlation can occur if the products are linked to some common source of uncertainty, for example, general economic conditions. For example, positive correlation is likely to be present if all of a firm's products tend to perform poorly in a depressed economy and perform well in a robust economy. Negative correlation is present when there is relatively little uncertainty with respect to total category sales but substantial uncertainty with respect to the allocation of those sales across the product line. For example,

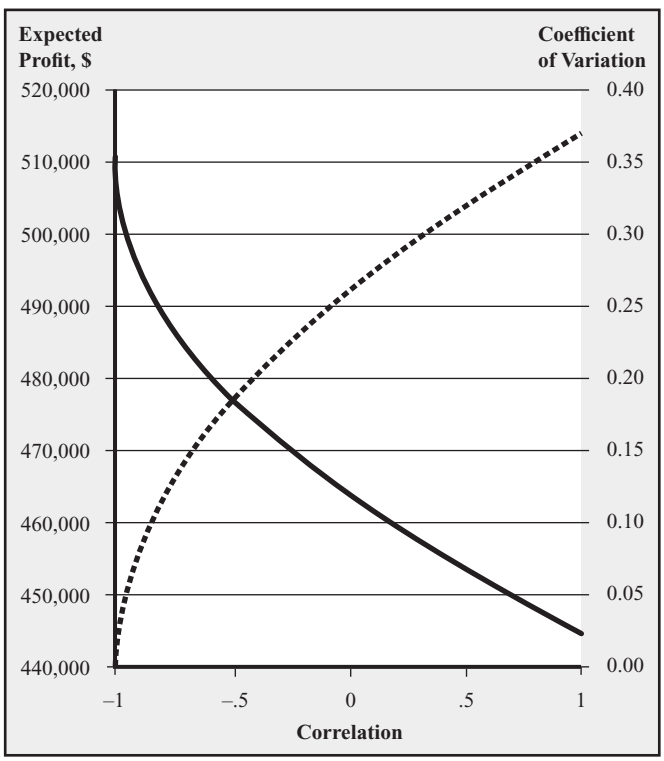


FIGURE 15.8
The Correlation between Surf and Dive Demand for the Hammer 3/2 and the Expected Profit of the Universal Hammer Wetsuit (decreasing curve) and the Coefficient of Variation of Total Demand (increasing curve)

a firm selling fashionable jackets may know pretty well how many jackets will sell in total but have considerable uncertainty over which colors will be hot this season.

To summarize, a key benefit of a universal design is the reduction in demand variability, which leads to better performance in terms of matching supply and demand (e.g., higher profit or lower inventory for a targeted service level). But there are drawbacks to a universal design strategy as well:

- A universal design may not provide the needed functionality to consumers with special needs. For example, most bicycle manufacturers produce road bikes designed for fast touring on well-paved roads and mountain bikes for tearing through rugged trails. They even sell hybrid bikes that have some of the features of a road bike as well as some of the features of a mountain bike. But it is not sufficient to just sell a hybrid bike because it would not satisfy the high-performance portions of the road and mountain bike segments. The lower functionality of a universal design for some segments implies that it might not capture the same total demand as a set of focused designs.
- A universal design may be more expensive or it may be cheaper to produce than focused products. Because a universal design is targeted to many different uses, either it has components that are not necessary to some consumers or it has components that are of better quality than needed by certain consumers. These extra components or the extra quality increases a universal design's cost relative to focused designs. However, it is often cheaper to manufacture or procure a large quantity of a single component than small quantities of a bunch of components; that is, there are economies of scale in production and procurement. In that sense, a universal design may be cheaper.
- A universal design may eliminate some brand/price segmentation opportunities. By definition, a universal design has a single brand/price, but a firm may wish to maintain distinct brands/prices. As with the concern regarding functionality, a single brand/price may not be able to capture the same demand as multiple brands/prices.

With respect to O'Neill's Hammer 3/2 wetsuit, it appears that the first two concerns regarding a universal design are not relevant: Given that the surf and dive Hammers are identical with the exception of the logo, their functionality should be identical as well, and there is no reason to believe their production costs should be much different. However, the universal Hammer wetsuit does eliminate the opportunity to maintain two different O'Neill logos, one geared for the surf market and one geared for the dive market. If it is important to maintain these separate identities (e.g., you might not want serious surfers to think they are purchasing the same product as recreational divers), then maybe two suits are needed. On the other hand, if you wish to portray a single image for O'Neill, then maybe it is even better to have a single logo, in which case two different wetsuits make absolutely no sense.

While we have concentrated on the benefits of serving demand with a universal design, this discussion provides a warning for firms that may be engaging in excessive product proliferation. Every firm wishes to be "customer focused" or "customer oriented," which suggests that a firm should develop products to meet all of the needs of its potential customers. Truly innovative new products that add to a firm's customer base should be incorporated into a firm's product assortment. But if extra product variety merely divides a fixed customer base into smaller pieces, then the demand–supply mismatch cost for each product will increase. Given that some of the demand–supply mismatch costs are indirect (e.g., loss of goodwill due to poor service), a firm might not even realize the additional costs it bears due to product proliferation. Every once in a while a firm realizes that its product assortment has gone amok and *product line rationalization* is sorely needed. The trick to assortment reductions is to "cut the fat, but leave the meat (and surely the bones)"; that is, products should only be dropped if they merely cannibalize demand from other products.

15.3 Lead Time Pooling: Consolidated Distribution and Delayed Differentiation

Location and product pooling, discussed in the previous two sections, have limitations: location pooling creates distance between inventory and customers and product pooling potentially degrades product functionality. This section studies two strategies that address those limitations: consolidated distribution and delayed differentiation. Both of those strategies use a form of risk pooling that we call lead time pooling.

Consolidated Distribution

The key weakness of location pooling is that inventory is moved away from customers, thereby preventing customers from physically seeing a product before purchase, thus increasing the time a customer must wait to receive a product and generally increasing the delivery cost. However, as we have learned, it also can be costly to position inventory near every customer. A major reason for this cost is the problem of having product in the wrong place. For example, with Medtronic's approximately 500 sales territories, it is highly unlikely that all 500 territories will stock out at the same time. If a stockout occurs in one territory, it is quite likely that there is some other territory that has inventory to spare, even maybe a nearby territory. This imbalance of inventory occurs because a firm faces two different kinds of uncertainty, even with a single product: uncertainty with respect to total demand (e.g., how many InSync pacemakers are demanded in the United States on a particular day) and uncertainty with respect to the allocation of that demand (e.g., how many InSync pacemakers are demanded in each territory in the United States on a particular day). The consolidated-distribution strategy attempts to keep inventory close to customers while hedging against the second form of uncertainty.

We'll demonstrate the consolidated-distribution strategy via a retail example. Imagine demand for a single product occurs in 100 stores and average weekly demand per store follows a Poisson distribution with a mean of 0.5 unit per week. Each store is replenished directly from a supplier with an eight-week lead time. To provide good customer service, the retailer uses the order-up-to model and targets a 99.5 percent in-stock probability. The top panel of Figure 15.9 displays a schematic of this supply chain. Let's evaluate the amount of inventory the retailer needs.

With an eight-week lead time and a mean demand of 0.5 unit per week, the expected demand over $l + 1$ periods is $(8 + 1) \times 0.5 = 4.5$. From the Poisson Distribution Function Table in Appendix B we see that with a mean of 4.5, the order-up-to level $S = 10$ yields an in-stock probability of 99.33 percent and $S = 11$ yields an in-stock probability of 99.76 percent, so we need to choose $S = 11$ for each store. According to the Poisson Loss Function Table in Appendix B, with mean demand of 4.5 units over $l + 1$ periods and an order-up-to level $S = 11$, the expected back order is 0.00356 unit per week. Hence, each of the 100 stores will have the following expected inventory:

$$\begin{aligned}\text{Expected inventory} &= S - \text{Expected demand over } l + 1 \text{ periods} \\ &\quad + \text{Expected back order} \\ &= 11 - 4.5 + 0.00356 \\ &= 6.50356\end{aligned}$$

The total inventory among the 100 stores is then $6.504 \times 100 = 650.4$ units.

Now suppose the retailer builds a distribution center to provide consolidated distribution. The distribution center receives all shipments from the supplier and then replenishes each of the retail stores; it allows for consolidated distribution. The lead time for the distribution

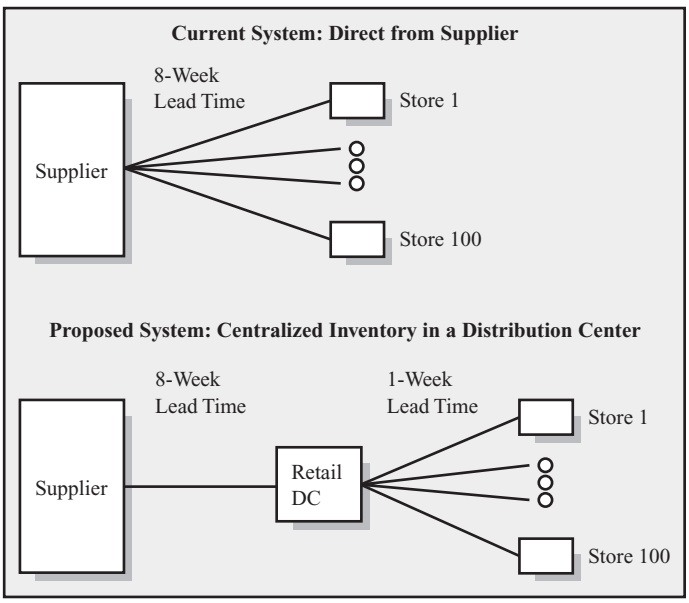


FIGURE 15.9
Two Retail Supply Chains, One with Direct Shipments from the Supplier, the Other with Consolidated Distribution in a Distribution Center
Expected weekly demand at each store is 0.5 unit and the target in-stock probability is 99.5 percent.

center remains eight weeks from the supplier. The lead time to replenish each of the retail stores is one week. To ensure a reliable delivery to the retail stores, the distribution center operates with a high in-stock probability, 99.5 percent. The bottom panel in Figure 15.9 displays the proposed supply chain with a distribution center.

The distribution center provides the retailer with a centralized location for inventory while still allowing the retailer to position inventory close to the customer. In contrast, the location pooling strategy would just create the centralized inventory location, eliminating the 100 stores close to customers. Therefore, this centralized-inventory strategy resembles location pooling without the major drawback of location pooling. But what does it do for the total inventory investment?

We can repeat the evaluation of the inventory investment for each store, assuming a 99.5 percent in-stock probability target and now a one-week lead time. From the Poisson Distribution Function Table, given expected demand over $l + 1$ periods is 1.0 unit, the order-up-to level $S = 4$ generates an in-stock probability of 99.63 percent. The resulting expected inventory per store is 3.00 units, nearly a 54 percent reduction in inventory from the direct-supply model (3.00 versus 6.5 units)! Because each store now receives a one-week lead time instead of an eight-week lead time, the inventory at the retail stores is dramatically reduced.

Now we need to evaluate the inventory at the distribution center. The demand at the distribution center equals the orders from the retail stores. On average, the retail stores order 0.5 unit per week; that is, the average inflow (i.e., order) into a store must equal the average outflow (i.e., demand), otherwise inventory either builds up continuously (if the inflow exceeds the outflow) or dwindles down to zero (if the outflow exceeds the inflow). Because the retail stores' total demand is $100 \times 0.5 = 50$ units per week, the average demand at the distribution center also must be 50 units per week.

While we can be very sure of our estimate of the distribution center's expected demand, the distribution center's standard deviation of demand is not immediately apparent. The standard deviation of demand at each retailer is $\sqrt{0.50} = 0.707$. (Recall that with Poisson demand, the standard deviation equals the square root of the mean.) Hence, if demand were independent across all stores, then the standard deviation of total demand would be

$0.707 \times \sqrt{100} = 7.07$. However, if there is positive correlation across stores, then the standard deviation would be higher, and with negative correlation the standard deviation would be lower. The only way to resolve this issue is to actually evaluate the standard deviation of total demand from historical sales data (the same data we used to estimate the demand rate of 0.5 unit per week at each store). Suppose we observe that the standard deviation of total weekly demand is 15. Hence, there is evidence of positive correlation in demand across the retail stores.

We now need to choose a distribution to represent demand at the distribution center. In this case, the Poisson is not the best choice. The standard deviation of a Poisson distribution is the square root of its mean, which in this case would be $\sqrt{50} = 7.07$. Because we have observed the standard deviation to be significantly higher, the Poisson distribution would not provide a good fit with the data. Our alternative, and a reasonable choice, is the normal distribution with mean 50 and standard deviation 15. Using the techniques from Chapter 14, we can determine that the distribution center's expected inventory is about 116 units if its target in-stock is 99.5 percent, the lead time is eight weeks, and weekly demand is normally distributed with mean 50 and standard deviation 15.

The only inventory that we have not counted so far is the pipeline inventory. In the direct-delivery model, there is pipeline inventory between the supplier and the retail stores. Using Little's Law, that pipeline inventory equals $0.5 \times 100 \times 8 = 400$ units. The consolidated-distribution model has the same amount of inventory between the supplier and the distribution center. However, with both models let's assume that pipeline inventory is actually owned by the supplier (e.g., the retailer does not start to pay for inventory until it is received). Hence, from the retailer's perspective, that inventory is not a concern. On the other hand, the retailer does own the inventory between the distribution center and the retail stores in the consolidated-distribution model. Again using Little's Law, there are $0.5 \times 100 \times 1 = 50$ units in that pipeline.

Table 15.4 summarizes the retailer's inventory in both supply chain structures. For comparison, the location pooling strategy is also included. With location pooling, all of the stores are eliminated and the retailer ships to customers from a central distribution center. Because that distribution center has an eight-week lead time and faces the same demand distribution as the DC in the consolidated-distribution strategy, its expected inventory is also 116 units.

We see from Table 15.4 that the consolidated-distribution strategy is able to reduce the expected inventory investment 28 percent [(650 − 466)/650] relative to the original direct-delivery structure. In fact, the advantage of the consolidated-distribution strategy is even better than this analysis suggests. The cost of holding one unit of inventory at a retail store is surely substantially higher than the cost of holding one unit in a distribution center: retail shelf space is more expensive than DC space, shrinkage is a greater concern, and so forth. Because the consolidated-distribution model reduces retail inventory by more than 50 percent, merely adding up the total inventory in the system underestimates the value of the consolidated-distribution model.

TABLE 15.4
Retail Inventory with Three Supply Chain Structures

	Direct Delivery Supply Chain	Consolidated-Distribution Supply Chain	Location Pooling
Expected total inventory at the stores	650	300	0
Expected inventory at the DC	0	116	116
Pipeline inventory between the DC and the stores	0	50	0
Total	650	466	116

Interestingly, the consolidated-distribution model outperforms direct delivery even though the total lead time from the supplier to the retail stores is increased by one week due to the routing of all inventory through the DC. Why is inventory reduced despite the longer total lead time? As mentioned earlier, in this system there are two types of uncertainty: uncertainty with total demand in a given week and uncertainty with the allocation of that demand over the retail stores. When inventory leaves the supplier, the retailer is essentially betting on how much inventory will be needed eight weeks later. However, in the direct-delivery model, the retailer also must predict *where* that inventory is needed; that is, the retailer must gamble on a total quantity and an allocation of that quantity across the retail stores. There is uncertainty with the total inventory needed, but even more uncertainty with where that inventory is needed. The consolidated-distribution model allows the retailer to avoid that second gamble: The retailer only needs to bet on the amount of inventory needed for the central distribution center. In other words, while the retailer must commit to a unit's final destination in the direct-delivery model, in the consolidated-distribution model the retailer delays that commitment until the unit arrives at the distribution center. It is precisely because the DC allows the retailer to avoid that second source of uncertainty that the consolidated-distribution model can outperform the direct-delivery model.

The consolidated-distribution model exploits what is often called *lead time pooling*. Lead time pooling can be thought of as combining the lead times for multiple inventory locations. Actually, it is easier to explain graphically: in Figure 15.9 we see that the 100 connections between the supplier and the retail stores in the direct-delivery model (four of which are actually drawn) are pooled into a single connection between the supplier and the DC in the consolidated-distribution model.

We saw that demand correlation influenced the effectiveness of product pooling and location pooling. Not surprisingly, demand correlation has the same effect here. The greater the correlation, the higher the standard deviation of demand at the distribution center. Figure 15.10 displays supply chain inventory with the consolidated-distribution model over a range of demand variability for the distribution center. As retail demand becomes

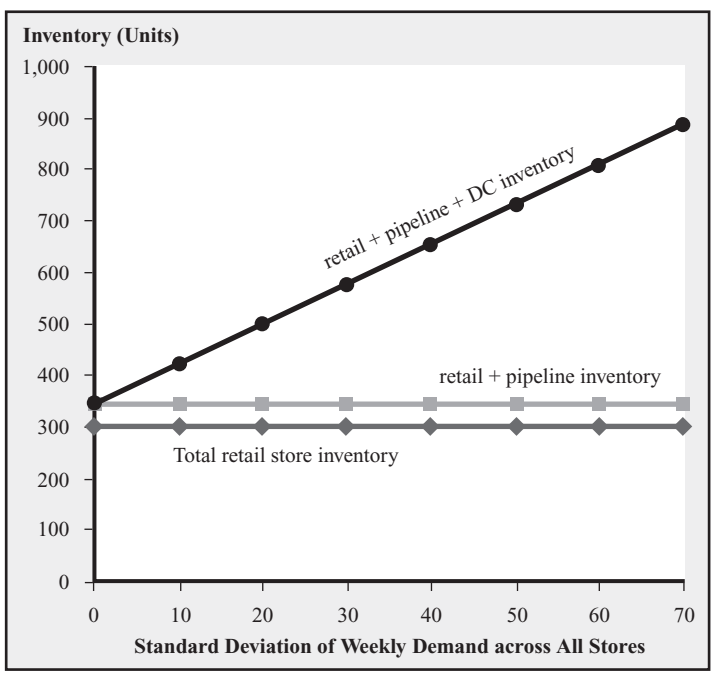


FIGURE 15.10
Inventory with the Consolidated-Distribution Supply Chain
Diamonds = total retail store inventory, squares = retail + pipeline inventory, circles = retail + pipeline + DC inventory.

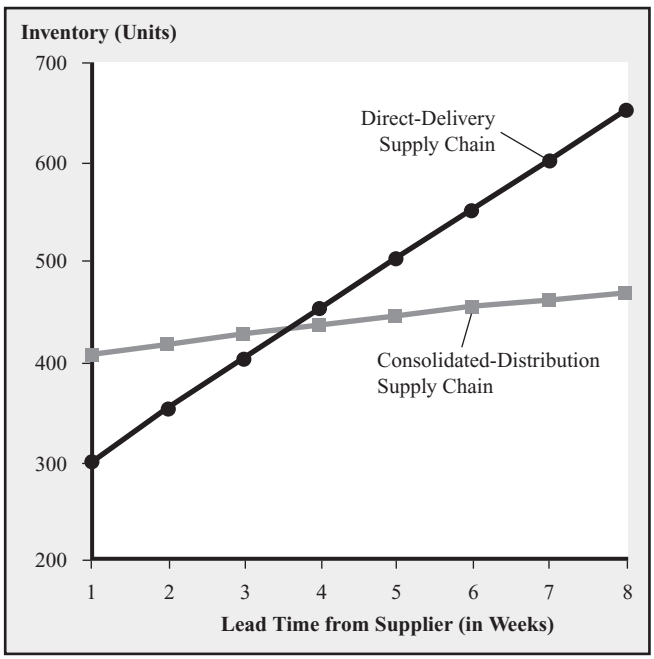


FIGURE 15.11
Inventory with the Consolidated-Distribution Supply Chain (squares) and the Direct-Delivery Supply Chain (circles) with Different Supplier Lead Times

more negatively correlated, the inventory in the consolidated-distribution model declines. However, we have seen that inventory can be reduced even with some positive correlation: The consolidated-distribution model outperforms direct delivery if the DC's standard deviation is about 40 or lower.

Another factor that determines the attractiveness of the consolidated-distribution model relative to the direct-delivery model is the lead time from the supplier. Figure 15.11 displays total supply chain inventory with both models for various supplier lead times. The direct-delivery model performs better than the consolidated-distribution model if the supplier's lead time is three weeks or fewer; otherwise, the consolidated-distribution model does better. This occurs because lead time pooling is most effective as the lead time increases. In particular, the lead time before the distribution center (i.e., from the supplier) should be longer than the lead time after the distribution center (i.e., to the stores).

To summarize, a central inventory location (i.e., a distribution center) within a supply chain can exploit lead time pooling to reduce the supply chain's inventory investment while still keeping inventory close to customers. This strategy is most effective if total demand is less variable than demand at the individual stores and if the lead time before the distribution center is much longer than the lead time after the distribution center.

While we have concentrated on the inventory impact of the consolidated distribution strategy, that strategy has other effects on the supply chain. We have not included the extra cost of operating the distribution center, even though we did mention that the holding cost for each unit of inventory at the distribution center is likely to be lower than at the retail stores. Furthermore, we have not included the extra transportation cost from the DC to the retailer. A common critique of this kind of supply chain is that it clearly increases the distance a unit must travel from the supplier to the retailer. However, there are some additional benefits of a distribution center that we also have not included.

A DC enables a retailer to take better advantage of temporary price discounts from the supplier; that is, it is easier to store a large buy at the DC than at the retail stores. (See the Trade Promotions and Forward Buying part of Section 17.1 for an analytical model of this issue.) The DC also will facilitate more frequent deliveries to the retail stores. With the

direct-delivery model, each store receives a shipment from each supplier. It is generally not economical to make partial truckload shipments, what is referred to as a "less-than-load" or LTL shipment. Therefore, in our example, the retailer receives weekly shipments from the supplier because the retailer would not be able to order a full truckload for each store on a more frequent basis.

But with a DC, more frequent shipments are economical. The DC allows the retailer to put products from multiple suppliers into a truck bound for a store. Because now a truck is filled with products from multiple suppliers, it can be filled more frequently. As a result, with the DC in the supply chain, each store might be able to receive a full truckload per day, whereas without the DC each store can only receive a shipment every week. (This argument also is used to justify the airlines' "hub-and-spoke" systems: It may be difficult to consistently fill a plane from Gainesville to Los Angeles on a daily basis, but Delta Airlines offers service between those two cities via its Atlanta hub because the Atlanta–Los Angeles leg can be filled with passengers flying from other southeast cities.) More frequent deliveries reduce inventory even further than our analysis suggests. (See Section 14.8 for more discussion.) Even the DC may be able to order more frequently from the supplier than weekly because the DC consolidates the orders from all of the retailers. In fact, while the lead time pooling benefit of a DC in this example is significant, it is quite possible that some of these other reasons for operating a DC are even more important.

Delayed Differentiation

Consolidated distribution is a strategy that uses lead time pooling to provide some of the benefits of location pooling without moving inventory far away from customers. Delayed differentiation is the analogous strategy with respect to product pooling; that is, delayed differentiation hedges the uncertainty associated with product variety without taking the variety away from customers. We'll illustrate delayed differentiation with our Hammer 3/2 example from O'Neill.

Recall that the Hammer 3/2 is sold by O'Neill in two versions: a surf wetsuit with the traditional wave logo silk-screened on the chest and a dive wetsuit with O'Neill's dive logo put in the same place. The product-pooling approach to this variety is to eliminate it: sell only one Hammer 3/2 suit with a single logo. However, that is an extreme solution and there may be reasons to maintain two different products.

The problem with two different products is that we might run out of surf Hammers while we have extra dive Hammers. In that situation, it would be great if we could just erase the dive logo and put on the surf logo, since the rest of the wetsuit is identical. Better yet, if we just stocked "logo-less" or generic wetsuits, then we could add the appropriate logo as demand arrives. That strategy is called *delayed differentiation* because we are delaying the differentiation of the wetsuit into its final form until after we observe demand.

Several things are necessary to make this delayed-differentiation strategy work. First, we need to be able to silk-screen the logo onto the generic wetsuit. This is a nontrivial issue. Currently the logo is silk-screened onto the chest piece before it is sewn into the suit. Silk-screening the logo onto a complete suit is substantially harder and may require some redesigning of the silk-screening process. Assuming we can overcome that technical difficulty, we still need to be able to add the silk screen quickly so that there is not much delay between the time a wetsuit is requested and when it is shipped. Hence, we'll need a sufficient amount of idle capacity in that process to ensure fast delivery even though demand may fluctuate throughout the season.

If these challenges are resolved, then we are left with deciding how many of the generic wetsuits to order and evaluating the resulting profit savings. In fact, we have already completed those steps. If we assume that we only silk-screen the logo onto wetsuits when we

receive a firm demand for a surf or dive wetsuit, then we never keep finished goods inventory; that is, we only have to worry about our generic wetsuit inventory. The demand for the generic wetsuit is identical to the demand for the universal wetsuit; that is, it is the sum of surf Hammer demand and dive Hammer demand. The economics of the generic suit are the same as well: They sell for the same price, they have the same production cost, and we'll assume they have the same salvage value. (In some cases, the salvage value of the generic suit might be higher or lower than the salvage value of the finished product, but in this case it is plausibly about the same.) Therefore, as with the universal design analysis, we need to decide how many generic wetsuits to order given they are sold for $190 each, they cost $110 each, they will be salvaged for $90 each, and demand is normally distributed with mean 6,384 and standard deviation 1,670.

Using our analysis from the section on product pooling, the optimal order quantity is 7,840 units with the delayed differentiation strategy and expected profit increases to $463,920. Although product pooling and delayed differentiation result in the same numerical analysis, the two strategies are different. Delayed differentiation still offers multiple wetsuits to consumers, so their demands are not pooled together as with a universal design. Instead, delayed differentiation works like lead time pooling with consolidated distribution: a key differentiating feature of the product is delayed until after better demand information is observed; with location pooling that feature is the product's final destination (i.e., store) and with delayed differentiation that feature is the product's logo. Furthermore, product pooling does not require a significant modification to the production process, whereas delayed differentiation does require a change to the silk-screening process. In other applications, delayed differentiation may require a more dramatic change to the process and/or the product design.

In general, delayed differentiation is an ideal strategy when

1. Customers demand many versions, that is, variety is important.
2. There is less uncertainty with respect to total demand than there is for individual versions.
3. Variety is created late in the production process.
4. Variety can be added quickly and cheaply.
5. The components needed to create variety are inexpensive relative to the generic component (i.e., the main body of the product).

Let's explain further each of the five points just mentioned. (1) If variety isn't important, then the firm should offer fewer variants or just a universal design. (2) There should be less uncertainty with total demand so there will be few demand–supply mismatches with the generic component. In general, the more negative correlation across product variants the better, since negative correlation reduces uncertainty in the total demand. (3) Just as we saw that consolidated distribution works best if the supplier lead time to the distribution center is long relative to the lead time from the distribution center to the retail stores, delayed differentiation is most valuable if there is a long lead time to produce the generic component and a short lead time to convert the generic component into a finished product. (4) If adding variety to the generic component is too slow, then the waiting time for customers may be unacceptable, thereby rendering delayed differentiation unacceptable. In addition, if adding variety at the end of the process is costly, then the inventory savings from delayed differentiation may not be worth the extra production cost. (5) Finally, delayed differentiation saves inventory of the generic component (e.g., the generic wetsuit) but does not save inventory of the differentiating components. Hence, delayed differentiation is most useful if the majority of the product's value is in the generic component.

Delayed differentiation is particularly appropriate when variety is associated with the cosmetic features of a product, for example, color, labels, and packaging. For example, suppose

a company such as Black and Decker sells power drills to both Home Depot and Walmart. Those are two influential retailers; as a result, they may wish to have slightly different packaging, and, in particular, they might wish to have different product codes on their packages so that consumers cannot make direct price comparisons. The power drill company could store drills in the two different packages, but that creates the possibility of having Home Depot drills available while Walmart drills are stocked out. Because it is relatively easy to complete the final packaging, the delayed-differentiation strategy only completes the packaging of drills after it receives firm orders from the retailers. Furthermore, packaging material is cheap compared to the drill, so while the firm doesn't want to have excessive inventory of drills, it isn't too costly to have plenty of packages available.

Retail paints provide another good example for the application of delayed differentiation. Consumers surely do not want a universal design when it comes to paint color, despite Henry Ford's famous theory of product assortment.[2] But at the same time, a store cannot afford to keep paint available in every possible shade, hue, tone, sheen, and color. One alternative is for paint to be held in a central warehouse and then shipped to customers as needed, that is, a location pooling strategy. Given the vast variety of colors, it is not clear that even a location pooling strategy can be economical. Furthermore, paint is very costly to ship directly to consumers, so that pretty much kills that idea. Instead, the paint industry has developed equipment so that a retailer can use generic materials to mix any color in their vast catalog. The final production process takes some time, but an acceptable amount of time for consumers (5 to 15 minutes). The in-store production equipment is probably more expensive than mixing paints at a factory, but again, the extra cost here is worth it. Hence, by redesigning the product to add variety at the very end of the production process (i.e., even after delivery to the retail store), paint companies are able to economically provide consumers with extensive variety.

Delayed differentiation can even be used if the "generic component" can be sold to some customers without additional processing. To explain, suppose a company sells two different quality levels of a product, for example, a fast and a slow printer or a fast and a slow microprocessor. These quality differences might allow a firm to price discriminate and thereby increase its overall margins. However, the quality difference might not imply radically different costs or designs. For example, it might be possible to design the fast and the slow printers such that a fast printer could be converted into a slow printer merely by adding a single chip or by flipping a single switch. Hence, the firm might hold only fast printers so they can serve demand for fast printers immediately. When demand for a slow printer occurs, then a fast printer is taken from inventory, the switch is flipped to make it a slow printer, and then it is shipped as a slow printer.

Delayed differentiation is indeed a powerful strategy. In fact, it bears a remarkable resemblance to another powerful strategy, make-to-order production (Chapter 13). With make-to-order production, a firm only begins making a product after it receives a firm order from a customer. Dell Inc. has used the make-to-order strategy with remarkable effectiveness in the personal computer industry. With delayed differentiation, a generic component is differentiated into a final product only after demand is received for that final product. So what is the difference between these two ideas? In fact, they are conceptually quite similar. Their difference is one of degree. Delayed differentiation is thought of as a strategy that stores nearly finished product and completes the remaining few production steps with essentially no delay. Make-to-order is generally thought to apply to a situation in which the remaining production steps from components to a finished unit are more substantial, therefore involving more than a trivial delay. Hence, delayed differentiation and make-to-order occupy two ends of the same spectrum with no clear boundary between them.

[2] Consumers can have any Model T they want, as long as it is black.

15.4 Capacity Pooling with Flexible Manufacturing[3]

Delayed differentiation takes advantage of completely flexible capacity at the end of the manufacturing process; that is, the final production step is capable of taking a generic component and converting it into any final product. Unfortunately, the luxury of complete flexibility is not always available or affordable to a firm, especially if one considers a larger portion of the manufacturing process. This section studies how a firm can use risk pooling with flexible capacity, but not necessarily completely flexible capacity. See also Section 11.7 for additional discussion on capacity flexibility.

To provide a context, consider the manufacturing challenge of an auto manufacturer such as General Motors. GM operates many different assembly plants and produces many different vehicles. Assembly capacity is essentially fixed in this industry over a substantial time horizon due to rigid labor contracts and the extensive capital requirements of an assembly plant. However, demand for individual vehicles can be quite variable: some products are perennially short on capacity, while others seem to always have too much capacity. To alleviate the resulting demand–supply mismatches, auto manufacturers continually strive for more manufacturing flexibility, that is, the ability to produce more than one vehicle type with the same capacity. GM could use flexible manufacturing to move capacity from slow-selling products to fast-selling products, thereby achieving higher sales and higher capacity utilization. But flexibility is not free: Tooling and assembly equipment capable of making more than one vehicle is more expensive than dedicated equipment and equipment capable of making any vehicle (complete flexibility) is extremely expensive. So how much flexibility does GM need and where should that flexibility be installed?

Let's define a specific problem that is representative of the challenge GM faces. There are 10 manufacturing plants and 10 vehicles (e.g., Chevy Malibu, GMC Yukon XL, etc). For now each plant is assigned to produce just one vehicle, that is, there is no flexibility in the network. Capacity for each vehicle is installed before GM observes the vehicle's demand in the market. Demand is uncertain: a normal distribution represents each vehicle's demand with mean 100 and standard deviation 40. For a slight twist on the distribution, let's assume the minimum demand is 20 and the maximum demand is 180; that is, the normal distribution is truncated so that excessively extreme outcomes are not possible.[4] Even though we impose upper and lower bounds on demand, demand is still quite uncertain, a level of uncertainty that is typical in the auto industry. One last point with respect to demand: We assume the demands for each vehicle are independent; therefore, the correlation between the demands for any two vehicles is zero.

Each plant has a capacity to produce 100 units. If demand exceeds capacity for a vehicle, then the excess is lost. If demand is less than capacity, then demand is satisfied but capacity is idle. Figure 15.12 displays this situation graphically: The left-hand side of the figure represents the 10 production plants; the right-hand side represents the 10 vehicle types; and the lines are "links" that indicate which plant is capable of producing which vehicles. In the "no flexibility" situation, each plant is capable of producing only one vehicle, so there is a total of 10 links. The configuration with the smallest amount of flexibility has 11 links, an example of which is displayed on the right-hand side of Figure 15.12. With 11 links, one plant is capable of producing two different vehicles. As we add more links, we add more flexibility. Total flexibility is achieved when we have 100 links, that is, every

[3] This section is based on the research reported in Jordon and Graves (1995).

[4] In other words, any outcome of the normal distribution that is either lower than 20 or higher than 180 is ignored and additional random draws are made until an outcome is received between 20 and 180. There is only a 4.6 percent chance that an outcome of a normal distribution is greater than two standard deviations from the mean (as in this case).

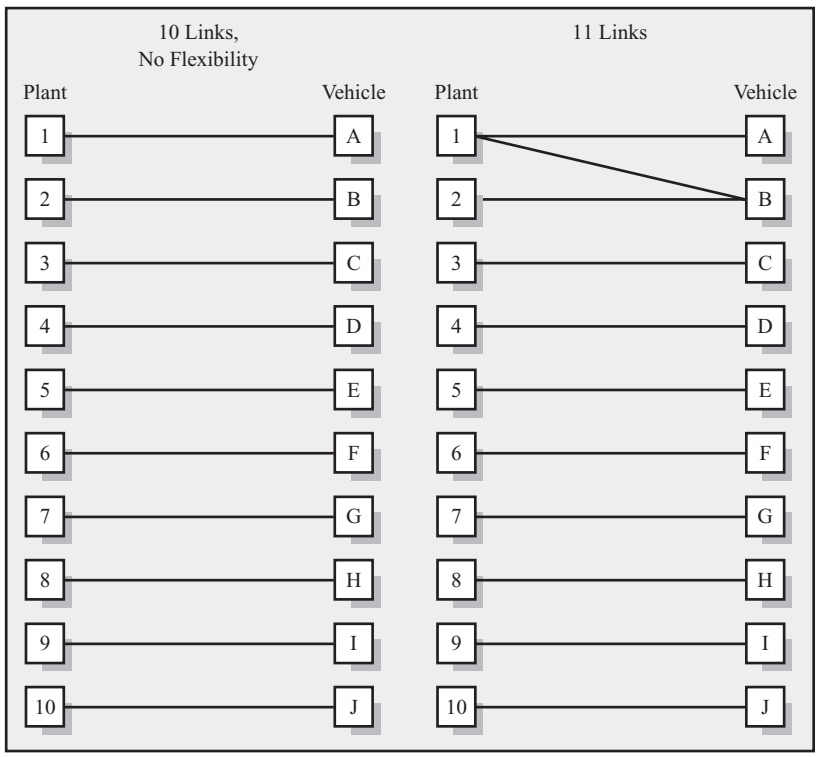


FIGURE 15.12
Two Configurations, One with No Flexibility (10 links) and One with Limited Flexibility (11 links)

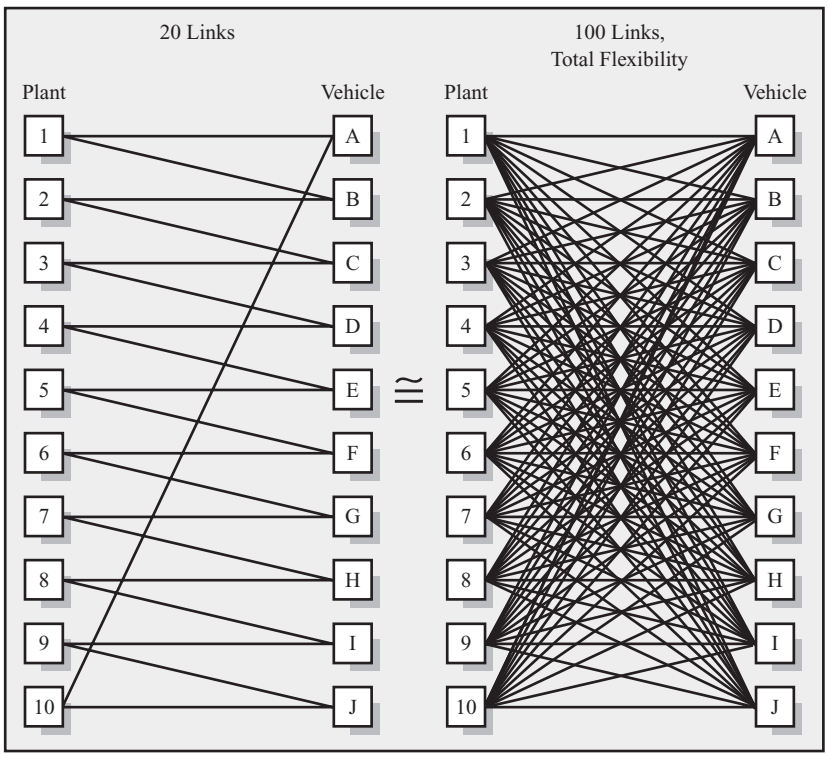
plant is able to produce every product. Figure 15.13 displays the full flexibility configuration as well as one of the possible configurations with 20 links.

With each configuration, we are interested in evaluating the expected unit sales and expected capacity utilization. Unfortunately, for most configurations, it is quite challenging to evaluate those performance measures analytically. However, we can obtain accurate estimates of those performance measures via simulation. Each iteration of the simulation draws random demand for each product and then allocates the capacity to maximize unit sales within the constraints of the feasible links. For example, in the configuration with 11 links displayed in Figure 15.12, suppose in one of the iterations that demand for vehicle A is 85 units and vehicle B is 125 units. In that case, plant 2 uses its entire 100 units of capacity to produce vehicle B and plant 1 uses its entire 100 units of capacity to produce 85 units of vehicle A and 15 units of vehicle B, thereby only losing 10 units of potential vehicle B sales. Our estimate of each performance measure is just its average across the iterations. After many iterations, our estimates will be quite accurate.

Via simulation we find that with no flexibility, expected unit sales are 853 units and expected capacity utilization is 85.3 percent. With 11 links, the expected unit sales increase to 858 units and capacity utilization increases to 85.8 percent. We do slightly better with this additional flexibility when demand for vehicle B exceeds plant 2's capacity and demand for vehicle A is below plant 1's capacity, because then plant 1 can use its capacity to produce both vehicles A and B (as in our previous example). Figure 15.14 provides data on the performance of configurations with 10 to 20 links and the full flexibility configuration.

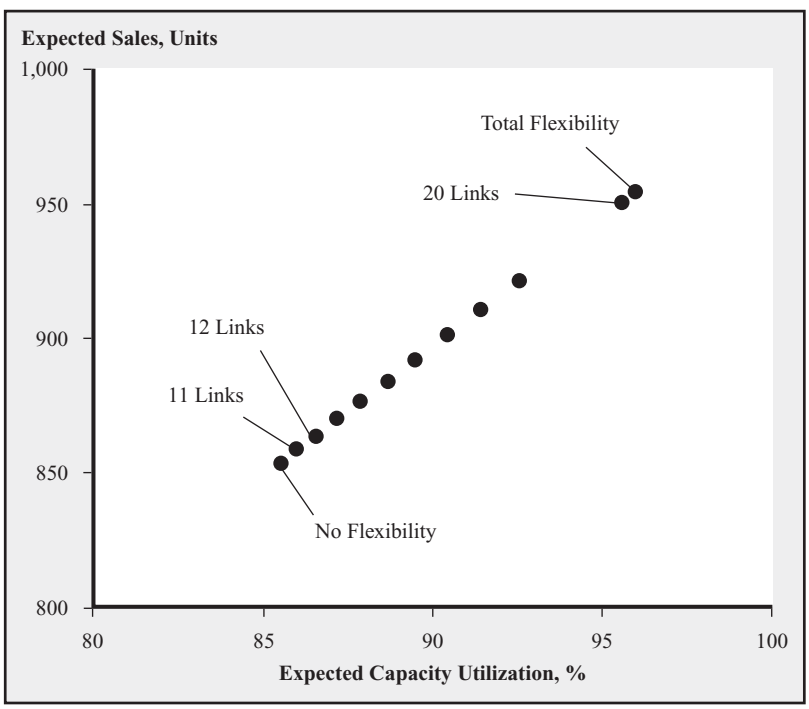
Figure 15.14 reveals that total flexibility is able to increase our performance measures considerably: Capacity utilization jumps to 95.4 percent and expected sales increase to 954 units. But what is more remarkable is that adding only 10 additional links produces nearly the same outcome as full flexibility, which has an additional 90 links: capacity utilization is 94.9 percent with 20 links and expected sales are 949 units. Apparently, there is very little incremental value to the additional flexibility achieved by adding the 11th

FIGURE 15.13
Flexibility Configurations with Approximately Equal Capability to Respond to Demand Uncertainty



through the 90th additional links to the no-flexibility configuration. In other words, given that installing flexibility is costly, it is unlikely that total flexibility will be economically rational. This result has a similar feel to our finding that with location pooling, the majority of the benefit is captured by pooling only a few locations.

FIGURE 15.14
Impact of Incrementally Adding Flexibility on Expected Sales and Capacity Utilization



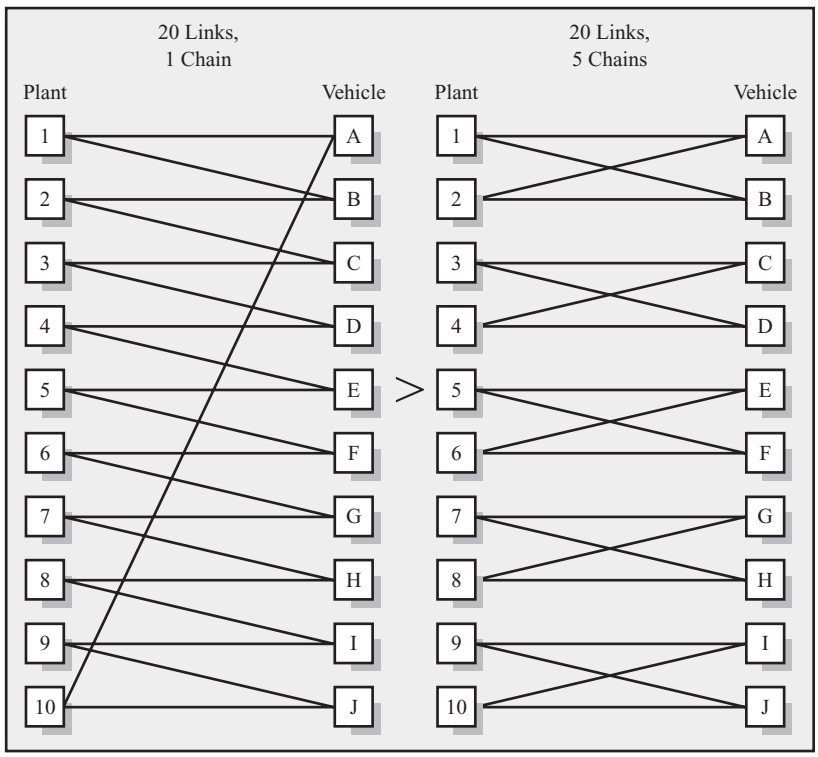


FIGURE 15.15
Flexibility Configurations with the Same Number of Links but Different Number of Chains

It may seem surprising that capacity pooling increases utilization, given that pooling server capacity in a queuing system has no impact on utilization, as discussed in Chapter 9. The key difference is that in a queuing system, demand is never lost; it just has to wait longer than it might want to be served. Hence, the amount of demand served is independent of how the capacity is structured. Here, demand is lost if there isn't a sufficient amount of capacity. Therefore, more flexibility increases the demand served, which increases the utilization of the capacity.

Although flexibility with 20 links can perform nearly as well as total flexibility with 100 links, not every configuration with 20 links performs that well. Figure 15.13 displays the particular 20-link configuration that nearly equals total flexibility. The effectiveness of that configuration can be explained by the concept of *chaining.* A chain is a group of plants and vehicles that are connected via links. For example, in the 11-link configuration displayed in Figure 15.12, the first two plants and vehicles form a single chain and the remaining plant–vehicle pairs form eight additional chains. With the 20-link configuration displayed in Figure 15.13, there is a single chain, as there is with the total flexibility configuration.

In general, flexibility configurations with the longest and fewest chains for a given number of links perform the best. Figure 15.15 displays two 20-link configurations, one with a single chain (the same one as displayed in Figure 15.13) and the other with five chains. We already know that the single chain configuration has expected sales of 949 units. Again via simulation, we discover that the 20-link configuration with five chains generates expected sales of only 896 units, which compares to the 853 expected unit sales with no-flexibility.

Long chains are beneficial because they facilitate the reallocation of capacity to respond to demand. For example, suppose demand for vehicle A is less than expected, but demand for vehicle G is very strong. If both vehicles are in the same chain, then plant 1's idle capacity can be shifted along the chain to help fill vehicle G's demand: plant 1 produces some vehicle B, plant 2 produces some of both vehicles B and C, and so forth so that both plants 6 and 7 can produce some vehicle G. If both of those vehicles are not part of the same chain (as in our five-chain configuration), then this swapping of capacity is not possible.

In addition to how flexibility is configured, there are two additional issues worth mentioning that influence the value of flexibility: correlation and total capacity. So far we have assumed that demands across vehicles are independent. We learned with the other risk-pooling strategies that risk pooling becomes more effective as demand becomes more negatively correlated. The same holds here: With pooled capacity, the uncertainty in total demand is more important than the uncertainty with individual products; hence, negative correlation is preferred. However, this does not mean that two negatively correlated products must be produced in the same plant. Instead, it is sufficient that two negatively correlated products are produced in the same chain. This is a valuable insight if the negatively correlated products are physically quite different (e.g., a full-size truck and a compact sedan) because producing them in the same chain might be far cheaper than producing them in the same plant.

The total available capacity also influences the effectiveness of flexibility. Suppose capacity for each plant were only 20 units. In that case, each plant would always operate at 100 percent utilization, so flexibility has no value. The end result is the same with the other extreme situation. If each plant could produce 180 units, then flexibility is again not needed because every plant is sure to have idle capacity. In other words, flexibility is more valuable when capacity and demand are approximately equal, as in our numerical examples.

Figure 15.16 further emphasizes that flexibility is most valuable with intermediate amounts of capacity: The biggest gap between the no-flexibility trade-off curve and the 20-link trade-off curve occurs when total capacity equals expected total demand, 1,000 units.

Figure 15.16 illustrates another observation: flexibility and capacity are substitutes. For example, to achieve expected sales of 950 units, GM can either install total capacity of 1,250 units with no flexibility or 1,000 units of capacity with 20-link flexibility. If capacity is cheap relative to flexibility, then the high-capacity–no-flexibility option may

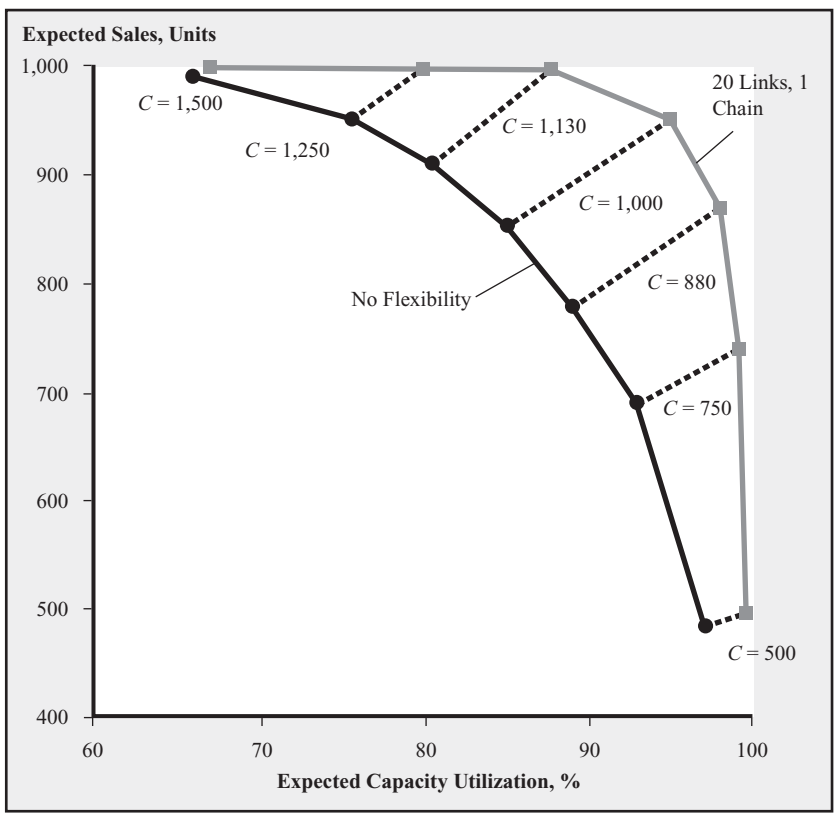


FIGURE 15.16
Expected Sales and Capacity Utilization
Shown are seven different capacities (C) and two configurations, one with no flexibility (10 links) and one with 20 links and one chain (displayed in Figure 15.15). In each case, the total capacity is equally divided among the 10 products and expected total demand is 1,000 units.

TABLE 15.5
Fiscal Year 2005 Results for Several Contract Manufacturers

Company	Revenue*	Cost of Goods*	Gross Margin
Hon-Hai Precision Industries	101,946	92,236	9.5%
Flextronics	28,680	27,166	5.3%
Jabil Circuits	13,409	12,148	9.4%
Celestica	6,526	6,012	7.9%
Sanmina-SCI	6,319	5,750	9.0%
Benchmark Electronics	2,402	2,174	9.5%
Plexus	2,013	1,770	12.1%

* In millions of dollars

be preferable. But if capacity is expensive relative to flexibility (especially given that we only need 10 additional links of flexibility), then the low-capacity–some-flexibility option may be better.

So far, our discussion has focused on a single firm and its flexibility within its own network of resources. However, if a firm cannot implement flexible capacity on its own, another option is to hire a firm that essentially provides this service for them. In fact, there is an entire industry doing just this—the contract manufacturing industry. These are companies that generally do not have their own products or brands. What they sell is capacity—flexible capacity that is used for all of their clients. For example, Flextronics could be assembling circuit boards for IBM, Hewlett-Packard, and Cisco Systems. The same equipment and often the same components are used by these multiple manufacturers, so, instead of each one investing in its own capacity and component inventory, Flextronics pools their needs. In other words, while any of these companies could produce its own circuit boards, because of capacity pooling. Flextronics is able to produce them with higher utilization and therefore lower cost. This added efficiency allows Flextronics to charge a margin, albeit a rather thin margin, as indicated in Table 15.5.

Figure 15.17 displays the revenue growth from seven leading electronics contract manufacturers. (There are contract manufacturers in other industries as well, such as pharmaceuticals.) It shows that the industry barely existed before 1992, and then there was a huge increase in sales leading up to 2000. The bursting of the telecom bubble gave the industry

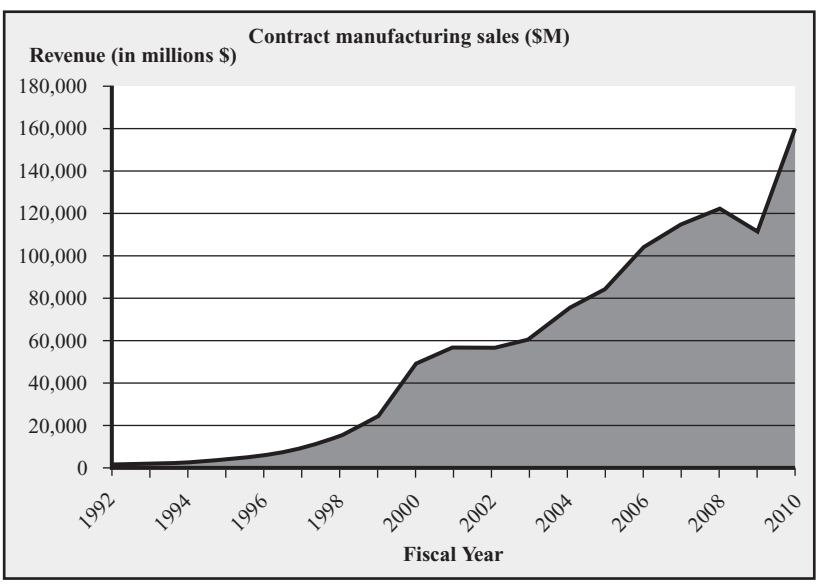


FIGURE 15.17
Total Revenue of Seven Leading Contract Manufacturers by Fiscal Year: Hon-Hai Precision Industries, Flextronics, Jabil Circuits, Celestica, Sanmina-SCI, Benchmark Electronics and Plexus.
Note: The fiscal years of these firms vary somewhat, so total revenue in a calendar year will be slightly different.

a pause in the early 2000s, and then it proceeded to gain momentum again, with the 2008–2009 dip caused by the worldwide recession. Hon-Hai Precision Industries is the clear leader in this group, as shown in Table 15.5. Apple is one of its more visible customers, and Apple's success has been a substantial contributor to Hon-Hai's revenue growth.

To summarize, this section considers the pooling of capacity via manufacturing flexibility. The main insights are

- A limited amount of flexibility can accommodate demand uncertainty nearly as well as total flexibility as long as the flexibility is configured to generate long chains.
- Flexibility should be configured so that negatively correlated products are part of the same chain but need not be produced in the same plant.
- Flexibility is most valuable when total capacity roughly equals expected demand.
- It may be possible to purchase flexibility by working with a contract manufacturer.

Therefore, it is generally neither necessary nor economically rational for a firm to sink the huge investment needed to achieve total flexibility. Flexibility is surely valuable, but it should not be installed haphazardly. Finally, while we have used the context of automobile manufacturing to illustrate these insights, they nevertheless apply to workers in service environments. For example, it is not necessary to cross-train workers so that they can handle every task (full flexibility). Instead, it is sufficient to train workers so that long chains of skills are present in the organization.

15.5 Summary

This chapter describes and explores several different strategies that exploit risk pooling to better match supply and demand. Each has its strengths and limitations. For example, location pooling is very effective at reducing inventory but moves inventory away from customers. Consolidated distribution is not as good as location pooling at reducing inventory, but it keeps inventory near customers. Product pooling with a universal design is also quite useful but might limit the functionality of the products offered. Delayed differentiation addresses that limitation but probably requires redesigning the product/process and may introduce a slight delay to fulfill demand. Capacity pooling can increase sales and capacity utilization but requires flexible capacity, which is probably not free and may be quite expensive. Hence, these are effective strategies as long as they are applied in the appropriate settings.

Even though we considered a variety of situations and models (e.g., order-up-to and newsvendor), we have developed some consistent observations:

- A little bit of risk pooling goes a long way. With location pooling, it is usually necessary to pool only a few locations, not all of them. With capacity pooling, a little bit of flexibility, as long as it is properly designed (i.e., long chains), yields nearly the same outcome as full flexibility.
- Risk-pooling strategies are most effective when demands are negatively correlated because then the uncertainty with total demand is much less than the uncertainty with any individual item/location. It follows that these strategies become less effective as demands become more positively correlated.
- Risk-pooling strategies do not help reduce pipeline inventory. That inventory can only be reduced by moving inventory through the system more quickly.
- Risk-pooling strategies can be used to reduce inventory while maintaining the same service (in-stock probability) or they can be used to increase service while holding the same inventory, or a combination of those improvements.

Table 15.6 provides a summary of the key notation and equations presented in this chapter.

TABLE 15.6
Summary of Notation and Key Equations in Chapter 15

The combination of two demands with the same mean and standard deviation yields

$$\text{Expected pooled demand} = 2 \times \mu$$

$$\text{Standard deviation of pooled demand} = \sqrt{2 \times (1 + \text{Correlation})} \times \sigma$$

$$\text{Coefficient of variation of pooled demand} = \sqrt{\frac{1}{2}(1 + \text{Correlation})} \times \left(\frac{\sigma}{\mu}\right)$$

15.6 Further Reading

In recent years, risk-pooling strategies have received considerable attention in the academic community as well as in practice.

Lee (1996) provides a technical treatment of the delayed-differentiation strategy. A more managerial description of delayed differentiation can be found in Feitzinger and Lee (1997). Brown, Lee, and Petrakian (2000) describe the application of delayed differentiation at a semiconductor firm. Simchi-Levi, Kaminsky, and Simchi-Levi (2003) and Chopra and Meindl (2004) cover risk-pooling strategies in the context of supply chain management.

Ulrich and Eppinger (2011) discuss the issues of delayed differentiation and product architecture from the perspective of a product development team.

Upton (1994, 1995) provides broad discussions on the issue of manufacturing flexibility.

15.7 Practice Problems

Q15.1* **(Egghead)** In 1997 Egghead Computers ran a chain of 50 retail stores all over the United States. Consider one type of computer sold by Egghead. Demand for this computer at each store on any given week was independently and normally distributed with a mean demand of 200 units and a standard deviation of 30 units. Inventory at each store is replenished directly from a vendor with a 10-week lead time. At the end of 1997, Egghead decided it was time to close their retail stores, put up an Internet site, and begin filling customer orders from a single warehouse.

a. By consolidating the demand into a single warehouse, what will be the resulting standard deviation of weekly demand for this computer faced by Egghead? Assume Egghead's demand characteristics before and after the consolidation are identical.

b. Egghead takes physical possession of inventory when it leaves the supplier and grants possession of inventory to customers when it leaves Egghead's shipping dock. In the consolidated distribution scenario, what is the pipeline inventory?

Q15.2* **(Two Products)** Consider two products, A and B. Demands for both products are normally distributed and have the same mean and standard deviation. The coefficient of variation of demand for each product is 0.6. The estimated correlation in demand between the two products is -0.7. What is the coefficient of variation of the total demand of the two products?

Q15.3* **(Fancy Paints)** Fancy Paints is a small paint store. Fancy Paints stocks 200 different SKUs (stock-keeping units) and places replenishment orders weekly. The order arrives one month (let's say four weeks) later. For the sake of simplicity, let's assume weekly demand for each SKU is Poisson distributed with mean 1.25. Fancy Paints maintains a 95 percent in-stock probability.

a. What is the average inventory at the store at the end of the week?

b. Now suppose Fancy Paints purchases a color-mixing machine. This machine is expensive, but instead of stocking 200 different SKU colors, it allows Fancy Paints to stock only five basic SKUs and to obtain all the other SKUs by mixing. Weekly demand for each SKU is normally distributed with mean 50 and standard deviation 8. Suppose Fancy Paints maintains a 95 percent in-stock probability for each of the five colors. How much inventory on average is at the store at the end of the week?

c. After testing the color-mixing machine for a while, the manager realizes that a 95 percent in-stock probability for each of the basic colors is not sufficient: Since mixing requires the presence of multiple mixing components, a higher in-stock probability for components is needed to maintain a 95 percent in-stock probability for the individual SKUs. The manager decides that a 98 percent in-stock probability for each of the five basic SKUs should be

(* indicates that the solution is at the end of the book)

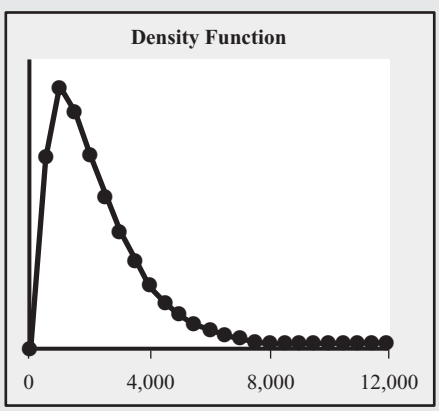
adequate. Suppose that each can costs $14 and 20 percent per year is charged for holding inventory (assume 50 weeks per year). What is the change in the store's holding cost relative to the original situation in which all paints are stocked individually?

Q15.4* **(Burger King)** Consider the following excerpts from a *Wall Street Journal* article on Burger King (Beatty, 1996):

> Burger King intends to bring smiles to the faces of millions of parents and children this holiday season with its "Toy Story" promotion. But it has some of them up in arms because local restaurants are running out of the popular toys . . . Every Kids Meal sold every day of the year comes with a giveaway, a program that has been in place for about six years and has helped Grand Metropolitan PLC's Burger King increase its market share. Nearly all of Burger King's 7,000 U.S. stores are participating in the "Toy Story" promotion . . . Nevertheless, meeting consumer demand still remains a conundrum for the giants. That is partly because individual Burger King restaurant owners make their tricky forecasts six months before such promotions begin. "It's asking you to pull out a crystal ball and predict exactly what consumer demand is going to be," says Richard Taylor, Burger King's director of youth and family marketing. "This is simply a case of consumer demand outstripping supply." The long lead times are necessary because the toys are produced overseas to take advantage of lower costs . . . Burger King managers in Houston and Atlanta say the freebies are running out there, too . . . But Burger King, which ordered nearly 50 million of the small plastic dolls, is "nowhere near running out of toys on a national level."

Let's consider a simplified analysis of Burger King's situation. Consider a region with 200 restaurants served by a single distribution center. At the time the order must be placed with the factories in Asia, demand (units of toys) for the promotion at each restaurant is forecasted to be gamma distributed with mean 2,251 and standard deviation 1,600. A discrete version of that gamma distribution is provided in the following table, along with a graph of the density function:

Q	$F(Q)$	$L(Q)$	Q	$F(Q)$	$L(Q)$
0	0.0000	2,251.3	6,500	0.9807	31.4
500	0.1312	1,751.3	7,000	0.9865	21.7
1,000	0.3101	1,316.9	7,500	0.9906	15.0
1,500	0.4728	972.0	8,000	0.9934	10.2
2,000	0.6062	708.4	8,500	0.9954	6.9
2,500	0.7104	511.5	9,000	0.9968	4.6
3,000	0.7893	366.6	9,500	0.9978	3.0
3,500	0.8480	261.3	10,000	0.9985	1.9
4,000	0.8911	185.3	10,500	0.9989	1.2
4,500	0.9224	130.9	11,000	0.9993	0.6
5,000	0.9449	92.1	11,500	0.9995	0.3
5,500	0.9611	64.5	12,000	1.0000	0.0
6,000	0.9726	45.1			



(* indicates that the solution is at the end of the book)

Suppose, six months in advance of the promotion, Burger King must make a single order for each restaurant. Furthermore, Burger King wants to have an in-stock probability of at least 85 percent.

a. Given those requirements, how many toys must each restaurant order?

b. How many toys should Burger King expect to have at the end of the promotion?

Now suppose Burger King makes a single order for all 200 restaurants. The order will be delivered to the distribution center and each restaurant will receive deliveries from that stockpile as needed. If demands were independent across all restaurants, total demand would be $200 \times 2{,}251 = 450{,}200$ with a standard deviation of $\sqrt{200} \times 1{,}600 = 22{,}627$. But it is unlikely that demands will be independent across restaurants. In other words, it is likely that there is positive correlation. Nevertheless, based on historical data, Burger King estimates the coefficient of variation for the total will be half of what it is for individual stores. As a result, a normal distribution will work for the total demand forecast.

c. How many toys must Burger King order for the distribution center to have an 85 percent in-stock probability?

d. If the quantity in part c is ordered, then how many units should Burger King expect to have at the end of the promotion?

e. If Burger King ordered the quantity evaluated in part a (i.e., the amount such that each restaurant would have its own inventory and generate an 85 percent in-stock probability) but kept that entire quantity at the distribution center and delivered to each restaurant only as needed, then what would the DC's in-stock probability be?

Q15.5* **(Livingston Tools)** Livingston Tools, a manufacturer of battery-operated, hand-held power tools for the consumer market (such as screwdrivers and drills), has a problem. Its two biggest customers are "big box" discounters. Because these customers are fiercely price competitive, each wants exclusive products, thereby preventing consumers from making price comparisons. For example, Livingston will sell the exact same power screwdriver to each retailer, but Livingston will use packing customized to each retailer (including two different product identification numbers). Suppose weekly demand of each product to each retailer is normally distributed with mean 5,200 and standard deviation 3,800. Livingston makes production decisions on a weekly basis and has a three-week replenishment lead time. Because these two retailers are quite important to Livingston, Livingston sets a target in-stock probability of 99.9 percent.

a. Based on the order-up-to model, what is Livingston's average inventory of each of the two versions of this power screwdriver?

b. Someone at Livingston suggests that Livingston stock power screwdrivers without putting them into their specialized packaging. As orders are received from the two retailers, Livingston would fulfill those orders from the same stockpile of inventory, since it doesn't take much time to actually package each tool. Interestingly, demands at the two retailers have a slight negative correlation, -0.20. By approximately how much would this new system reduce Livingston's inventory investment?

Q15.6 **(Restoration Hardware)** Consider the following excerpts from a *New York Times* article (Kaufman, 2000):

> Despite its early promise . . . Restoration has had trouble becoming a mass-market player. . . . What went wrong? High on its own buzz, the company expanded at breakneck speed, more than doubling the number of stores, to 94, in the year and a half after the stock offering . . . Company managers agree, for example, that Restoration's original inventory system, which called for all furniture to be kept at stores instead of at a central warehouse, was a disaster.

Let's look at one Restoration Hardware product, a leather chair. Average weekly sales of this chair in each store is Poisson with mean 1.25 units. The replenishment lead time is 12 weeks. (This question requires using Excel to create Poisson distribution and loss function tables that are not included in the appendix. See Appendix C for the procedure to evaluate a loss function table.)

(* indicates that the solution is at the end of the book)

a. If each store holds its own inventory, then what is the company's annual inventory turns if the company policy is to target a 99.25 percent in-stock probability?

b. Suppose Restoration Hardware builds a central warehouse to serve the 94 stores. The lead time from the supplier to the central warehouse is 12 weeks. The lead time from the central warehouse to each store is one week. Suppose the warehouse operates with a 99 percent in-stock probability, but the stores maintain a 99.25 percent in-stock probability. If only inventory at the retail stores is considered, what are Restoration's annual inventory turns?

Q15.7** **(Study Desk)** You are in charge of designing a supply chain for furniture distribution. One of your products is a study desk. This desk comes in two colors: black and cherry. Weekly demand for each desk type is normal with mean 100 and standard deviation 65 (demands for the two colors are independent). The lead time from the assembly plant to the retail store is two weeks and you order inventory replenishments weekly. There is no finished goods inventory at the plant (desks are assembled to order for delivery to the store).

a. What is the expected on-hand inventory of desks at the store (black and cherry together) if you maintain a 97 percent in-stock probability for each desk color?

You notice that only the top part of the desk is black or cherry; the remainder (base) is made of the standard gray metal. Hence, you suggest that the store stock black and cherry tops separately from gray bases and assemble them when demand occurs. The replenishment lead time for components is still two weeks. Furthermore, you still choose an order-up-to level for each top to generate a 97 percent in-stock probability.

b. What is the expected on-hand inventory of black tops?

c. How much less inventory of gray bases do you have on average at the store with the new in-store assembly scheme relative to the original system in which desks are delivered fully assembled? (*Hint:* Remember that each assembled desk requires one top and one base.)

Q15.8 **(O'Neill)** One of O'Neill's high-end wetsuits is called the Animal. Total demand for this wetsuit is normally distributed with a mean of 200 and a standard deviation of 130. In order to ensure an excellent fit, the Animal comes in 16 sizes. Furthermore, it comes in four colors, so there are actually 64 different Animal SKUs (stock-keeping units). O'Neill sells the Animal for $350 and its production cost is $269. The Animal will be redesigned this season, so at the end of the season leftover inventory will be sold off at a steep markdown. Because this is such a niche product, O'Neill expects to receive only $100 for each leftover wetsuit. Finally, to control manufacturing costs, O'Neill has a policy that at least five wetsuits of any size/color combo must be produced at a time. Total demand for the smallest size (extra small-tall) is forecasted to be Poisson with mean 2.00. Mean demand for the four colors are black = 0.90, blue = 0.50, green = 0.40, and yellow = 0.20.

a. Suppose O'Neill already has no extra small-tall Animals in stock. What is O'Neill's expected profit if it produces one batch (five units) of extra small-tall black Animals?

b. Suppose O'Neill announces that it will only sell the Animal in one color, black. If O'Neill suspects this move will reduce total demand by 12.5 percent, then what now is its expected profit from the black Animal?

Q15.9* **(Consulting Services)** A small economic consulting firm has four employees, Alice, Bob, Cathy, and Doug. The firm offers services in four distinct areas, Quotas, Regulation, Strategy, and Taxes. At the current time Alice is qualified for Quotas, Bob does Regulation, and so on. But this isn't working too well: the firm often finds it cannot compete for business in one area because it has already committed to work in that area while in another area it is idle. Therefore, the firm would like to train the consultants to be qualified in more than one area. Which of the following assignments is likely to be most beneficial to the firm?

a.

	Alice	Bob	Cathy	Doug
Qualified areas:	Quotas	Regulation	Strategy	Taxes
	Regulation	Taxes	Quotas	Strategy

(* indicates that the solution is at the end of the book)

b.

	Alice	**Bob**	**Cathy**	**Doug**
Qualified areas:	Quotas Regulation	Regulation Quotas	Strategy Taxes	Taxes Strategy

c.

	Alice	**Bob**	**Cathy**	**Doug**
Qualified areas:	Quotas Regulation	Regulation Quotas	Strategy Regulation	Taxes Quotas

d.

	Alice	**Bob**	**Cathy**	**Doug**
Qualified areas:	Quotas Strategy	Regulation Taxes	Strategy Quotas	Taxes Regulation

e.

	Alice	**Bob**	**Cathy**	**Doug**
Qualified areas:	Quotas Strategy	Regulation Taxes	Strategy Quotas	Taxes Regulation

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 16

Revenue Management with Capacity Controls

The operations manager constantly struggles with a firm's supply process to better match it to demand. In fact, most of our discussion in this text has concentrated on how the supply process can be better organized, structured, and managed to make it more productive and responsive. But if supply is so inflexible that it cannot be adjusted to meet demand, then another approach is needed. In particular, this chapter takes the opposite approach: Instead of matching supply to demand, we explore how demand can be adjusted to match supply. The various techniques for achieving this objective are collected under the umbrella term *revenue management,* which is also referred to as *yield management.* Broadly speaking, revenue management is the science of maximizing the revenue earned from a fixed supply.

This chapter discusses two specific techniques within revenue management: *protection levels/booking limits* and *overbooking.* (We will see that protection levels and booking limits are really two different concepts that implement the same technique.) Those techniques perform revenue management via capacity controls; that is, they adjust over time the availability of capacity. Prices are taken as fixed, so protection levels and overbooking attempt to maximize revenue without changing prices.

We begin the chapter with a brief introduction to revenue management: its history, its success stories, and some "margin arithmetic" to explain why it can be so powerful. We next illustrate the application of protection levels and overbooking to an example from the hotel industry. The final sections discuss the implementation of these techniques in practice and summarize insights.

16.1 Revenue Management and Margin Arithmetic

Revenue management techniques were first developed in the airline industry in the early 1980s. Because each flown segment is a perishable asset (once a plane leaves the gate, there are no additional opportunities to earn additional revenue on that particular flight), the airlines wanted to maximize the revenue they earned on each flight, which is all the more important given the razor-thin profit margins in the industry. For example, a typical airline operates with about 73 percent of its seats filled but needs to fill about 70 percent of its seats to breakeven: on a 100-seat aircraft, the difference between making and losing money is measured by a handful of passengers.

Firms that implement revenue management techniques generally report revenue increases in the range of 3 to 7 percent with relatively little additional capital investment. The importance of that incremental revenue can be understood with the use of "margin arithmetic." A firm's net profit equation is straightforward:

$$\text{Profit} = R \times M - F = \text{Net profit \%} \times R$$

where

$$\begin{aligned} R &= \text{Revenue} \\ M &= \text{Gross margin as a percentage of revenue} \\ F &= \text{Fixed costs} \\ \text{Net profit \%} &= \text{Net profit as a percentage of revenue} \end{aligned}$$

A firm's net profit as a percentage of its revenue (Net profit %) is generally in the range of 1 to 10 percent.

Now let's suppose we implement revenue management and increase revenue. Let Revenue increase be the percentage increase in revenue we experience, which, as has already been mentioned, is typically in the 3 to 7 percent range. Our percentage change in profit is then

$$\begin{aligned} \text{\% change in profit} &= \frac{[(100\% + \text{Revenue increase}) \times R \times M - F] - [R \times M - F]}{R \times M - F} \\ &= \frac{\text{Revenue increase} \times R \times M}{R \times M - F} \\ &= \frac{\text{Revenue increase} \times M}{\text{Net profit \%}} \end{aligned}$$

(The second line above cancels out terms in the numerator such as the fixed costs. The third line replaces the denominator with Net profit % $\times R$ and then cancels R from both the numerator and denominator.) Table 16.1 presents data evaluated with the above equation for various gross margins, revenue increases, and net profits as a percentage of revenues. The table illustrates that a seemingly small increase in revenue can have a significant impact on profit, especially when the gross margin is large. Thus, a 3 to 7 percent increase in revenue can easily generate a 50 to 100 percent increase in profit, especially in a high-gross-margin setting; revenue management indeed can be an important set of tools. We next illustrate in detail two of the tools in that set with an example from the hotel industry.

TABLE 16.1
Percentage Change in Profit for Different Gross Margins, Revenue Increases, and Net Profits as a Percentage of Revenue

Net Profit % = 2%					Net Profit % = 6%				
	Revenue increase					Revenue increase			
Gross Margin	1%	2%	5%	8%	Gross Margin	1%	2%	5%	8%
100%	50%	100%	250%	400%	100%	17%	33%	83%	133%
90	45	90	225	360	90	15	30	75	120
75	38	75	188	300	75	13	25	63	100
50	25	50	125	200	50	8	17	42	67
25	13	25	63	100	25	4	8	21	33
15	8	15	38	60	15	3	5	13	20

16.2 Protection Levels and Booking Limits

The Park Hyatt Philadelphia at the Bellevue, located at Walnut and Broad in downtown Philadelphia, has 118 king/queen rooms that it offers to both leisure and business travelers.[1] Leisure travelers are more price sensitive and tend to reserve rooms well in advance of their stay. Business travelers are generally willing to pay more for a room, in part because they tend to book much closer to the time of their trip and in part because they wish to avoid the additional restrictions associated with the discount fare (e.g., advance purchase requirements and more restrictive cancellation policies). With leisure travelers in mind, the Hyatt offers a $159 discount fare for a midweek stay, which contrasts with the regular fare of $225. We'll refer to these as the low and high fares and use the notation $r_l = 159$ and $r_h = 225$ (r stands for revenue and the subscript indicates l for low fare or h for high fare).

Suppose today is April 1, but we are interested in the Hyatt's bookings on May 29th, which is a midweek night. The Hyatt knows that there will be plenty of travelers willing to pay the low fare, so selling all 118 rooms by May 29th is not a problem. However, all else being equal, the Hyatt would like those rooms to be filled with high-fare travelers rather than low-fare travelers. Unfortunately, there is little chance that there will be enough demand at the high fare to fill the hotel and the lost revenue from an empty room is significant: Once May 29th passes, the Hyatt can never again earn revenue from that capacity. So the Hyatt's challenge is to extract as much revenue as possible from these two customer segments for its May 29th rooms; that is, we wish to maximize revenue.

The objective to maximize revenue implicitly assumes that the variable cost of an occupied room is inconsequential. The zero-variable cost assumption is reasonable for an airline. It is probably less appropriate for a hotel, given that an occupied room requires additional utilities and cleaning staff labor. Nevertheless, we stick with the traditional maximize-revenue objective in this chapter. If the variable cost of a customer is significant, then the techniques we present can be easily modified to implement a maximize-profit objective. (For example, see Practice Problems Q16.8 and Q16.10.)

Returning to our example, the Hyatt could just accept bookings in both fare classes as they occur until either it has 118 reservations or May 29th arrives; the first-come, first-served regime is surely equitable. With that process, it is possible the Hyatt has all 118 rooms reserved one week before May 29th. Unfortunately, because business travelers tend to book late, in that situation it is likely some high-fare travelers will be turned away in that last week; the Hyatt is not allowed to cancel a low-fare reservation to make room for a high-fare traveler. Turning away a high-fare reservation is surely a lost revenue opportunity.

There is a better way than just accepting reservations on a first-come, first-served basis. Instead, the Hyatt could reserve a certain number of rooms just for the high-fare travelers, that is, to protect some rooms for last-minute bookings. This is formalized with the concept of protection levels and booking limits.

The *protection level* for a fare is the number of rooms that are reserved for that fare or higher. We let Q represent our protection level for the high fare. If $Q = 35$, then we protect 35 rooms for the high fare. What does it mean to "protect" 35 rooms? It means that at all

[1] The Park Hyatt in Philadelphia does have 118 king/queen rooms, but the demand and fare data in this case are disguised. Furthermore, the revenue management techniques described in the chapter are meant to be representative of how the Park Hyatt could do revenue management, but should not be taken to represent the Park Hyatt's actual operating procedures.

times there must always be *at least* 35 rooms that could be reserved with the high fare. For example, suppose there were 83 rooms reserved at the low fare, 30 rooms reserved at the high fare, and 5 unreserved rooms. Because there are enough unreserved rooms to allow us to possibly have 35 high-fare rooms, we have not violated our protection level.

But now suppose the next traveler requests a low-fare reservation. If we were to allow that reservation, then we would no longer have enough unreserved rooms to allow at least 35 high-fare rooms. Therefore, according to our protection level rule, we would not allow that low-fare reservation. In fact, the limit of 83 has a name; it is called a booking limit: The *booking limit* for a fare is the maximum number of reservations allowed at that fare or lower. There is a relationship between the high-fare protection level and the low-fare booking limit:

$$\text{High-fare protection level} = \text{Capacity} - \text{Low-fare booking limit} \qquad \textbf{(16.1)}$$

In order to have at least 35 rooms available for the high fare (its protection level), the Hyatt cannot allow any more than 83 reservations at the low fare (its booking limit) as long as the total number of allowed reservations (capacity) is 118.

You might now wonder about the protection level for the low fare and the booking limit for the high fare. There is no need to protect any rooms at the low fare because the next best alternative is for the room to go empty. So the protection level for the low fare is 0. Analogously, we are willing to book as many rooms as possible at the high fare because there is no better alternative, so the booking limit on the high fare should be set to at least 118. (As we will see in the next section, we may even wish to allow more than 118 bookings.)

Given that we have defined a booking limit to be the maximum number of reservations allowed for a fare class *or lower,* we have implicitly assumed that our booking limits are *nested.* With *nested booking limits,* it is always true that if a particular fare class is open (i.e., we are willing to accept reservations at that fare class), then we are willing to accept all higher fare classes as well. It is also true that if a particular fare class is closed, then all lower fare classes are closed as well. For reasons beyond the scope of this discussion, nested booking limits may not be optimal. Nevertheless, because nested booking limits make intuitive sense, most revenue management systems operate with nested booking limits. So, throughout our discussion, we shall assume nested booking limits.

So now let's turn to the issue of choosing a booking limit for the low fare or, equivalently, a protection level for the high fare. As in many operational decisions, we again face the "too much–too little" problem. If we protect too many rooms for the high-fare class, then some rooms might remain empty on May 29th. To explain, suppose one week before May 29th we have 83 low-fare bookings but only 10 high-fare bookings. Because we have reached the low-fare booking limit, we "close down" that fare and only accept high-fare bookings in the last week. If only 20 additional high-fare bookings arrive, then on May 29th we have five unreserved rooms, which we might have been able to sell at the low fare. Nevertheless, those five rooms go empty. So protecting too many rooms for a fare class can lead to empty rooms.

But the Hyatt can also protect too few rooms. Suppose one week before May 29th we have 80 low-fare bookings and 35 high-fare bookings. Because only 35 rooms are protected for the high fare, the remaining three unreserved rooms could be taken at the low fare. If they are reserved at the low fare, then some high-fare travelers might be turned away; that is, the Hyatt might end up selling a room at the low fare that could have been sold at a high fare. If the protection level were three rooms higher, then those three

unreserved rooms could only go at the high fare. Therefore, because the low-fare bookings tend to come before the high-fare bookings, it is possible to protect too few rooms for the high fare.

Our discussion so far suggests the Hyatt could use the newsvendor model logic to choose a protection level. (Peter Belobaba of MIT first developed this approach and labeled it the "Expected Marginal Seat Revenue" analysis. See Belobaba, 1989) To implement the model, we need a forecast of high-fare demand and an assessment of the underage and overage costs. Let's say the Hyatt believes a Poisson distribution with mean 27.3 represents the number of high-fare travelers on May 29th. (This forecast could be constructed using booking data from similar nights, similar times of the year, and managerial intuition.) Table 16.2 provides a portion of the distribution function for that Poisson distribution.

Now we need an overage cost C_o and an underage cost C_u. The underage cost is the cost per unit of setting the protection level too low (i.e., "under" protecting). If we do not protect enough rooms for the high fare, then we sell a room at the low fare that could have been sold at the high fare. The lost revenue is the difference between the two fares, that is, $C_u = r_h - r_l$.

The overage cost is the cost per unit of setting the protection level too high (i.e., "over" protecting). If we set the protection level too high, it means that we did not need to protect so many rooms for the high-fare customers. In other words, demand at the high fare is less than Q, our protection level. If Q were lower, then we could have sold another room at the low fare. Hence, the overage cost is the incremental revenue of selling a room at the low fare: $C_o = r_l$. According to the newsvendor model, the optimal protection level (i.e., the one that maximizes revenue, which is also the one that minimizes the overage and underage costs) is the Q such that the probability the high-fare demand is less than or equal to Q equals the critical ratio, which is

$$\frac{C_u}{C_o + C_u} = \frac{r_h - r_l}{r_l + (r_h - r_l)} = \frac{r_h - r_l}{r_h} = \frac{225 - 159}{225} = 0.2933$$

In words, we want to find the Q such that there is a 29.33 percent probability high-fare demand is Q or lower. From Table 16.2, we see that $F(23) = 0.2381$ and $F(24) = 0.3040$, so the optimal protection level is $Q = 24$ rooms. (Recall the round-up rule: When the critical ratio falls between two values in the distribution function table, choose the entry that leads to the higher decision variable.) The corresponding booking limit for the low fare is $118 - 24 = 94$ rooms.

TABLE 16.2
The Distribution and Loss Function for a Poisson with Mean 27.3

Q	F(Q)	L(Q)	Q	F(Q)	L(Q)	Q	F(Q)	L(Q)
10	0.0001	17.30	20	0.0920	7.45	30	0.7365	1.03
11	0.0004	16.30	21	0.1314	6.55	31	0.7927	0.77
12	0.0009	15.30	22	0.1802	5.68	32	0.8406	0.56
13	0.0019	14.30	23	0.2381	4.86	33	0.8803	0.40
14	0.0039	13.30	24	0.3040	4.10	34	0.9121	0.28
15	0.0077	12.31	25	0.3760	3.40	35	0.9370	0.19
16	0.0140	11.31	26	0.4516	2.78	36	0.9558	0.13
17	0.0242	10.33	27	0.5280	2.23	37	0.9697	0.09
18	0.0396	9.35	28	0.6025	1.76	38	0.9797	0.06
19	0.0618	8.39	29	0.6726	1.36	39	0.9867	0.04

In some situations, it is more convenient to express a booking limit as an *authorization level:* The authorization level for a fare class is the percentage of available capacity that can be reserved at that fare or lower. For example, a booking limit of 94 rooms corresponds to an authorization level of 80 percent (94/118) because 80 percent of the Hyatt's rooms can be reserved at the low fare. The process of evaluating protection levels and booking limits is summarized in Exhibit 16.1.

If the Hyatt uses a protection level of 24 rooms, then the Hyatt's expected revenue is higher than if no protection level is used. How much higher? To provide some answer to that question, we need to make a few more assumptions. First, let's assume that there is ample low-fare demand. In other words, we could easily book all 118 rooms at the low fare. Second, let's assume the low-fare demand arrives before any high-fare bookings. Hence, if we do not protect any rooms for the high fare, then the low-fare customers will reserve all 118 rooms before any high-fare customer requests a reservation.

Given our assumptions, the Hyatt's revenue without any protection level would be $118 \times \$159 = \$18{,}762$: all 118 rooms are filled at the low fare. If we protect 24 rooms, then we surely fill 94 rooms at the low fare, for an expected revenue of $94 \times \$159 = \$14{,}946$. What is the expected revenue from the 24 protected rooms? Given that high-fare demand is Poisson with mean 27.3, from Table 16.2 we see that we can expect to turn away 4.1 high-fare bookings, that is, the loss function is $L(24) = 4.1$. In other words, we can expect to lose 4.1 high-fare bookings. Our expected high-fare bookings is analogous to expected sales in the newsvendor model, so

$$\begin{aligned}\text{Expected high-fare bookings} &= \text{Expected high-fare demand} - \text{Expected lost sales}\\ &= 27.3 - 4.1\\ &= 23.2\end{aligned}$$

In other words, we expect to have 23.2 high-fare reservations if we protect 24 rooms and high-fare demand is Poisson with mean 27.3. Therefore, because the Hyatt protects fewer rooms than expected demand, the Hyatt can expect to sell most of the rooms it protects with very few empty rooms. To be precise, of the 24 protected rooms, only 0.8 of them is expected to be empty:

$$\begin{aligned}\text{Expected number of empty rooms} &= Q - \text{Expected high-fare bookings}\\ &= 24 - 23.2\\ &= 0.8\end{aligned}$$

This makes sense. The incremental revenue of selling a high fare is only \$66, but the cost of an empty room is \$159, so a conservative protection level is prudent.

If the Hyatt expects to sell 23.2 rooms at the high fare, then the revenue from those rooms is $23.2 \times \$225 = \$5{,}220$. Total revenue when protecting 24 rooms is then $\$14{,}946 + \$5{,}220 = \$20{,}166$. Hence, our expected revenue increases by $(20{,}166 - 18{,}762)/18{,}762 = 7.5$ percent. As a point of reference, we can evaluate the *maximum expected revenue,* which is achieved if we sell to every high-fare customer and sell all remaining rooms at the low fare:

$$\begin{aligned}\text{Maximum expected revenue} &= 27.3 \times \$225 + (118 - 27.3) \times \$159\\ &= \$20,564\end{aligned}$$

Thus, the difference between the maximum expected revenue and the revenue earned by just selling at the low fare is $\$20{,}564 - \$18{,}762 = \$1{,}802$. The Hyatt's revenue with a

Exhibit 16.1

EVALUATING THE OPTIMAL PROTECTION LEVEL FOR THE HIGH FARE OR THE OPTIMAL BOOKING LIMIT FOR THE LOW FARE WHEN THERE ARE TWO FARES AND REVENUE MAXIMIZATION IS THE OBJECTIVE

Step 1. Evaluate the critical ratio:

$$\text{Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_h - r_l}{r_h}$$

Step 2. Find the Q such that $F(Q)$ = Critical ratio, where $F(Q)$ is the distribution function of high-fare demand:

a. If $F(Q)$ is given in table form, then find the Q in the table such that $F(Q)$ equals the critical ratio. If the critical ratio falls between two entries in the table, choose the entry with the higher Q.

b. If high-fare demand is normally distributed with mean μ and standard deviation σ, then find the z-statistic in the Standard Normal Distribution Function Table such that $\Phi(z)$ = Critical ratio. If the critical ratio falls between two entries in the table, choose the entry with the higher z. Finally, convert the chosen z into Q: $Q = \mu + z \times \sigma$.

Step 3. The optimal high-fare protection level is Q evaluated in Step 2. The optimal low-fare booking limit is Capacity $-$ Q, where Capacity is the number of allowed reservations.

protection level falls short of the maximum expected revenue by only \$20,564 − \$20,166 = \$398. Hence, a protection level for the high fare allows the Hyatt to capture about 78 percent (1 − \$398/\$1,802) of its potential revenue improvement.

A revenue increase of 7.5 percent is surely substantial given that it is achieved without the addition of capacity. Nevertheless, we must be reminded of the assumptions that were made. We assumed there is ample demand for the low fare. If low-fare demand is limited, then a protection level for the high fare is less valuable and the incremental revenue gain is smaller. For example, if the sum of low- and high-fare demand is essentially always lower than 118 rooms, then there is no need to protect the high fare. More broadly, revenue management with protection levels is most valuable when operating in a capacity-constrained situation.

The second key assumption is that low-fare demand arrives before high-fare demand. If some high-fare demand "slips in" before the low-fare demand snatches up all 118 rooms, then the revenue estimate without a protection level, \$18,762, is too low. In other words, even if we do not protect any rooms for the high fare, it is possible that we would still obtain some high-fare bookings.

Although we would need to look at actual data to get a more accurate sense of the potential revenue improvement by using protection levels, our estimate is in line with the typical revenue increases reported in practice due to revenue management, 3 to 7 percent.

Now that we have considered a specific example of booking limits at a hotel, it is worth enumerating the characteristics of a business that are conducive to the application of booking limits.

• *The same unit of capacity can be used to sell to different customer segments.* It is easy for an airline to price discriminate between leisure and business travelers when the

capacity that is being sold is different, for example, a coach cabin seat and a first-class seat. Those are clearly distinguishable products/services. Booking limits are applied when the capacity sold to different segments is identical; for example, a coach seat on an aircraft or a king/queen room in the Hyatt sold at two different fares.

• *There are distinguishable customer segments and the segments have different price sensitivity.* There is no need for protection levels when the revenue earned from all customers is the same, for example, if there is a single fare. Booking limits are worthwhile if the firm can earn different revenue from different customer segments with the same type of capacity. Because the same unit of capacity is being sold, it is necessary to discriminate between the customer segments. This is achieved with *fences:* additional restrictions that are imposed on the low fare that prevent high-fare customers from purchasing with the low fare. Typical fences include advanced purchase requirements, Saturday night stay requirements, cancellation fees, change fees, and so forth. Of course, one could argue that these fences make the low and high fares different products; for example, a full-fare coach ticket is not the same product as a supersaver coach ticket even if they both offer a seat in the coach cabin. True, these are different products in the broad sense, but they are identical products with respect to the capacity they utilize.

• *Capacity is perishable.* An unused room on May 29th is lost forever, just as an unused seat on a flight cannot be stored until the next flight. In contrast, capacity in a production facility can be used to make inventory, which can be sold later whenever capacity exceeds current demand.

• *Capacity is restrictive.* If the total demand at the leisure and business fares is rarely greater than 118 rooms, then the Hyatt has no need to establish protection levels or booking limits. Because capacity is expensive to install and expensive to change over time, it is impossible for a service provider to always have plenty of capacity. (Utilization would be so low that the firm would surely not be competitive and probably not viable.) But due to seasonality effects, it is possible that the Hyatt has plenty of capacity at some times of the year and not enough capacity at other times. Booking limits are not needed during those lull times but are quite useful during the peak demand periods.

• *Capacity is sold in advance.* If we were allowed to cancel a low-fare reservation whenever someone requested a high-fare reservation (i.e., bump a low-fare passenger off the plane without penalty), then we would not need to protect seats for the high fare: We would accept low-fare bookings as they arrive and then cancel as many as needed to accommodate the high-fare travelers. Similarly, we do not need protection levels if we were to conduct an auction just before the flight departs. For example, imagine a situation in which all potential demand would arrive at the airport an hour or so before the flight departs and then an auction is conducted to determine who would earn a seat on that flight. This is a rather silly way to sell airline seats, but in other contexts there is clearly a movement toward more auctionlike selling mechanisms. Because the auction ensures that capacity is sold to the highest bidders, there is no need for protection levels.

• *A firm wishes to maximize revenue, has the flexibility to charge different prices, and may withhold capacity from certain segments.* A hotel is able to offer multiple fares and withhold fares. In other words, even though the practice of closing a discount fare means the principle of first-come, first-served is violated, this practice is generally not viewed as unethical or unscrupulous. However, there are settings in which the violation of first-come, first-served, or the charging of different prices, or the use of certain fences is not acceptable to consumers, for example, access to health care.

• *A firm faces competition from a "discount competitor."* The low fares charged by People Express, a low-frills airline started after deregulation, were a major motivation for

the development of revenue management at American Airlines. In order to compete in the low-fare segment, American was forced to match People Express's fares. But American did not want to have its high-fare customers paying the low fare. Booking limits and low-fare fences were the solution to the problem: American could compete at the low-fare segment without destroying the revenue from its profitable high-fare customers. People Express did not install a revenue management system and quickly went bankrupt after American's response.

16.3 Overbooking

In many service settings, customers are allowed to make reservations and then either are allowed to cancel their reservations with relatively short notice, or just fail to show up to receive their service. For example, on May 28th, the Hyatt might have all of its 118 rooms reserved for May 29th but then only 110 customers might actually show up, leaving eight rooms empty and not generating any revenue. Overbooking, described in this section, is one solution to the no-show problem. If the Hyatt chooses to overbook, then that means the Hyatt accepts more than 118 reservations even though a maximum of 118 guests can be accommodated. Overbooking is also common in the airline industry: In the United States, airlines deny boarding to about one million passengers annually (Stringer, 2002). Furthermore, it has been estimated that prohibiting overbooking would cost the world's airlines $3 billion annually due to no-shows (Cross, 1995).

Let the variable Y represent the number of additional reservations beyond capacity that the Hyatt is willing to accept, that is, up to $118 + Y$ reservations are accepted. Overbooking can lead to two kinds of outcomes. On a positive note, the number of no-shows can be greater than the number of overbooked reservations, so all the actual customers can be accommodated and more customers are accommodated than would have been without overbooking. For example, suppose the Hyatt accepts 122 reservations and there are six no-shows. As a result, 116 rooms are occupied, leaving only two empty rooms, which is almost surely fewer empty rooms than if the Hyatt had only accepted 118 reservations.

On the negative side, the Hyatt can get caught overbooking. For example, if 122 reservations are accepted, but there are only two no-shows, then 120 guests hold reservations for 118 rooms. In that situation, two guests need to be accommodated at some other hotel and the Hyatt probably must give some additional compensation (e.g., cash or free future stay) to mitigate the loss of goodwill with those customers.

In deciding the proper amount of overbooking, there is a "too much–too little" trade-off: Overbook too much and the hotel angers some customers, but overbook too little and the hotel has the lost revenue associated with empty rooms. Hence, we can apply the newsvendor model to choose the appropriate Y. We first need a forecast of the number of customers that will not show up based on historical data. Let's say the Hyatt believes for the May 29th night that the no-show distribution is Poisson with mean 8.5. Table 16.3 provides the distribution function.[2]

[2] A careful reader will notice that our distribution function for no-shows is independent of the number of reservations made. In other words, we have assumed the average number of no-shows is 8.5 whether we make 118 reservations or 150 reservations. Hence, a more sophisticated method for choosing the overbooking quantity would account for the relationship between the number of reservations allowed and the distribution function of no-shows. While that more sophisticated method is conceptually similar to our procedure, it is also computationally cumbersome. Therefore, we shall stick with our heuristic method. Fortunately, our heuristic method performs well when compared against the more sophisticated algorithm.

TABLE 16.3
Poisson Distribution Function with Mean 8.5

Q	$F(Q)$	Q	$F(Q)$
0	0.0002	10	0.7634
1	0.0019	11	0.8487
2	0.0093	12	0.9091
3	0.0301	13	0.9486
4	0.0744	14	0.9726
5	0.1496	15	0.9862
6	0.2562	16	0.9934
7	0.3856	17	0.9970
8	0.5231	18	0.9987
9	0.6530	19	0.9995

Next, we need underage and overage costs. If the Hyatt chooses Y to be too low, then there will be empty rooms on May 29th (i.e., the Hyatt "under" overbooked). If the Hyatt indeed has plenty of low-fare demand, then those empty rooms could have at least been sold for r_l = \$159, so the underage cost is $C_u = r_l = 159$. Surprisingly, the underage cost does not depend on whether customers are allowed to cancel without penalty or not. To explain, suppose we accepted 120 reservations, but there are three no-shows. If reservations are refundable, we collected revenue from 117 customers (because the three no-shows are given a refund) but could have collected revenue from the one empty room. If reservations are not refundable, we collect revenue from 120 customers, but, again, we could have collected revenue from the one empty room. In each case our incremental revenue is \$159 from the one additional room we could have sold had we accepted one more reservation.

If the Hyatt chooses Y to be too high, then there will be more guests than rooms. The guests denied a room need to be accommodated at some other hotel and Hyatt offers other compensation. The total cost to Hyatt for each of those guests is estimated to be about \$350, so the overage cost is $C_o = 350$. *Note:* This cost is net of any revenue collected from the customer. For example, if the reservation is not refundable, then the Hyatt incurs \$509 in total costs due to the denial of service, for a net cost of \$350 (\$509−\$159), whereas if the reservation is refundable, then the Hyatt incurs \$350 in total costs due to the denial of service. Either way, the Hyatt is \$350 worse off for each customer denied a room.

The critical ratio is

$$\frac{C_u}{C_o + C_u} = \frac{159}{350 + 159} = 0.3124$$

Looking in Table 16.3, we see that $F(6) = 0.2562$ and $F(7) = 0.3856$, so the optimal quantity to overbook is $Y = 7$. In other words, the Hyatt should allow up to $118 + 7 = 125$ reservations for May 29th. Exhibit 16.2 summarizes the process of evaluating the optimal quantity to overbook.

If the Hyatt chooses to overbook by seven reservations and if the Hyatt indeed receives 125 reservations, then there is about a 26 percent chance ($F(6) = 0.2562$) that the Hyatt will find itself overbooked on May 29th. Because it is not assured that the Hyatt will receive that many reservations, the actual frequency of being overbooked would be lower.

A natural question is how should the Hyatt integrate its protection-level/booking-limit decision with its overbooking decision. The following describes a reasonable heuristic.

Exhibit 16.2

THE PROCESS TO EVALUATE THE OPTIMAL QUANTITY TO OVERBOOK

Step 1. Evaluate the critical ratio:

$$\text{Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_l}{\text{Cost per bumped customer} + r_l}$$

Step 2. Find the Y such that $F(Y)$ = Critical ratio, where $F(Y)$ is the distribution function of no-shows:

a. If $F(Y)$ is given in table form, then find the Y in the table such that $F(Y)$ equals the critical ratio. If the critical ratio falls between two entries in the table, choose the entry with the higher Y.

b. If no-shows are normally distributed with mean μ and standard deviation σ, then find the z-statistic in the Standard Normal Distribution Function Table such that $\Phi(z)$ = Critical ratio. If the critical ratio falls between two entries in the table, choose the entry with the higher z. Finally, convert the chosen z into Y: $Y = \mu + z \times \sigma$.

Step 3. Y is the optimal amount to overbook; that is, the number of allowed reservations is Y + Capacity, where Capacity is the maximum number of customers that can actually be served.

If the Hyatt is willing to overbook by seven rooms, that is, $Y = 7$, then its effective capacity is $118 + 7 = 125$ rooms. Based on the forecast of high-fare demand and the underage and overage costs associated with protecting rooms for the high-fare travelers, we determined that the Hyatt should protect 24 rooms for the high fare. Using equation (16.1), that suggests the booking limit for the low fare should be

$$\begin{aligned}\text{Low-fare booking limit} &= \text{Capacity} - \text{High-fare protection level} \\ &= 125 - 24 \\ &= 101\end{aligned}$$

The high-fare booking limit would then be 125, that is, the Hyatt accepts up to 101 low-fare reservations and up to 125 reservations in total.

16.4 Implementation of Revenue Management

Although the applications of revenue management described in this chapter present a reasonably straightforward analysis, in practice there are many additional complications encountered in the implementation of revenue management. A few of the more significant complications are discussed below.

Demand Forecasting

We saw that forecasts are a necessary input to the choice of both protection levels and overbooking quantities. As a result, the choices made are only as good as the inputted forecasts; as the old adage says, "garbage in, garbage out." Fortunately, reservation systems generally provide a wealth of information to formulate these forecasts. Nevertheless,

the forecasting task is complicated by the presence of seasonality, special events (e.g., a convention in town), changing fares (both the firm's own fares as well as the competitors' fares), and truncation (once a booking limit is reached, most systems do not capture the lost demand at that fare level), among others. Furthermore, it is possible that the revenue management decisions themselves might influence demand and, hence, the forecasts used to make those decisions. As a result, with any successful revenue management system, a considerable amount of care and effort is put into the demand forecasting task.

Dynamic Decisions

Our analysis provided a decision for a single moment in time. However, fares and forecasts change with time and, as a result, booking limits need to be reviewed frequently (generally daily). In fact, sophisticated systems take future adjustments into consideration when setting current booking limits.

Variability in Available Capacity

A hotel is a good example of a service firm that generally does not have much variation in its capacity: it is surely difficult to add a room to a hotel and the number of rooms that cannot be occupied is generally small. The capacity of an airline's flight is also rigid but maybe less so than a hotel's capacity because the airline can choose to switch the type of aircraft used on a route. However, a car rental company's capacity at any given location is surely variable and not even fully controllable by the firm. Hence, those firms also must forecast the amount of capacity they think will be available at any given time.

Reservations Coming in Groups

If there is a convention in town for May 29th, then the Hyatt may receive a single request for 110 rooms at the low fare. Although this request violates the booking limit, the booking limit was established assuming reservations come one at a time. It is clearly more costly to turn away a single block of 110 reservations than it is to turn away one leisure traveler.

Effective Segmenting of Customers

We assumed there are two types of customers: a low-fare customer and a high-fare customer. In reality, this is too simplistic. There surely exist customers that are willing to pay the high fare, but they are also more than willing to book at the low fare if given the opportunity. Hence, fences are used to separate out customers by their willingness to pay. Well-known fences include advance purchase requirements, cancellation fees, change fees, Saturday night stay requirements, and so on. But these fences are not perfect; that is, they do not perfectly segment out customers. As a result, there is often spillover demand from one fare class to another. It is possible that more effective fences exist, but some fences might generate stiff resistance from customers. For example, a firm could regulate a customer's access to various fare classes based on his or her annual income, or the average price the customer paid in past service encounters, but those schemes will surely not receive a warm reception.

Multiple Fare Classes

In our application of revenue management, we have two fare classes: a low fare and a high fare. In reality there can be many more fare classes. With multiple fare classes, it becomes necessary to forecast demand for each fare class and to establish multiple booking limits.

Software Implementation

While the investment in revenue management software is often reasonable relative to the potential revenue gain, it is nevertheless not zero. Furthermore, revenue management systems often have been constrained by the capabilities of the reservation systems they must work with. In other words, while the revenue management software might be able to make a decision as to whether a fare class should be open or closed (i.e., whether to accept a request for a reservation at a particular fare), it also must be able to communicate that decision to the travel agent or customer via the reservation system. Finally, there can even be glitches in the revenue management software, as was painfully discovered by American Airlines. Their initial software had an error that prematurely closed down the low-fare class on flights with many empty seats (i.e., it set the low-fare class booking limit too low). American Airlines discovered the error only when they realized that the load on those flights was too low (the load is the percent of seats occupied; it is the utilization of the aircraft). By that time it was estimated $50 million in revenue had been lost. Hence, properly chosen booking limits can increase revenue, but poorly chosen booking limits can decrease revenue. As a result, careful observation of a revenue management system is always necessary.

Variation in Capacity Purchase: Not All Customers Purchase One Unit of Capacity

Even if two customers pay the same fare, they might be different from the firm's perspective. For example, suppose one leisure traveler requests one night at the low fare whereas another requests five nights at the low fare. While these customers pay the same amount for a given night, it is intuitive that turning away the second customer is more costly. In fact, it may even be costlier than turning away a single high-fare reservation.

Airlines experience a challenge similar to a hotel's multinight customer. Consider two passengers traveling from Chicago (O'Hare) to New York (JFK) paying the discount fare. For one passenger JFK is the final destination, whereas the other passenger will fly from JFK to London (Heathrow) on another flight with the same airline. The revenue management system should recognize that a multileg passenger is more valuable than a single-leg customer. But booking limits just defined for each fare class on the O'Hare–JFK segment do not differentiate between these two customers. In other words, the simplest version of revenue management does *single-leg* or *single-segment control* because the decision rules are focused on the fares of a particular segment in the airline's network. Our example from the Hyatt could be described as *single-night control* because the focus is on a room for one evening.

One solution to the multileg issue is to create a booking limit for each fare class–itinerary combination, not just a booking limit for each fare class on each segment. This is called *origin-destination control,* or *O-D control* for short. For example, suppose there are three fare classes, Y, M, Q (from highest to lowest), on two itineraries, O'Hare–JFK and O'Hare–Heathrow (via JFK):

Fare Class	O'Hare to JFK	O'Hare to Heathrow
Y	$724	$1,610
M	475	829
Q	275	525

Six booking limits could be constructed to manage the inventory on the O'Hare–JFK leg. For example:

Fare Class	O'Hare to JFK	O'Hare to Heathrow
Y		100
M		68
Y	60	
Q		40
M	35	
Q	20	

Hence, it would be possible to deny a Q fare request to an O'Hare–JFK passenger while accepting a Q fare request to an O'Hare–Heathrow passenger: There could be 20 Q fare reservations on the O'Hare–JFK itinerary but fewer than 40 reservations between the M and Q fares on the O'Hare–JFK itinerary and the Q fare on the O'Hare–Heathrow itinerary. If there were only three booking limits on that leg, then all Q fare requests are either accepted or rejected, but it is not possible to accept some Q fare requests while denying others.

While creating a booking limit for each fare class–itinerary combination sounds like a good idea, unfortunately, it is not a practical idea for most revenue management applications. For example, there could be thousands of possible itineraries that use the O'Hare–JFK leg. It would be a computational nightmare to derive booking limits for such a number of itineraries on each possible flight leg, not to mention an implementation challenge. One solution to this problem is *virtual nesting.* With virtual nesting, a limited number of *buckets* are created, each with its own booking limit, each with its own set of fare class–itinerary combinations. Fare class–itinerary combinations are assigned to buckets in such a way that the fare class–itinerary combinations within the same bucket have similar value to the firm, while fare class–itinerary combinations in different buckets have significantly different values.

For example, four buckets could be created for our example, labeled 0 to 3:

Bucket	Itinerary	Fare class
0	O'Hare to Heathrow	Y
1	O'Hare to Heathrow	M
	O'Hare to JFK	Y
2	O'Hare to Heathrow	Q
	O'Hare to JFK	M
3	O'Hare to JFK	Q

The O'Hare–JFK Y fare is combined into one bucket with the O'Hare–Heathrow M fare because they generate similar revenue ($724 and $829), whereas the O'Hare–Heathrow Y fare is given its own bucket due to its much higher revenue ($1,610). Thus, with virtual nesting, it is possible to differentiate among the customers on the same leg willing to pay the same fare. Furthermore, virtual nesting provides a manageable solution if there are many different fare classes and many different types of customers (e.g., customers flying different itineraries or customers staying a different number of nights in a hotel).

While virtual nesting was the first solution implemented for this issue, it is not the only solution. A more recent, and more sophisticated, solution is called *bid-price control.* Let's explain bid-price controls in the context of our airline example. The many different itineraries that use the O'Hare–JFK segment generate different revenue to the airline, but they all use the same unit of capacity, a coach seat on the O'Hare to JFK flight. With bid-price control, each type of capacity on each flight segment is assigned

a *bid price.* Then, a fare class–itinerary combination is accepted as long as its fare exceeds the sum of the bid prices of the flight legs in its itinerary. For example, the bid prices could be

	O'Hare to JFK	JFK to Heathrow
Bid price	$290	$170

Hence, an O'Hare–JFK itinerary is available as long as its fare exceeds $290 and an O'Hare–Heathrow itinerary (via JFK) is available as long as its fare exceeds $290 + $170 = $460. Therefore, on the O'Hare–JFK itinerary, the Y and M fare classes would be open (fares $724 and $475 respectively); while on the O'Hare–Heathrow itinerary, all fares would be available (because the lowest Q fare, $525, exceeds the total bid price of $460).

With bid-price control, there is a single bid price on each flight segment, so it is a relatively intuitive and straightforward technique to implement. The challenge with bid-price control is to find the correct bid prices. That challenge requires the use of sophisticated optimization techniques.

16.5 Summary

Revenue management is the science of using pricing and capacity controls to maximize revenue given a relatively fixed supply/capacity. This chapter focuses on the capacity control tools of revenue management: protection levels/booking limits and overbooking. Protection levels/booking limits take advantage of the price differences between fares and the generally staggered nature of demand arrivals; that is, low-fare reservations made by leisure travelers usually occur before high fare reservations made by business travelers. By establishing a booking limit for low fares, it is possible to protect enough capacity for the later-arriving high fares. Overbooking is useful when customer reservations are not firm; if a portion of the customers can be expected to not use the capacity they reserved, then it is wise to accept more reservations than available capacity.

The science of revenue management is indeed quite complex and continues to be an extremely active area of research. Despite these challenges, revenue management has been proven to be a robust and profitable tool, as reflected in the following quote by Robert Crandall, former CEO of AMR and American Airlines (Smith, Leimkuhler, and Darrow, 1992):

> I believe that revenue management is the single most important technical development in transportation management since we entered the era of airline deregulation in 1979 . . . The development of revenue management models was a key to American Airlines' survival in the post-deregulation environment. Without revenue management we were often faced with two unsatisfactory responses in a price competitive marketplace. We could match deeply discounted fares and risk diluting our entire inventory, or we could not match and certainly lose market share. Revenue management gave us a third alternative—match deeply discounted fares on a portion of our inventory and close deeply discounted inventory when it is profitable to save space for later-booking higher value customers. By adjusting the number of reservations which are available at these discounts, we can adjust our minimum available fare to account for differences in demand. This creates a pricing structure which responds to demand on a flight-by-flight basis. As a result, we can more effectively match our demand to supply.

Table 16.4 provides a summary of the key notation and equations presented in this chapter.

TABLE 16.4
Summary of Key Notation and Equations in Chapter 16

Choosing protection levels and booking limits:

With two fares, r_h = high fare and r_l = low fare, the high-fare protection level Q has the following critical ratio:

$$\text{Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_h - r_l}{r_h}$$

(Find the Q such that the critical ratio is the probability high-fare demand is less than or equal to Q.)

Low-fare booking limit = Capacity − Q

Choosing an overbooking quantity Y:

Let r_l be the low fare. The optimal overbooking quantity Y has the following critical ratio:

$$\text{Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_l}{\text{Cost per bumped customer} + r_l}$$

16.6 Further Reading

For a brief history of the development of revenue management, see Cross (1995). For a more extensive history, see Cross (1997). Cross (1997) also provides a detailed overview of revenue management techniques.

See Talluri and van Ryzin (2004) for an extensive treatment of the state of the art in revenue management for both theory and practice. Two already-published reviews on the theory of revenue management are McGill and van Ryzin (1999) and Weatherford and Bodily (1992).

Applications of revenue management to car rentals, golf courses, and restaurants can be found in Geraghty and Johnson (1997), Kimes (2000) and Kimes, Chase, Choi, Lee, and Ngonzi (1998).

16.7 Practice Problems

Q16.1* **(The Inn at Penn)** The Inn at Penn hotel has 150 rooms with standard queen-size beds and two rates: a full price of $200 and a discount price of $120. To receive the discount price, a customer must purchase the room at least two weeks in advance (this helps to distinguish between leisure travelers, who tend to book early, and business travelers, who value the flexibility of booking late). For a particular Tuesday night, the hotel estimates that the demand from leisure travelers could fill the whole hotel while the demand from business travelers is distributed normally with a mean of 70 rooms and a standard deviation of 29.

a. Suppose 50 rooms are protected for full-price rooms. What is the booking limit for the discount rooms?

b. Find the optimal protection level for full-price rooms (the number of rooms to be protected from sale at a discount price).

c. The Sheraton declared a fare war by slashing business travelers' prices down to $150. The Inn at Penn had to match that fare to keep demand at the same level. Does the optimal protection level increase, decrease, or remain the same? Explain your answer.

d. What number of rooms (on average) remain unfilled if we establish a protection level of 61 for the full-priced rooms?

e. If The Inn were able to ensure that every full-price customer would receive a room, what would The Inn's expected revenue be?

f. If The Inn did not choose to protect any rooms for the full price and leisure travelers book before business travelers, then what would The Inn's expected revenue be?

g. Taking the assumptions in part f and assuming now that The Inn protects 50 rooms for the full price, what is The Inn's expected revenue?

Q16.2* **(Overbooking The Inn at Penn)** Due to customer no-shows, The Inn at Penn hotel is considering implementing overbooking. Recall from Q16.1 that The Inn at Penn has 150 rooms, the full fare is $200, and the discount fare is $120. The forecast of no-shows

(* indicates that the solution is at the end of the book)

is Poisson with a mean of 15.5. The distribution and loss functions of that distribution are as follows:

Y	*F*(*Y*)	*L*(*Y*)	*Y*	*F*(*Y*)	*L*(*Y*)	*Y*	*F*(*Y*)	*L*(*Y*)
8	0.0288	7.52	14	0.4154	2.40	20	0.8944	0.28
9	0.0552	6.55	15	0.5170	1.82	21	0.9304	0.18
10	0.0961	5.61	16	0.6154	1.33	22	0.9558	0.11
11	0.1538	4.70	17	0.7052	0.95	23	0.9730	0.06
12	0.2283	3.86	18	0.7825	0.65	24	0.9840	0.04
13	0.3171	3.08	19	0.8455	0.44	25	0.9909	0.02

The Inn is sensitive about the quality of service it provides alumni, so it estimates the cost of failing to honor a reservation is $325 in lost goodwill and explicit expenses.

a. What is the optimal overbooking limit, that is, the maximum reservations above the available 150 rooms that The Inn should accept?

b. If The Inn accepts 160 reservations, what is the probability The Inn will not be able to honor a reservation?

c. If The Inn accepts 165 reservations, what is the probability The Inn will be fully occupied?

d. If The Inn accepts 170 reservations, what is the expected total cost incurred due to bumped customers?

Q16.3* **(WAMB)** WAMB is a television station that has 25 thirty-second advertising slots during each evening. It is early January and the station is selling advertising for Sunday, March 24. They could sell all of the slots right now for $4,000 each, but, because on this particular Sunday the station is televising the Oscar ceremonies, there will be an opportunity to sell slots during the week right before March 24 for a price of $10,000. For now, assume that a slot not sold in advance *and* not sold during the last week is worthless to WAMB. To help make this decision, the salesforce has created the following probability distribution for last-minute sales:

Number of Slots, *x*	Probability Exactly *x* Slots Are Sold
8	0.00
9	0.05
10	0.10
11	0.15
12	0.20
13	0.10
14	0.10
15	0.10
16	0.10
17	0.05
18	0.05
19	0.00

a. How many slots should WAMB sell in advance?

b. In practice, there are companies willing to place standby advertising messages: if there is an empty slot available (i.e., this slot was not sold either in advance or during the last week), the standby message is placed into this slot. Since there is no guarantee that such a slot will be available, standby messages can be placed at a much lower cost. Now suppose that if a slot is not sold in advance *and* not sold during the last week, it will be used for a standby promotional message that costs advertisers $2,500. Now how many slots should WAMB sell in advance?

(* indicates that the solution is at the end of the book)

c. Suppose WAMB chooses a booking limit of 10 slots on advanced sales. In this case, what is the probability there will be slots left over for stand-by messages?

d. One problem with booking for March 24 in early January is that advertisers often withdraw their commitment to place the ad (typically this is a result of changes in promotional strategies; for example, a product may be found to be inferior or an ad may turn out to be ineffective). Because of such opportunistic behavior by advertisers, media companies often overbook advertising slots. WAMB estimates that in the past the number of withdrawn ads has a Poisson distribution with mean 9. Assume each withdrawn ad slot can still be sold at a standby price of $2,500 although the company misses an opportunity to sell these slots at $4,000 a piece. Any ad that was accepted by WAMB but cannot be accommodated (because there isn't a free slot) costs the company $10,000 in penalties. How many slots (at most) should be sold?

e. Over time, WAMB saw a steady increase in the number of withdrawn ads and decided to institute a penalty of $1,000 for withdrawals. (Actually, the company now requires a $1,000 deposit on any slot. It is refunded only if WAMB is unable to provide a slot due to overbooking.) The expected number of withdrawn ads is expected to be cut in half (to only 4.5 slots). Now how many slots (at most) should be sold?

Q16.4* **(Designer Dress)** A fashion retailer in Santa Barbara, California, presents a new designer dress at one of the "by invitation only" fashion shows. After the show, the dress will be sold at the company's boutique store for $10,000 apiece. Demand at the boutique is limited due to the short time the dress remains fashionable and is estimated to be normal with mean 70 and standard deviation 40. There were only 100 dresses produced to maintain exclusivity and high price. It is the company's policy that all unsold merchandise is destroyed.

a. How many dresses remain unsold on average at the end of the season?

b. What is the retailer's expected revenue?

c. Fashion companies often sell a portion of new merchandise at exhibitions for a discount while the product is still "fresh" in the minds of the viewers. The company decides to increase revenues by selling a certain number of dresses at a greatly discounted price of $6,000 during the show. Later, remaining dresses will be available at the boutique store for a normal price of $10,000. Typically, all dresses offered at the show get sold, which, of course, decreases demand at the store: it is now normal with mean 40 and standard deviation 25. How many dresses should be sold at the show?

d. Given your decision in part c, what is expected revenue?

e. Given your decision in part c, how many dresses are expected to remain unsold?

Q16.5* **(Overbooking PHL-LAX)** On a given Philadelphia–Los Angeles flight, there are 200 seats. Suppose the ticket price is $475 on average and the number of passengers who reserve a seat but do not show up for departure is normally distributed with mean 30 and standard deviation 15. You decide to overbook the flight and estimate that the average loss from a passenger who will have to be bumped (if the number of passengers exceeds the number of seats) is $800.

a. What is the maximum number of reservations that should be accepted?

b. Suppose you allow 220 reservations. How much money do you expect to pay out in compensation to bumped passengers?

c. Suppose you allow 220 reservations. What is the probability that you will have to deal with bumped passengers?

Q16.6 **(PHL-LAX)** Consider the Philadelphia–Los Angeles flight discussed in Q16.5. Assume the available capacity is 200 seats and there is no overbooking. The high fare is $675 and the low fare is $375. Demand for the low fare is abundant while demand for the high fare is normally distributed with a mean of 80 and standard deviation 35.

a. What is the probability of selling 200 reservations if you set an optimal protection level for the full fare?

(* indicates that the solution is at the end of the book)

b. Suppose a protection level of 85 is established. What is the average number of lost high-fare passengers?

c. Continue to assume a protection level of 85 is established. What is the expected number of unoccupied seats?

d. Again assume a protection level of 85 is established. What is the expected revenue from the flight?

Q16.7** **(Annenberg)** Ron, the director at the Annenberg Center, is planning his pricing strategy for a musical to be held in a 100-seat theater. He sets the full price at $80 and estimates demand at this price to be normally distributed with mean 40 and standard deviation 30. Ron also decides to offer student-only advance sale tickets discounted 50 percent off the full price. Demand for the discounted student-only tickets is usually abundant and occurs well before full price ticket sales.

a. Suppose Ron sets a 50-seat booking limit for the student-only tickets. What is the number of full-price tickets that Ron expects to sell?

b. Based on a review of the show in another city, Ron updates his demand forecast for full-price tickets to be normal with mean 60 and standard deviation 40, but he does not change the prices. What is the optimal protection level for full-price seats?

c. Ron realizes that having many empty seats negatively affects the attendees' value from the show. Hence, he decides to change the discount given on student-only tickets from 50 percent off the full price to 55 percent off the full price and he continues to set his protection level optimally. (The demand forecast for full-price tickets remains as in b, normal with mean 60 and standard deviation 40.) How will this change in the student-only discount price affect the expected number of empty seats? (Will they increase, decrease, or remain the same or it is not possible to determine what will happen?)

d. Ron knows that on average eight seats (Poisson distributed) remain empty due to no-shows. Ron also estimates that it is 10 times more costly for him to have one more attendee than seats relative to having one empty seat in the theater. What is the maximum number of seats to sell in excess of capacity?

Q16.8 **(Park Hyatt)** Consider the example of the Park Hyatt Philadelphia discussed in the text. Recall that the full fare is $225, the expected full-fare demand is Poisson with mean 27.3, the discount fare is $159, and there are 118 king/queen rooms. Now suppose the cost of an occupied room is $45 per night. That cost includes the labor associated with prepping and cleaning a room, the additional utilities used, and the wear and tear on the furniture and fixtures. Suppose the Park Hyatt wishes to maximize expected profit rather than expected revenue. What is the optimal protection level for the full fare?

Q16.9 **(MBA Admissions)** Each year the admissions committee at a top business school receives a large number of applications for admission to the MBA program and they have to decide on the number of offers to make. Since some of the admitted students may decide to pursue other opportunities, the committee typically admits more students than the ideal class size of 720 students. You were asked to help the admissions committee estimate the appropriate number of people who should be offered admission. It is estimated that in the coming year the number of people who will not accept the admission offer is normally distributed with mean 50 and standard deviation 21. Suppose for now that the school does not maintain a waiting list, that is, all students are accepted or rejected.

a. Suppose 750 students are admitted. What is the probability that the class size will be at least 720 students?

b. It is hard to associate a monetary value with admitting too many students or admitting too few. However, there is a mutual agreement that it is about two times more expensive to have a student in excess of the ideal 720 than to have fewer students in the class. What is the appropriate number of students to admit?

c. A waiting list mitigates the problem of having too few students since at the very last moment there is an opportunity to admit some students from the waiting list. Hence, the admissions committee revises its estimate: It claims that it is five times more expensive

to have a student in excess of 720 than to have fewer students accept among the initial group of admitted students. What is your revised suggestion?

Q16.10** **(Air Cargo)** An air cargo company must decide how to sell its capacity. It could sell a portion of its capacity with long-term contracts. A long-term contract specifies that the buyer (the air cargo company's customer) will purchase a certain amount of cargo space at a certain price. The long-term contract rate is currently $1,875 per standard unit of space. If long-term contracts are not signed, then the company can sell its space on the spot market. The spot market price is volatile, but the expected future spot price is around $2,100. In addition, spot market demand is volatile: sometimes the company can find customers; other times it cannot on a short-term basis. Let's consider a specific flight on a specific date. The company's capacity is 58 units. Furthermore, the company expects that spot market demand is normally distributed with mean 65 and standard deviation 45. On average, it costs the company $330 in fuel, handling, and maintenance to fly a unit of cargo.

a. Suppose the company relied exclusively on the spot market, that is, it signed no long-term contracts. What would be the company's expected profit?
b. Suppose the company relied exclusively on long-term contracts. What would be the company's expected profit?
c. Suppose the company is willing to use both the long-term and the spot markets. How many units of capacity should the company sell with long-term contracts to maximize *revenue?*
d. Suppose the company is willing to use both the long-term and the spot markets. How many units of capacity should the company sell with long-term contracts to maximize *profit?*

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 17

Supply Chain Coordination

Supply chain performance depends on the actions taken by all of the organizations in the supply chain; one weak link can negatively affect every other location in the chain. While everyone supports in principle the objective of optimizing the supply chain's performance, each firm's primary objective is the optimization of its own performance. And unfortunately, as shown in this chapter, self-serving behavior by each member of the supply chain can lead to less than optimal supply chain performance. In those situations, the firms in the supply chain can benefit from better operational coordination.

In this chapter we explore several challenges to supply chain coordination. The first challenge is the *bullwhip effect:* the tendency for demand variability to increase, often considerably, as you move up the supply chain (from retailer, to distributor, to factory, to raw material suppliers, etc.). Given that variability in any form is problematic for effective operations, it is clear the bullwhip effect is not a desirable phenomenon. We identify the causes of the bullwhip effect and propose several techniques to combat it.

A second challenge to supply chain coordination comes from the *incentive conflicts* among the supply chain's independent firms: An action that maximizes one firm's profit might not maximize another firm's profit. For example, one firm's incentive to stock more inventory, or to install more capacity, or to provide faster customer service, might not be the same as another firm's incentive, thereby creating some conflict between them. We use a stylized example of a supply chain selling sunglasses to illustrate the presence and consequences of incentive conflicts. Furthermore, we offer several remedies to this problem.

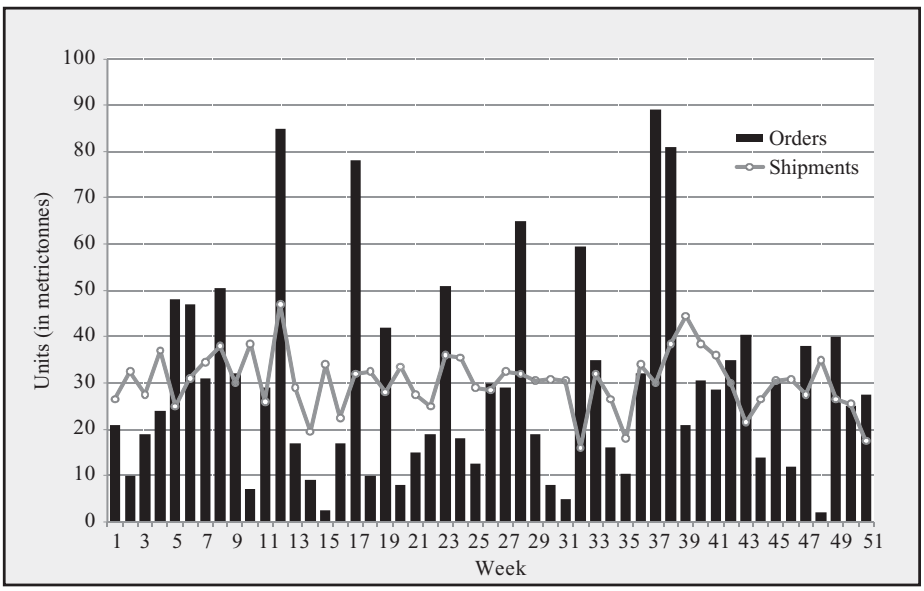
17.1 The Bullwhip Effect: Causes and Consequences

Barilla is a leading Italian manufacturer of pasta. Figure 17.1 plots outbound shipments of pasta from one of its Cortese distribution center over a one-year period along with the orders Cortese placed on Barilla's upstream factories. Think of the outbound shipments as what was demanded of Cortese by its downstream customers and the orders as what Cortese demanded from its upstream suppliers. Clearly, Cortese's demand on its upstream suppliers is more volatile than the demand Cortese faces from its customers.

This pattern, in which a stage in the supply chain amplifies the volatility of its orders relative to its demand, is called the *bullwhip effect*. If there are several stages (or levels) in the supply chain (e.g., retailer, wholesaler, distributor, factory), then this amplification can feed

FIGURE 17.1
Barilla's Cortese Distribution Center Orders and Shipments

Source: Harvard Business School, Barilla Spa case.



on itself—one level further amplifies the amplified volatility of its downstream customer. This accentuation of volatility resembles the increased amplitude one observes as a whip is cracked—hence the name, the bullwhip effect. In fact, Procter & Gamble coined the term to describe what they observed in their diaper supply chain: They knew that final demand for diapers was reasonably stable (consumption by babies), but the demands requested on their diaper factories were extremely variable. Somehow variability was propagating up their supply chain.

The bullwhip effect does not enhance the performance of a supply chain: Increased volatility at any point in the supply chain can lead to product shortages, excess inventory, low utilization of capacity, and/or poor quality. It impacts upstream stages in the supply chain, which must directly face the impact of variable demand, but it also indirectly affects downstream stages in the supply chain, which must cope with less reliable replenishments from upstream stages. Hence, it is extremely important that its causes be identified so that cures, or at least mitigating strategies, can be developed.

Figure 17.1 provides a real-world example of the bullwhip effect, but to understand the causes of the bullwhip effect, it is helpful to bring it into the laboratory, that is, to study it in a controlled environment. Our controlled environment is a simple supply chain with two levels. The top level has a single supplier and the next level has 20 retailers, each with one store. Let's focus on a single product, a product in which daily demand has a Poisson distribution with mean 1.0 unit at each retailer. Hence, total consumer demand follows a Poisson distribution with mean 20.0 units. (Recall that the sum of Poisson distributions is also a Poisson distribution.) Figure 17.2 displays this supply chain.

Before we can identify the causes of the bullwhip effect, we must agree on how we will measure and identify it. We use the following definition:

> *The bullwhip effect is present in a supply chain if the variability of demand at one level of the supply chain is greater than the variability of demand at the next downstream level in the supply chain, where variability is measured with the coefficient of variation.*

For example, if the coefficient of variation in the supplier's demand (which is the sum of the retailers' orders) is greater than the coefficient of variation of the retailers' total demand, then the bullwhip effect is present in our supply chain.

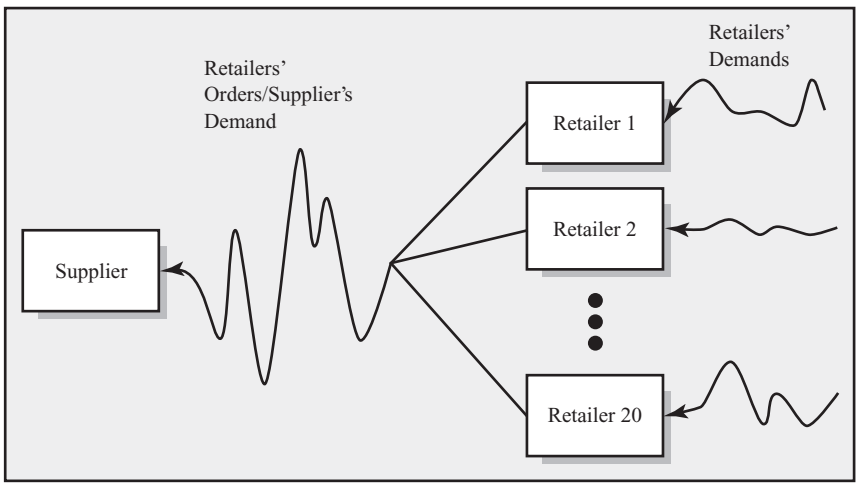


FIGURE 17.2
A Supply Chain with One Supplier and 20 Retailers
Daily demand at each retailer follows a Poisson distribution with mean 1.0 unit.

We already know how to evaluate the coefficient of variation in the retailers' total demand: Total demand is Poisson with mean 20, so the standard deviation of demand is $\sqrt{20} = 4.47$ and the coefficient of variation is $4.47/20 = 0.22$. The coefficient of variation of the supplier's demand (i.e., the coefficient of variation of the retailers' orders) depends on how the retailers place orders with the supplier.

Interestingly, while the way in which the retailers submit orders to the supplier can influence the standard deviation of the retailers' orders, it cannot influence the mean of the retailers' orders. To explain, due to the law of the conservation of matter, what goes into a retailer must equal what goes out of the retailer on average; otherwise, the amount inside the retailer will not be stable: If more goes in than goes out, then the inventory at the retailer continues to grow, whereas if less goes in than goes out, then inventory at the retailer continues to fall. Hence, no matter how the retailers choose to order inventory from the supplier, the mean of the supplier's demand (i.e., the retailers' total order) equals the mean of the retailers' total demand. In this case, the supplier's mean demand is 20 units per day, just as the mean of consumer demand is 20 units per day. We can observe this in Figure 17.1 as well: Cortese's average shipment is about 30 tonnes and their average order is also about 30 tonnes.

To evaluate the coefficient of variation in the supplier's demand, we still need to evaluate the standard deviation of the supplier's demand, which does depend on how the retailers submit orders. Let's first suppose that the retailers use an order-up-to policy to order replenishments from the supplier.

A key characteristic of an order-up-to policy is that the amount ordered in any period equals the amount demanded in the previous period (see Chapter 14). As a result, if all of the retailers use order-up-to policies with daily review, then their daily orders will match their daily demands. In other words, there is no bullwhip effect!

> *If all retailers use an order-up-to policy (with a constant order-up-to level* S *), then the standard deviation of the retailers' orders in one period equals the standard deviation of consumer demand in one period; that is, there is no bullwhip effect.*

So we started our experiment with the intention of finding a cause of the bullwhip effect and discovered that the bullwhip effect need not occur in practice. It does not occur when every member at the same level of the supply chain implements a *"demand-pull"* inventory policy each period, that is, their orders each period exactly match their demands. Unfortunately, firms do not always adopt such "distortion-free" inventory management.

In fact, they may have good individual reasons to deviate from such behavior. It is those deviations that cause the bullwhip effect. We next identify five of them.

Order Synchronization

Suppose the retailers use order-up-to policies, but they order only once per week. They may choose to order weekly rather than daily because they incur a fixed cost per order and therefore wish to reduce the number of orders they make. (See Section 14.8.) Hence, at the start of each week, a retailer submits to the supplier an order that equals the retailer's demand from the previous week. But because we are interested in the supplier's *daily* demand, we need to know on which day of the week each retailer's week begins. For simplicity let's assume there are five days per week and the retailers are evenly spaced out throughout the week; that is, four of the 20 retailers submit orders on Monday, four submit orders on Tuesday, and so forth. Figure 17.3 displays a simulation outcome of this scenario. From the figure it appears that the variability in consumer demand is about the same as the variability in the supplier's demand. In fact, if we were to simulate many more periods and evaluate the standard deviations of those two data series, we would, in fact, discover that the standard deviation of consumer demand *exactly* equals the standard deviation of the supplier's demand. In other words, we still have not found the bullwhip effect.

But we made a critical assumption in our simulation. We assumed the retailers' order cycles were evenly spaced throughout the week: the same number of retailers order on Monday as on Wednesday as on Friday. But that is unlikely to be the case in practice: firms tend to prefer to submit their orders on a particular day of the week or a particular day of the month. To illustrate the consequence of this preference, let's suppose the retailers tend to favor the beginning and the end of the week: nine retailers order on Monday, five on Tuesday, one on Wednesday, two on Thursday, and three on Friday. Figure 17.4 displays the simulation outcome with that scenario.

We have discovered the bullwhip effect! The supplier's daily demand is clearly much more variable than consumer demand. For this particular sample, the coefficient of variation of the supplier's demand is 0.78 even though the coefficient of variation of consumer demand is only 0.19: the supplier's demand is about four times more variable than consumer demand! And this is not the result of a particularly strange demand pattern; that is,

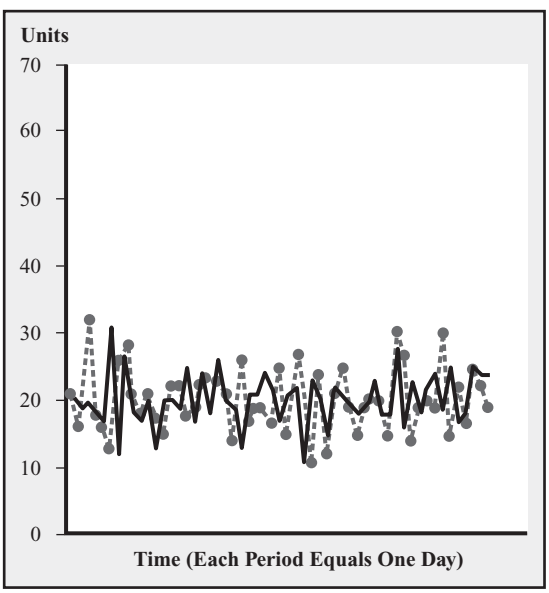


FIGURE 17.3
Simulated Daily Consumer Demand (solid line) and Daily Supplier Demand (circles)
Supplier demand equals the sum of the retailers' orders.

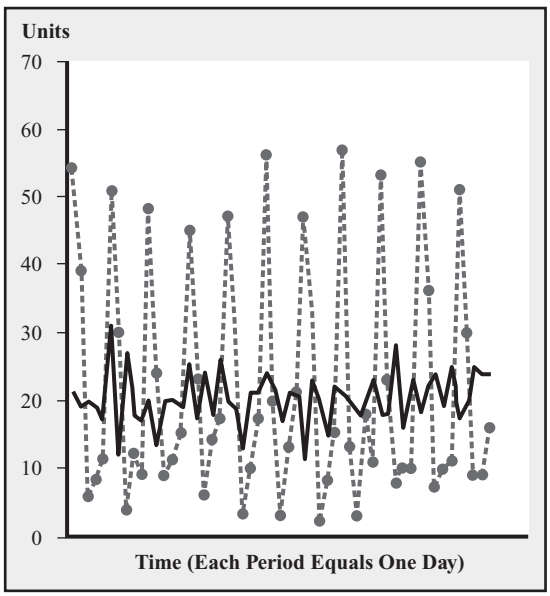


FIGURE 17.4
Simulated Daily Consumer Demand (solid line) and Supplier Demand (circles) When Retailers Order Weekly
Nine retailers order on Monday, five on Tuesday, one on Wednesday, two on Thursday, and three on Friday.

the same qualitative result is obtained if a very long interval of time is simulated. In fact, for comparison, you can note that the consumer demand in Figure 17.4 is identical to consumer demand in Figure 17.3.

Not only do we now observe the bullwhip effect, we have just identified one of its causes, *order synchronization:* If the retailers' order cycles become even a little bit synchronized, that is, they tend to cluster around the same time period, then the bullwhip effect emerges. While the retailers order on average to match average consumer demand, due to their order synchronization there will be periods in which they order considerably more than the average and periods in which they order considerably less than the average, thereby imposing additional demand volatility on the supplier.

Order synchronization also can be observed higher up in the supply chain. For example, suppose the supplier implements a materials requirement planning (MRP) system to manage the replenishment of component inventory. (This is a computer system that determines the quantity and timing of component inventory replenishments based on future demand forecasts and production schedules.) Many firms implement their MRP systems on a monthly basis. Furthermore, many implement their systems to generate replenishment orders in the first week of the month. So a supplier's supplier may receive a flood of orders for its product during the first week of the month and relatively little demand later in the month. This has been called *MRP jitters* or the *hockey stick phenomenon* (the graph of demand over the month looks like a series of hockey sticks, a flat portion and then a spike up).

Order Batching

We argued that the retailers might wish to order weekly rather than daily to avoid incurring excessive ordering costs. This economizing on ordering costs also can be achieved by *order batching:* each retailer orders so that each order is an integer multiple of some batch size. For example, now let's consider a scenario in which each retailer uses a batch size of 15 units. This batch size could represent a case or a pallet or a full truckload. Let's call it a pallet. By ordering only in increments of 15 units, that is, in pallet quantities, the retailer can facilitate the movement of product around the warehouse and the loading of product onto trucks. How does the retailer decide when to order a pallet? A natural rule is to order a batch whenever the accumulated demand since the last order exceeds the batch size.

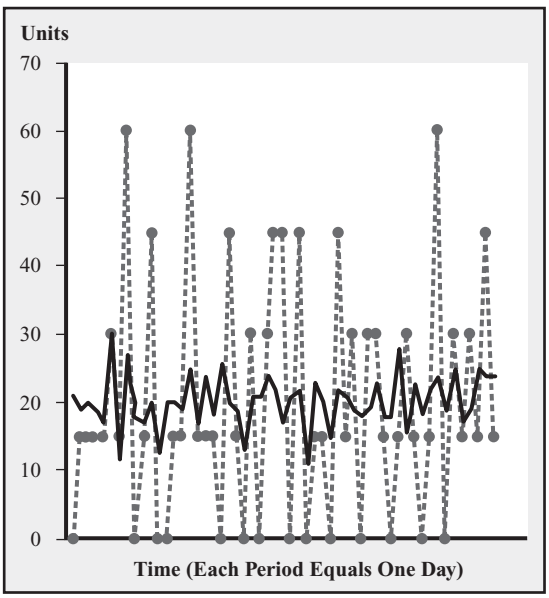


FIGURE 17.5
Simulated Daily Consumer Demand (solid line) and Supplier Demand (circles) When Retailers Order in Batches of 15 Units
Every 15th demand, a retailer orders one batch from the supplier that contains 15 units.

Therefore, in this example, every 15th demand triggers an order for a pallet. Naturally, ordering in batches economizes on the number of orders the retailer must make:

$$\text{Average number of periods between orders} = \frac{\text{Batch size}}{\text{Mean demand per period}}$$

In this situation, the retailer orders on average every 15/1 = 15 periods.

Figure 17.5 displays a simulation outcome with batch ordering. Because the retailers only order in pallet quantities, the supplier's demand equals a multiple of 15: on some days there are no orders, on most days one pallet is ordered by some retailer, on a few days there are up to four pallets ordered.

We again observe the bullwhip effect: The variability of the supplier's demand is considerably greater than the variability of consumer demand. To be specific, the supplier's demand has a coefficient of variation equal to 0.87 in this example, which contrasts with the 0.19 coefficient of variation for consumer demand. Thus, we have identified a second cause of the bullwhip effect, *order batching:* The bullwhip effect emerges when retailers order in batches that contain more than one unit (e.g., pallet quantities or full truckload quantities). Again, the retailers' total order on average equals average consumer demand, but not the variability of their orders. This occurs because, due to the batch quantity requirement, the retailer's order quantity in a period generally does not match the retailer's demand in that period: it tends to be either greater than or less than consumer demand. In other words, the batch quantity requirement forces the retailer to order in a way that is more variable than consumer demand even though, on average, it equals consumer demand.

Trade Promotions and Forward Buying

Suppliers in some industries offer their retailers *trade promotions:* a discount off the wholesale price that is available only for a short period of time. Trade promotions cause retailers to buy on-deal, also referred to as a *forward buy,* which means they purchase much more than they need to meet short-term needs. Trade promotions are a key tool for a supplier when the supplier wants to engage in the practice of *channel stuffing:* providing incentives to induce retailers (the channel) to hold more inventory than needed for the

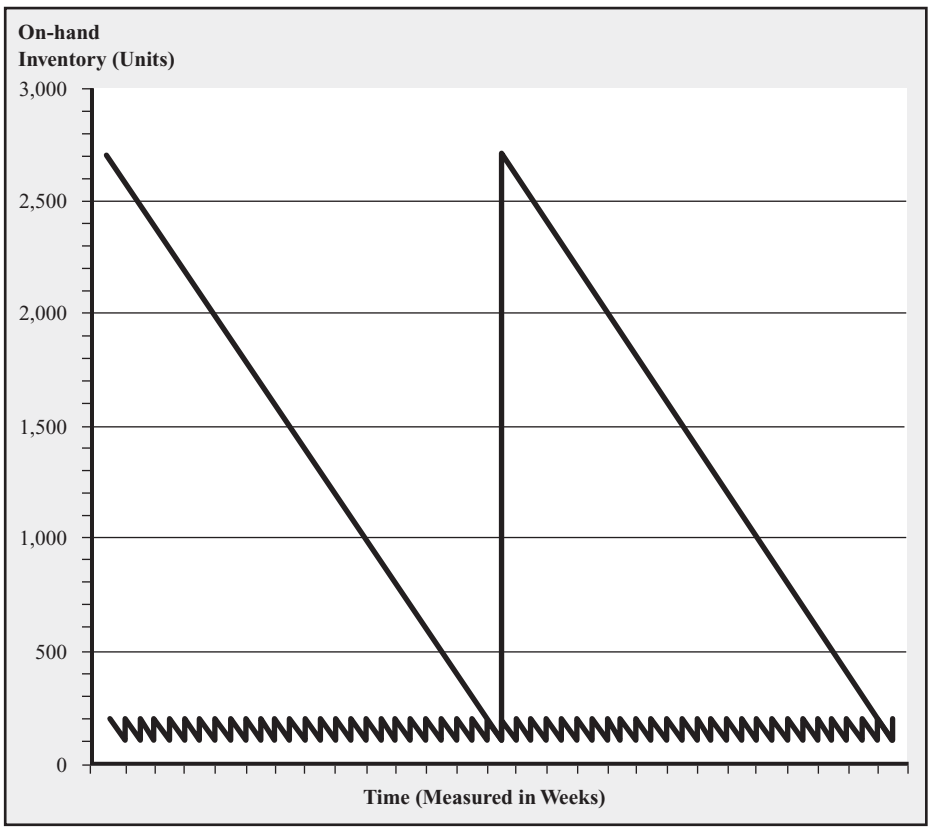


FIGURE 17.6
On-Hand Inventory of Chicken Noodle Soup at a Retailer under Two Procurement Strategies
The first strategy, called demand-pull (lower sawtooth), has the retailer ordering 100 cases each week. The second strategy, called forward buying (upper sawtooth), has the retailer ordering 2,600 cases twice per year.

short term. Because with trade promotions many retailers purchase at the same time (order synchronization) and because they order in large quantities (order batching), trade promotions are capable of creating an enormous bullwhip. Let's illustrate this with another simple scenario.

Suppose a supplier sells chicken noodle soup; let's consider one of the supplier's retailers. The supplier's regular price of chicken noodle soup is $20 per case, but twice a year the supplier offers an 8 percent discount for cases purchased during a one-week period, for example, the first week in January and the first week in July. The retailer sells on average 100 cases of soup per week and likes to carry a one-week safety stock, that is, the retailer does not let its inventory fall below 100 cases. To avoid unnecessary complications, let's further assume that the retailer's order at the beginning of a week is delivered immediately and demand essentially occurs at a constant rate. The retailer's annual holding cost rate is 24 percent of the dollar value of its inventory.

We now compare the retailer's profit with two different ordering strategies. With the first strategy, the retailer orders every week throughout the year; with the second strategy, the retailer orders only twice per year—during the trade promotion. We call the first strategy *demand-pull* because the retailer matches orders to current demand. The second strategy is called *forward buying* because each order covers a substantial portion of future demand. Figure 17.6 displays the retailer's on-hand inventory over the period of one year with both ordering strategies.

With demand-pull, the retailer's inventory "saw-tooths" between 200 and 100 units, with an average of 150 units. With forward buying, the retailer's inventory also "saw-tooths" but now between 2,700 and 100, with an average of 1,400 units. Note, although throughout the text we measure inventory at the end of each period, here, we are measuring average

inventory throughout time. That is, we take average inventory to be the midpoint between the peak of each sawtooth and the trough of each sawtooth. This approach is easier to evaluate and leads to the same qualitative results (and from a practical perspective, nearly the same quantitative result as well).

Let's now evaluate the retailer's total cost with each strategy. With demand-pull, the retailer's average inventory is 150 units. During the two promotion weeks, the average inventory in dollars is $150 \times \$18.4 = \$2{,}760$ because the promotion price is $\$20 \times (1 - 0.08) = \$18.40$. During the remaining 50 weeks of the year, the average inventory in dollars is $150 \times \$20 = \$3{,}000$. The weighted average inventory in dollars is

$$\frac{(\$2{,}760 \times 2) + (\$3{,}000 \times 50)}{52} = \$2{,}991$$

The annual holding cost on that inventory is $\$2{,}991 \times 24\% = \$718$.

The purchased cost during the year is

$$(\$20 \times 100 \times 50) + (\$18.40 \times 100 \times 2) = \$103{,}680$$

because 100 units are purchased at the regular price over 50 weeks of the year and 100 units are purchased at the discount price during the two promotion weeks of the year. The demand-pull strategy's total cost is $\$718 + \$103{,}680 = \$104{,}398$.

The analysis of the forward buying strategy is analogous to the demand-pull strategy. A summary is provided in Table 17.1.

From Table 17.1 we see that forward buying is more profitable to the retailer than weekly ordering with demand-pull: the forward buying total cost is 2.4 percent less than the demand-pull strategy, which is a considerable amount in the grocery industry. We can conclude that a relatively small trade promotion can rationally cause a retailer to purchase a significant volume of product. In fact, the retailer may wish to purchase enough product to cover its demand until the supplier's next promotion. In contrast, it is highly unlikely that an 8 percent discount would induce consumers to purchase a six-month supply of chicken noodle soup; rational retailers are more price sensitive than consumers.

The impact of the trade promotion on the supplier is not good. Imagine the supplier sells to many retailers, all taking advantage of the supplier's trade promotion. Hence, the retailers' orders become synchronized (they order during the same trade promotion weeks of the year) and they order in very large batch quantities (much more than is needed to cover their immediate needs). In other words, trade promotions combine order synchronization and order batching to generate a significant bullwhip effect.

Interestingly, with the forward buying strategy, the retailer does not ever purchase at the regular price. Hence, if the supplier were to offer the retailer the $18.40 price

TABLE 17.1
Analysis of Total Holding and Procurement Costs for Two Ordering Strategies
In demand-pull, the retailer orders every week; in forward buying, the retailer orders twice per year during the supplier's trade promotions.

	Demand-Pull	Forward Buying
Annual purchase (units)	5,200	5,200
Average inventory (units)	150	1,400
Average inventory	$2,991	$25,760
Holding cost (24% of average inventory cost)	$718	$6,182
Units purchased at regular price	5,000	0
Units purchased at discount price	200	5,200
Total purchase cost	$103,680	$95,680
Total holding plus procurement cost	$104,398	$101,862

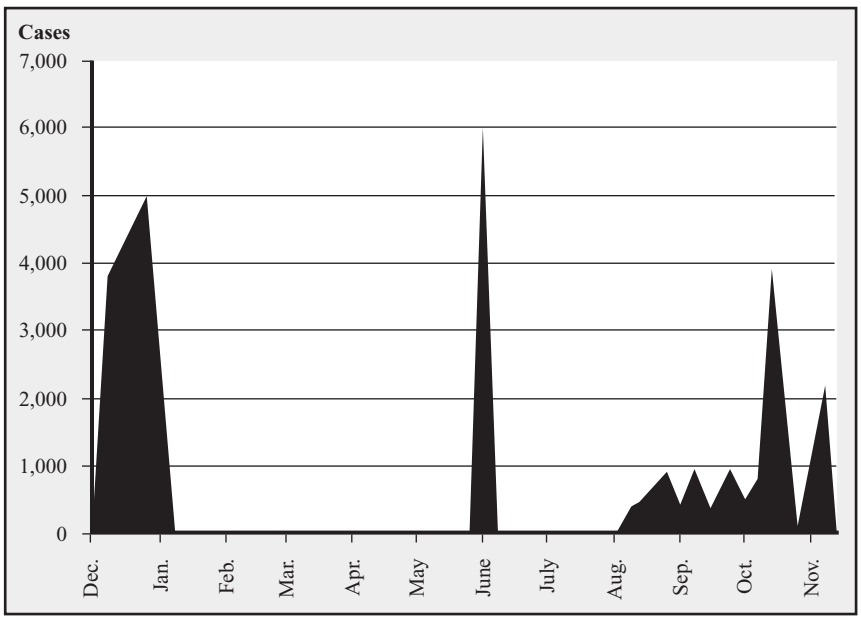


FIGURE 17.7
One Retailer's Purchases of Campbell's Chicken Noodle Soup over One Year

throughout the year (instead of just during the two trade promotion weeks), then the supplier's revenue would be the same. However, the retailer could then order on a weekly basis, thereby reducing the retailer's holding cost. It is not too difficult to calculate that the retailer's total cost in this constant-price scenario is $96,342, which is 5.4 percent less than the forward buying cost and 7.7 percent less than the original demand-pull strategy. Thus, due to forward buying, the supply chain's costs are about 5 percent higher than they need be without providing any benefit to the firms in the supply chain (the retailer surely does not benefit from holding extra inventory and the supplier does not benefit from higher revenue).

While our analysis has been with a theoretical supply chain of chicken noodle soup, Campbell Soup would concur that this analysis is consistent with their experience. For example, Figure 17.7 presents data on one retailer's purchases of Campbell's Chicken Noodle Soup over the course of the year. This product is traditionally promoted in January and June even though consumers primarily eat soup during the winter months.[1] As a result, this retailer requires substantial storage space to hold its forward buys. Other retailers may lack the financial and physical capabilities to be so aggressive with forward buying, but they nevertheless will take advantage of trade promotions to some extent. This is confirmed by Figure 17.8, which shows total consumption and shipments of Campbell's Chicken Noodle Soup over a one-year period: Shipments are clearly more volatile than consumption, thereby indicating the presence of the bullwhip effect.

Due to the trade promotion spike in demand in January of every year, Campbell Soup must put its chicken deboning plants on overtime from September through October, its canning plant works overtime November through December, and its shipping facility works overtime throughout January. All of these activities add to production costs, and all because of a spike in demand caused by the company's own pricing.

The negative effects of forward buying also are not limited to the supplier's operational efficiency. Some retailers purchase on-deal with no intention of selling those units

[1] Campbell's traditionally raises the price of its Chicken Noodle Soup during the summer, so the June buy avoids the imminent price increase. While this is technically not a promotion, the analysis is quite similar and the effect is essentially the same as a trade promotion.

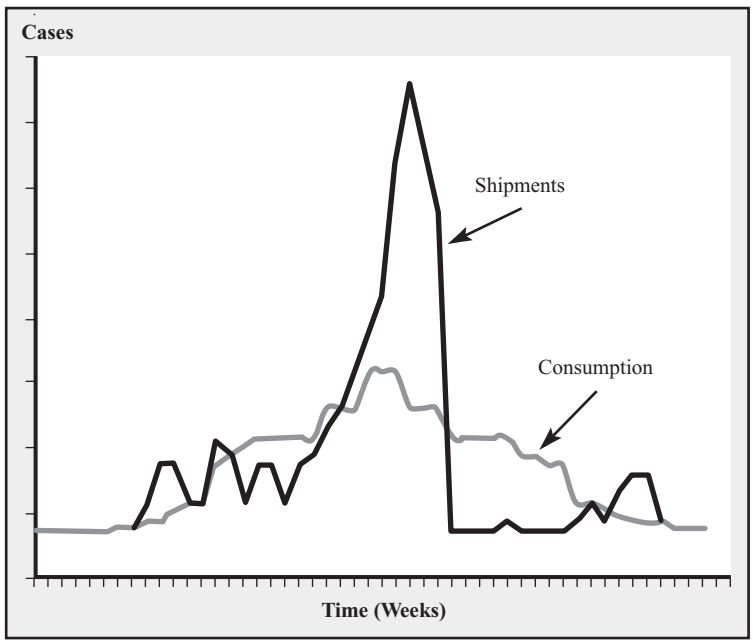


FIGURE 17.8
Total Shipments to Retailers and Consumption by Consumers of Campbell's Chicken Noodle Soup over a One-Year Period (roughly July to July)

to consumers. Instead, they intend on selling to other retailers that cannot take advantage of the deal due to either physical or capital constraints. Those retailers that sell to other retailers are called *diverters* and that practice is called *diversion.* In addition to extra handling (which reduces quality and leads to spoilage), diversion needlessly adds to transportation costs. It also should be mentioned that diversion occurs when a supplier attempts to lower its price in one region of the country while maintaining a higher price in another region, possibly because the supplier faces a regional competitor in the former region. That form of diversion was greatly reduced in the grocery industry when several national grocery chains emerged (Kroger, Safeway, etc.) in the late 1980s and early 1990s. Those national chains insisted that they would receive a single low price from their suppliers, thereby preventing regional price discrimination.

Reactive and Overreactive Ordering

So far in our experimental supply chains, we have assumed the retailer knows what expected demand is in each period even though demand could be stochastic. This is a reasonable assumption for well-established products such as chicken noodle soup. But for many other products, a retailer might not know expected demand with certainty. And this uncertainty creates a complication for the retailer's inventory management.

Suppose the retailer observes higher-than-usual demand in one period. How should the retailer react to this observation? One explanation for this outlier is that it occurred merely due to random fluctuation. In that case, the retailer probably should not change her expectation of future demand and so not change how she manages inventory. But there is another explanation for the outlier: It could signal that demand has shifted, suggesting the product's actual expected demand is higher than previously thought. If that explanation is believed, then the retailer should increase her order quantity to cover the additional future demand; otherwise she will quickly stock out. In other words, it is rational for a retailer to increase her order quantity when faced with an unusually high demand observation. Analogously, the retailer should decrease her order quantity when faced with an unusually low demand observation because future demand may be weaker than previously thought.

Hence, when a retailer cannot be sure that demand is stable over time, a retailer should rationally react aggressively to possible shifts in demand.

These reactions by the retailer contribute to the bullwhip effect. Suppose the retailer's high-demand observation is really due to random fluctuation. As a result, future demand will not be higher than expected even though the retailer reacted to this information by ordering more inventory. Hence, the retailer will need to reduce future orders so that the excess inventory just purchased can be drawn down. Ordering more than needed now and less than needed later implies the retailer's orders are more volatile than the retailer's demand, which is the bullwhip effect.

While it can be rational to react to extreme demand observations, it is also human nature to *over*react to such information, that is, to act too aggressively. For example, a high-demand signal may rationally warrant a 125 percent increase in a retailer's order quantity, but a retailer may "play it safe" and order 150 percent more just in case. Unfortunately, the retailer might not realize the consequence of this action. Suppose the retailer is replenished by a wholesaler, who is replenished by a distributor, who is replenished by a supplier. The retailer sees a blip in demand and so reacts with a larger order. The retailer's order is the wholesaler's demand, and so the wholesaler sees an even larger blip in demand. The wholesaler reacts and increases his order, which surprises the distributor. So the distributor reacts with an increased order, so large that the supplier only concludes that demand has accelerated substantially. In other words, overreactions can propagate up the supply chain, thereby generating a bullwhip effect.

Shortage Gaming

Under normal circumstances, a retailer will only order as much inventory as needed to cover short-term needs, in particular, the inventory needed to cover demand until the next possible replenishment. But it is not always known when the next possible replenishment will occur. If demand is increasing and capacity is constrained, then a retailer may anticipate a long wait for the next possible replenishment. A rational response is to order plenty of inventory, while inventory is potentially available, in case future replenishment opportunities do not materialize.

Imagine a supply chain with one supplier, a hot-selling product, limited capacity, and multiple retailers. Each retailer knows capacity is tight: While it is possible the supplier will have enough capacity to fill all of the retailers' orders, it is quite likely the supplier will not have enough capacity. The retailers also know that if the supplier runs out of capacity, then the supplier will allocate that scarce capacity to the retailers. The supplier may very well use a proportional allocation scheme: a retailer's share of the capacity is proportional to the retailer's order quantity relative to the total order quantity. For example, if a retailer orders 10 units and the other retailers order a total of 40 units, then the retailer will get a one-fifth share of the capacity ($10 / (10 + 40)$). When this situation occurs with a product, it is often said that the product is *on allocation;* that is, the supplier must allocate capacity because the total amount demanded by retailers exceeds available capacity.

Knowing that a product may be put on allocation, what should a retailer's ordering strategy be? Returning to our example, the retailer wants 10 units but anticipates only one-fifth of that order will be delivered. Hence, if 10 units are ordered, only 2 units will be received, far less than the retailer wants. An obvious solution is to instead order 50 units: if the retailer receives one-fifth of the order, and 50 units are ordered, then the retailer will receive the desired quantity, 10 units. But the other retailers are probably thinking the same thing. So they too may order much more than needed in anticipation of receiving only a fraction of their order. This behavior of ordering more than needed due to the anticipation of a possible capacity shortage is called *shortage gaming* or *order inflation.*

Shortage gaming can result in quite a mess for the supply chain. Some retailers may receive far less than they could sell (because they did not inflate their order enough) while others might actually receive much more than they can sell (because they inflated their order too much). For instance, the retailer in our example can order 50 units and actually receive 12 units, still only a fraction of the retailer's order, but 2 units more than wanted. Furthermore, order inflation contributes to the bullwhip effect: Once a supplier's customers believe that capacity may be constrained, the supplier's customers may inflate their orders substantially, thereby creating excessive volatility in the supplier's demand. Interestingly, this may occur even if there is enough capacity to satisfy the retailers' desired quantity; all that is needed to create order inflation is the belief among the retailers that they may not get their full order.

A supplier also can exacerbate the bullwhip effect with her own actions via shortage gaming. For example, suppose a supplier allows retailers to return unsold inventory. This was a common practice in the PC industry: Suppliers such as IBM would allow distributors to return any PC at any time for a full refund and IBM would even pay for shipping costs. With little risk associated with having too much inventory, distributors focused on the risk of having too little inventory, especially if they had less inventory than they wanted due to a capacity shortage (which was common). Hence, distributors actively participated in shortage gaming.

In the PC industry, it was also common to allow distributors to submit orders that could be canceled without penalty before the order was delivered. In effect, the distributor would be allowed to return an order even before receiving the order. Again, this practice mitigated the distributors' risk of excess ordering, so the focus turned to the risk of not receiving enough product. Distributors would submit excessively large orders knowing full well that they would later cancel a portion of their order. The amount that they would later cancel would depend on how well the product was selling and the available capacity. Not surprisingly, these *phantom orders,* as they are called in the industry (orders that are submitted even though a larger portion of them will disappear, like a phantom), create a bullwhip effect and substantial headaches for the supplier: the supplier receives plenty of orders but does not know what fraction of them will materialize into actual accepted deliveries.

17.2 Bullwhip Effect: Mitigating Strategies

This section discusses how firms have changed their business practices to combat the bullwhip effect. In the grocery industry, many of these changes came with the *Efficient Consumer Response* initiative that was initiated in the early 1990s. The claim was that this set of business practices, if fully implemented, could reduce U.S. grocery industry costs by $30 billion.

Not surprisingly, effective change begins with an understanding of root causes. In the case of the bullwhip effect, we identified five causes in the previous section: order synchronization, order batching, trade promotions, overreactive ordering, and shortage gaming.

Sharing Information

Greater information sharing about actual demand between the stages of the supply chain is an intuitive step toward reducing the bullwhip effect. As we saw in the simulations reported in the previous section, the pattern of retail orders may have very little resemblance to the pattern of retail demand. As a result, when retail orders are fluctuating wildly, it can be extremely difficult for a supplier to correctly forecast demand trends and it is not surprising at all if the supplier overreacts to those data. By giving the supplier frequent access to actual consumer demand data, the supplier can better assess trends in demand and plan accordingly.

But sharing current demand data is often not enough to mitigate the bullwhip effect. Demand also can be influenced by retailer actions on pricing, merchandizing, promotion, advertising, and assortment planning. As a result, a supplier cannot accurately forecast sales for a product unless the supplier knows what kind of treatment that product will receive from its retailers. Without that information, the supplier may not build sufficient capacity for a product that the retailers want to support, or the supplier may build too much capacity of a product that generates little interest among the retailers. Both errors may be prevented if the supplier and retailers share with each other their intentions. This sharing process is often labeled *collaborative planning, forecasting, and replenishment,* or CPFR for short.

While it is quite useful for a retailer to share information with its upstream suppliers, it also can be useful for a supplier to share information on availability with its downstream retailers. For example, a supplier may be aware of a component shortage that will lead to a shortage in a product that a retailer intends to promote. By sharing that information, the retailer could better allocate its promotional effort. It also can be useful to share information when the supplier knows that a capacity shortage will not occur, thereby preventing some shortage gaming.

Smoothing the Flow of Product

It is important to recognize that information sharing is quite helpful for reducing the bullwhip effect, but it is unlikely to eliminate it. The bullwhip effect is also a result of physical limitations in the supply chain like order synchronization and order batching.

Order synchronization can be reduced by eliminating reasons why retailers may wish to order at the same time (such as trade promotions). Coordinating with retailers to schedule them on different order cycles also helps.

Reducing order batching means smaller and more frequent replenishments. Unfortunately, this objective conflicts with the desire to control ordering, transportation, and handling costs. The fixed cost associated with each order submitted to the supplier can be reduced with the use of computerized automatic replenishment systems for deciding when and how much to order. In addition, some kind of technology standard, like *electronic data interchange* (EDI), is needed so that orders can be transmitted in an electronic format that can be received by the supplier.

Transportation costs can conflict with small batches because the cost of a truck shipment depends little on the amount that is shipped. Hence, there are strong incentives to ship in full truckloads. There are also economies of scale in handling inventory, which is why it is cheaper to ship in cases than in individual units and cheaper to move pallets rather than individual cases. So the trick is to find a way to have more frequent replenishments while still controlling handling and transportation costs.

One solution is for multiple retailers to consolidate their orders with a supplier through a distributor. By ordering from a distributor rather than directly from a supplier, a retailer can receive the supplier's products on a more frequent basis and still order in full truckloads. The difference is that with direct ordering, the retailer is required to fill a truck with the supplier's products whereas by going through a distributor, the retailer can fill a truck with product from multiple suppliers that sell through that distributor.

Eliminating Pathological Incentives

As we saw in the previous section, trade promotions provide an extremely strong incentive for a retailer to forward buy and forward buying creates a substantial bullwhip effect. A constant wholesale price completely eliminates this incentive. Furthermore, a constant wholesale price might not even cost the supplier too much in revenue, especially if the majority of the retailers never purchased at the regular price.

However, there are perceived negatives associated with eliminating trade promotions. Suppliers began using trade promotions to induce retailers to offer consumer promotions with the objective of using these consumer promotions to increase final consumer demand. And, in fact, trade promotion did succeed somewhat along these lines: Most retailers would cut the retail price during a trade promotion, thereby passing on at least a portion of the deal to consumers. Hence, if trade promotions can no longer be used to induce retailers to conduct consumer promotions, and if consumer promotions are deemed to be necessary, then suppliers must develop some other tool to generate the desired consumer promotions.

Generous returns and order cancellation policies are the other self-inflicted pathological incentives because they lead to shortage gaming and phantom ordering. One solution is to either eliminate these policies or at least make them less generous. For example, the supplier could agree to only partially refund returned units or the supplier could limit the number of units that can be returned or the supplier could limit the time in which they can be returned. The supplier also could impose an order cancellation penalty or require a nonrefundable deposit when orders are submitted.

Shortage gaming also can be eliminated by forgoing retailer orders altogether. To explain how this could work, suppose a supplier knows that a product will be on allocation, which means that each retailer will want more than it can receive. So the supplier does not even bother collecting retailer orders. Instead, the supplier could announce an allocation to each retailer proportional to the retailer's past sales. In the auto industry, this scheme is often called *turn-and-earn:* if a dealer turns a vehicle (i.e., sells a vehicle), then the dealer earns the right to another vehicle. Turn-and-earn allocation achieves several objectives: it ensures the supplier's entire capacity is allocated; it allocates more capacity to the higher-selling retailers, which makes intuitive sense; and it motivates retailers to sell more of the supplier's product. For example, in the auto industry, a supplier can use the allocation of a hot-selling vehicle to encourage a dealer to increase its sales effort for all vehicles so that the dealer can defend its allocation. While this extra motivation imposed on dealers is probably beneficial to the auto manufacturers, it is debatable whether it benefits dealers.

Using Vendor-Managed Inventory

Procter & Gamble and Walmart were among the first companies to identify the bullwhip effect and to take multiple significant steps to mitigate it. (Campbell's Soup was another early innovator in North America.) The set of changes they initiated are often collected under the label *Vendor-Managed Inventory,* or VMI for short. While many firms have now implemented their own version of VMI, VMI generally includes the following features:

- The retailer no longer decides when and how much inventory to order. Instead, the supplier decides the timing and quantity of shipments to the retailer. The firms mutually agree on an objective that the supplier will use to guide replenishment decisions (e.g., a target in-stock probability). The supplier's "reach" into the retailer can vary: In some applications, the supplier merely manages product in the retailer's distribution center and the retailer retains responsibility of replenishments from the distribution center to the stores. In other applications, the supplier manages inventory all the way down to the retailer's shelves. The scope of the supplier's reach also can vary by application: Generally, the supplier only controls decisions for its own products, but in some cases the supplier assumes responsibility for an entire category, which generally includes making replenishment decisions for the supplier's competitor's products on behalf of the retailer.
- If the supplier is going to be responsible for replenishment decisions, the supplier also needs information. Hence, with VMI the retailer shares with the supplier demand data (e.g., distribution center withdrawals and/or retail store point-of-sale data, POS data for short). The supplier uses those data as input to an automatic replenishment system; that is,

a computer program that decides the timing and quantity of replenishments for each product and at each location managed. In addition to normal demand movements, the supplier must be made aware of potential demand shifts that can be anticipated. For example, if the retailer is about to conduct a consumer promotion that will raise the base level of demand by a factor of 20, then the supplier needs to be aware of when that promotion will occur. These computer-guided replenishment systems are often referred to as *continuous replenishment* or *continuous product replenishment.* However, these are somewhat misnomers since product tends to be replenished more frequently but not continuously.

- The supplier and the retailer eliminate trade promotions. This is surely necessary if the retailer is going to give the supplier control over replenishment decisions because a retailer will not wish to forgo potential forward-buying profits. Hence, the adoption of VMI usually includes some agreement that the supplier will maintain a stable price and that price will be lower than the regular price to compensate the retailer for not purchasing on a deal.

The innovations included in VMI are complementary and are effective at reducing the bullwhip effect. For example, transferring replenishment control from the retailer to the supplier allows the supplier to control the timing of deliveries, thereby reducing, if not eliminating, any order synchronization effects. VMI also allows a supplier to ship in smaller lots than the retailer would order, thereby combating the order-batching cause of the bullwhip. For example, prior to the adoption of VMI, many of Campbell Soup's customers would order three to five pallets of each soup type at a time, where a pallet typically contains about 200 cases. They would order in multiple pallets to avoid the cost of frequent ordering. With VMI Campbell Soup decided to ship fast-moving soups in pallet quantities and slower-moving varieties in mixed pallet quantities (e.g., in one-half- or one-quarter-pallet quantities). Frequent ordering was not an issue for Campbell Soup because they implemented an automatic replenishment system. But Campbell Soup was still concerned about handling and transportation costs. As a result, with VMI Campbell Soup continued to ship in full truckloads, which are about 20 pallets each. However, with VMI each of the 20 pallets could be a different product, whereas before VMI there would be fewer than 20 products loaded onto each truck (because more than one pallet would be ordered for each product). Hence, with VMI it was possible to maintain full truckloads while ordering each product more frequently because each product was ordered in smaller quantities.

In some cases VMI also assists with order batching because it allows the supplier to combine shipments to multiple retailers. Before VMI it would be essentially impossible for two retailers to combine their order to construct a full truckload. But if the supplier has a VMI relationship with both retailers, then the supplier can combine their orders onto a truck as long as the retailers are located close to each other. By replenishing each retailer in smaller than full truckload batches, the supplier reduces the bullwhip effect while still maintaining transportation efficiency.

VMI also can combat the overreaction cause of the bullwhip effect. Because demand information is shared, the supplier is less likely to overreact to changes in the demand. In addition, because VMI is implemented with computer algorithms that codify replenishment strategies, a VMI system is not as emotionally fickle as a human buyer.

While VMI changes many aspects of the supply chain relationship between a supplier and retailer, some aspects of that relationship are generally not disturbed. For example, VMI eliminates trade promotions, but it does not necessarily seek to eliminate consumer promotions. Consumer promotions also can contribute to the bullwhip effect, but there are several reasons why they do not tend to increase volatility as much as trade promotions: Not every retailer runs a consumer promotion at the same time, so order synchronization is not as bad as with a trade promotion, and consumers do not forward buy as much as retailers. In addition, while some companies are willing to forgo trade promotions, only a

few are willing to forgo consumer promotions as well: Consumer promotions are viewed as a competitive necessity.

The Countereffect to the Bullwhip Effect: Production Smoothing

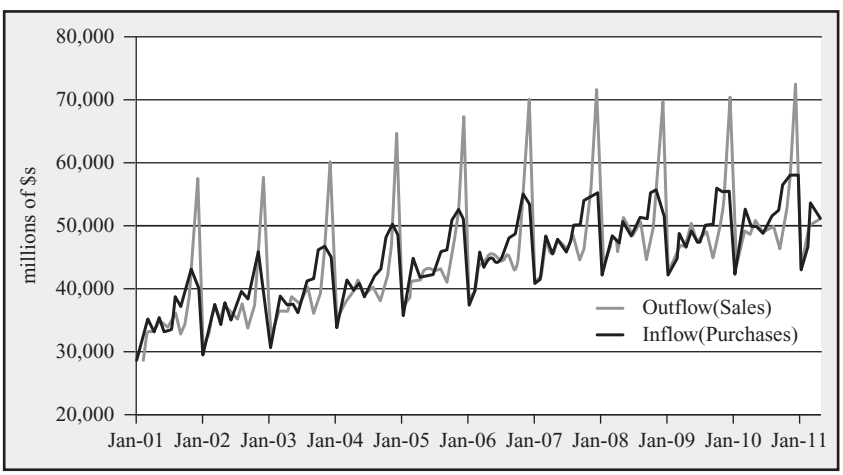
Due to the numerous causes of the bullwhip effect, one might expect that the bullwhip effect is a potential problem in nearly any supply chain. But this leads to the following questions: Does the bullwhip effect indeed exist in every supply chain? Is there any natural force that counteracts the bullwhip effect? The short answers are no, the bullwhip effect need not exist everywhere, because there is indeed a force that works to reduce it.

Figure 17.9 shows the monthly inflow and outflow of goods for general merchandisers (such as Walmart, Target, and Kohl's) in the United States over a 10-year period. Outflow of goods is analogous to demand—it is the dollar volume of goods that leaves general merchandisers, presumably into the hands of final customers. The inflow of goods is the dollar volume of goods purchased by general merchandisers. The figure reveals that the inflow of goods is actually less variable than the outflow of goods. Put another way, the demand seen by the suppliers of the general merchandisers (the inflow series) is less variable than the demand seen by the general merchandisers themselves (the outflow series)—we do not observe the bullwhip effect (at least at the aggregate level of an entire industry and at the monthly time interval). Why?

Looking at these retailers' demand, we see a noticeable fourth-quarter spike each year, which is particularly strong in November and especially in December. Intuitively, this is the annual holiday season sales surge. This annual spike presents retailers with a significant operational challenge—not only do customers need to be helped, shelves need to be replenished. Replenishing on a just-in-time basis requires a substantial amount of labor, but hiring that many seasonal workers for such a short time would be very expensive (just November and December). Instead, retailers start the process of moving product into their warehouses and stores at the start of the quarter, September and October. Each year, as Figure 17.9 reveals, retailers have a net inflow of goods during those months—inflows are greater than outflows (i.e., they build up their inventory). This prepositioning of inventory allows them to smooth out the inflow of product, thereby reducing the amount of work that needs to be done at the very busiest time of the year. In effect, retailers engage in production smoothing–build inventory during slow times and draw down inventory during hectic times so that the burden on your workforce is not too great. Apparently, it is cheaper to preposition inventory than it is to have large fluctuations in the number of employees.

FIGURE 17.9
Inflow and Outflow of Goods to U.S. General Merchandisers

Source: U.S. Census Bureau, Monthly retail trade data.



Due to this production-smoothing strategy, the suppliers to these retailers actually experience less volatility in their demand than the retailers do.

In general, when a retailer (or any other firm) faces highly seasonal demand (i.e., predictably variable demand), that retailer will have an incentive to engage in production smoothing. This, as we have seen, will act as a force to counteract the bullwhip effect. Whether this force is strong enough to eliminate the bullwhip effect or not depends on how seasonal demand is and how strong the bullwhip forces are. For general merchandisers, the holiday sales spike is sufficiently large and predictable that it overwhelms the bullwhip forces, at least when measured at the industry level and with monthly data. For individual retailers, individual products, and shorter time intervals (weekly or daily), the bullwhip effect may reemerge.

Although seasonality tends to dampen (or eliminate) the bullwhip effect, seasonality is still (almost by definition) a source of variability in the supply chain. But while it creates variability, it does not contribute to amplification—even the suppliers to general merchandisers experience considerable variability and seasonality in their demand, but it is less than the variability faced by their downstream customers.

17.3 Incentive Conflicts in a Sunglasses Supply Chain

The bullwhip effect deteriorates supply chain performance by propagating demand variability up the supply chain. But optimal supply chain performance is also not guaranteed in the absence of the bullwhip effect. This section considers the incentive conflicts that can occur between two firms in a supply chain even without the presence of the bullwhip effect. We illustrate these conflicts with a detailed example based on a supply chain for sunglasses.

Zamatia Ltd. (pronounced zah-MAH-tee-ah, to the cognoscenti) is an Italian upscale maker of eyewear. UV Inc., short for Umbra Visage, is one of their retailers in the United States. To match UV's stylish assortment, UV only operates small boutique stores located in trendy locations. We focus on one of their stores located in Miami Beach, Florida. Zamatia manufactures its sunglasses in Europe and Asia, so the replenishment lead time to the United States is long. Furthermore, the selling season for sunglasses is short and styles change significantly from year to year. As a result, UV receives only one delivery of Zamatia glasses before each season. As with any fashion product, some styles sell out quickly while others are left over at the end of the season.

Consider Zamatia's entry-level sunglasses for the coming season, the Bassano. UV purchases each one of those pairs of sunglasses from Zamatia for \$75 and retails them for \$115. Zamatia's production and shipping costs per pair are \$35. At the end of the season, UV generally needs to offer deep discounts to sell remaining inventory; UV estimates that it will only be able to fetch \$25 per leftover Bassano at the Miami Beach store. UV's Miami Beach store believes this season's demand for the Bassano can be represented by a normal distribution with a mean of 250 and a standard deviation of 125.

UV's procurement quantity decision can be made with the use of the newsvendor model (Chapter 12). Let Q be UV's order quantity. UV's underage cost per unit is $C_u = \$115 - \$75 = \$40$, that is, each lost sale due to underordering costs UV the opportunity cost of \$40. UV's overage cost per unit is $C_o = \$75 - \$25 = \$50$; the consequence of leftover inventory is substantial. UV's critical ratio is

$$\frac{C_u}{C_o + C_u} = \frac{40}{50 + 40} = \frac{4}{9} = 0.4444$$

Hence, to maximize expected profit, UV should choose an order quantity such that 44.4 percent is the probability there is some leftover inventory and 55.6 percent is the probability there is a stockout.

From the Standard Normal Distribution Function Table, we find $\Phi(-0.14) = 0.4443$ and $\Phi(-0.13) = 0.4483$, so the optimal z-statistic is -0.13 and the optimal order quantity is

$$Q = \mu + z \times \sigma = 250 - 0.13 \times 125 = 234$$

Using the equations and procedures described in Chapter 12, we also are able to evaluate several performance measures for UV's store:

$$\begin{aligned} \text{Expected sales (units)} &= 192 \\ \text{Expected leftover inventory} &= 42 \\ \text{Expected profit} &= \$5,580 \end{aligned}$$

Zamatia's profit from selling the Bassano at UV's Miami Beach store is $234 \times \$40 = \$9{,}360$, where 234 is the number of Bassano sunglasses that UV purchases and \$40 is Zamatia's gross margin ($\$75 - \$35 = \$40$).

While Zamatia might be quite pleased with this situation (it does earn \$9,360 relative to UV's \$5,580), it should not be. The total supply chain's profit is \$14,940, but it could be higher. To explain, suppose we choose an order quantity to maximize the supply chain's profit, that is, the combined expected profits of Zamatia and UV. In other words, what order quantity would a firm choose if the firm owned both Zamatia and UV? We call this the *supply chain optimal quantity* because it is the quantity that maximizes the *integrated supply chain.*

We can still use the newsvendor model to evaluate the supply chain's order quantity decision and performance measures. Each lost sale costs the supply chain the difference between the retail price and the production cost, $\$115 - \$35 = \$80$; that is, the supply chain's underage cost is $C_u = 80$. Each leftover Bassano costs the supply chain the difference between the production cost and the salvage value, $\$35 - \$25 = \$10$; that is, the supply chain's overage cost is $C_o = 10$. The supply chain's critical ratio is

$$\frac{C_u}{C_o + C_u} = \frac{80}{10 + 80} = 0.8889$$

The appropriate z-statistic for that critical ratio is 1.23 because $\Phi(1.22) = 0.8888$ and $\Phi(1.23) = 0.8907$. The supply chain's expected profit-maximizing order quantity is then

$$Q = \mu + z \times \sigma = 250 + 1.23 \times 125 = 404$$

which is considerably higher than UV's order of 234 units. The supply chain's performance measures can then be evaluated assuming the supply chain optimal order quantity, 404 units:

$$\begin{aligned} \text{Expected sales (units)} &= 243 \\ \text{Expected leftover inventory} &= 161 \\ \text{Expected profit} &= \$17,830 \end{aligned}$$

Thus, while Zamatia and UV currently earn an expected profit of \$14,940, their supply chain could enjoy an expected profit that is about 19 percent higher, \$17,830.

Why does the current supply chain perform significantly worse than it could? The obvious answer is that UV does not order enough Bassanos: UV orders 234 of them, but the supply chain's optimal order quantity is 404 units. But why doesn't UV order enough? Because UV is acting in its own self-interest to maximize its own profit. To explain further, UV must pay Zamatia \$75 per pair of sunglasses and so UV acts as if the cost to

TABLE 17.2
UV's Order Quantity *Q* and Performance Measures for Several Possible Wholesale Price Contracts

	Wholesale Price			
	$35	**$65**	**$75**	**$85**
C_u	$80	$50	$40	$30
C_o	$10	$40	$50	$60
Critical ratio	0.8889	0.5556	0.4444	0.3333
z	1.23	0.14	−0.13	−0.43
Q	404	268	234	196
Expected sales	243	209	192	169
Expected leftover inventory	161	59	42	27
Umbra's expected profit	$17,830	$8,090	$5,580	$3,450
Zamatia's expected profit	$0	$8,040	$9,360	$9,800
Supply chain's profit	$17,830	$16,130	$14,940	$13,250

produce each Bassano is $75, not the actual $35. From UV's perspective, it does not matter if the actual production cost is $35, $55, or even $0; its "production cost" is $75. UV correctly recognizes that it only makes $40 on each sale but loses $50 on each leftover pair. Hence, UV is prudent to order cautiously.

UV's trepidation with respect to ordering is due to a phenomenon called *double marginalization.* Because UV's profit margin ($40) is one of two profit margins in the supply chain, and necessarily less than the supply chain's total profit margin ($80), UV orders less than the supply chain optimal quantity. In other words, because UV only earns a portion ($40) of the total benefit of each sale ($80), UV is not willing to purchase as much inventory as would be optimal for the supply chain.

This example illustrates an important finding:

> *Even if every firm in a supply chain chooses actions to maximize its own expected profit, the total profit earned in the supply chain may be less than the entire supply chain's maximum profit.*

In other words, rational and self-optimizing behavior by each member of the supply chain does not necessarily lead to optimal supply chain performance. So what can be done about this? That is the question we explore next.

There is an obvious solution to get UV to order more Bassanos: Zamatia could reduce the wholesale price. A lower wholesale price increases UV's underage cost (gross margin) and decreases the overage cost (loss on leftover inventory), thereby making stockouts costlier and leftover inventory less consequential. More technically, reducing the wholesale price increases UV's critical ratio, which leads UV to order more. Table 17.2 provides some data on supply chain performance with various wholesale prices.

We indeed see that if Zamatia were to reduce its wholesale price from $75 to $65, then UV would increase its Bassano order from 234 to 268 units. UV is quite happy: Its profit increases from $5,580 to $8,090. Furthermore, the supply chain's profit increases from $14,905 to $16,130. In fact, why stop with a $10 wholesale price reduction? If Zamatia were to reduce the wholesale price down to the production cost, $35, then (1) UV orders the supply chain optimal quantity, 404 units, and (2) the supply chain's profit is optimal, $17,830! That strategy is called *marginal cost pricing* because the supplier only charges the retailer the marginal cost of production.

But while marginal cost pricing is terrific for UV and the supply chain, it is disastrous for Zamatia: by definition, Zamatia's profit plunges to zero with marginal cost pricing.

We now see a classic tension within a supply chain: An increase in one firm's profit might come at the expense of a decrease in the other firm's profit. Some might refer to this distributive situation as a *zero-sum game,* but in fact it is even worse! In a zero-sum

game, two parties negotiate over how to split a fixed reward (in this case, the total profit), but in this situation the total amount to be allocated between Zamatia and UV is not even fixed: Increasing Zamatia's profit may result in a smaller total profit to be shared.

With respect to the allocation of supply chain profit, firms should care about two things:

1. The size of a firm's piece of the "pie," where the pie refers to the supply chain's total profit.
2. The size of the total "pie," that is, the supply chain's total profit.

Number 1 is obvious: every firm always wants a larger piece of the pie. Number 2 is less obvious. For a fixed piece of the pie, why should a firm care about the size of the pie, that is, the size of the other firm's piece? "Petty jealousy" is not the answer. The answer is that it is always easier to divide a bigger pie: If a pie gets bigger, then it is possible to give everyone a bigger piece, that is, everyone can be better off if the pie is made bigger. In practice this is often referred to as a *win-win* deal, that is, both parties are better off.

Turning back to our discussion of the wholesale price for Zamatia and UV, we see that arguing over the wholesale price is akin to arguing over each firm's piece of the pie. And in the process of arguing over how to divide the pie, the firms may very well end up destroying part of the pie, thereby serving no one. What these firms need is a tool that first maximizes the size of the pie ($17,830) and then allows them to decide how to divide it between them without damaging any part of it. Such a tool is discussed in the next section.

17.4 Buy-Back Contracts

Without changing the wholesale price, Zamatia would get UV to order more Bassano sunglasses if Zamatia could mitigate UV's downside risk of leftover inventory: UV loses a considerable amount ($50) on each unit it is stuck with at the end of the season. One solution is for Zamatia to buy back from UV all leftover sunglasses for a full refund of $75 per pair; that is, Zamatia could offer UV a *buy-back contract,* also called a *returns policy.*

Unfortunately, buy-back contracts introduce new costs to the supply chain. In particular, UV must ship leftover inventory back to Zamatia, which it estimates costs about $1.50 per pair. And then there is the issue of what Zamatia will do with these leftover Bassano sunglasses when it receives them. One possibility is that Zamatia just throws them out, thereby "earning" a zero salvage value on each leftover Bassano. However, Zamatia may be able to sell a portion of its leftover inventory to a European retailer that may be experiencing higher sales or Zamatia may be able to collect some revenue via an outlet store. It is even possible that Zamatia has higher salvage revenue from each Bassano at the end of the season than UV. But let's suppose Zamatia is able to earn $26.50 per Bassano at the end of the season. Hence, from the perspective of the supply chain, it does not matter whether UV salvages these sunglasses at the end of the season (which earns $25) or if Zamatia salvages these sunglasses at the end of the season (which also earns $25, net of the shipping cost). In contrast, Zamatia and UV might care which firm does the salvaging of leftover inventory. We later expand upon this issue.

Let's begin the analysis of UV's optimal order quantity given the buy-back contract. UV's underage cost with this buy-back contract is still the opportunity cost of a lost sale, which is $C_u = \$115 - \$75 = \$40$. However, UV's overage cost has changed. Now UV only loses $1.50 per leftover pair due to Zamatia's generous full refund returns policy, $C_o = \$1.50$. UV's critical ratio is

$$\frac{C_u}{C_o + C_u} = \frac{40}{1.5 + 40} = 0.9639$$

With a critical ratio of 0.9639, the optimal z-statistic is 1.8 (i.e., $\Phi(1.79) = 0.9633$ and $\Phi(1.8) = 0.9641$), so UV's optimal order quantity is now

$$Q = \mu + z \times \sigma = 250 + 1.8 \times 125 = 475$$

We can evaluate UV's expected profit and discover that it has increased from \$5,580 (with no refund on returns) to \$9,580 with the returns policy. Furthermore, with an order quantity of 475 units, UV's expected leftover inventory is 227 units.

Zamatia has surely provided an incentive to UV to increase its order quantity, but is this offer also good for Zamatia? Zamatia's expected profit has several components: It sells 475 units to UV at the beginning of the season, which generates $475 \times \$75 = \$35{,}625$ in revenue; its production cost is $475 \times \$35 = \$16{,}625$; it expects to pay UV $227 \times \$75 = \$17{,}025$ to buy back the expected 227 units of leftover inventory; and it collects $227 \times \$26.5 = \$6{,}016$ in salvage revenue. Combining those components together yields an expected profit of \$7,991 for Zamatia, which is *lower* than Zamatia's profit without the returns policy, \$9,350.

How did Zamatia go wrong with this buy-back contract? Zamatia did encourage UV to order more Bassano sunglasses by reducing UV's exposure to leftover inventory risk. But Zamatia reduced that risk so much that UV actually ordered more than the supply chain optimal quantity, thereby setting Zamatia up for a large bill when leftover inventory gets shipped back. Is there a compromise between the wholesale price contract with too little inventory and the full refund buy-back contract with too much inventory? (Of course there is.)

Instead of giving a full refund on returned inventory, Zamatia could give a partial refund. For example, suppose Zamatia offers to buy back inventory from UV for \$65 per pair. This is still not a bad deal for UV. Its underage cost remains $C_u = 40$, but now its overage cost is $C_o = \$1.50 + \$75 - \$65 = \$11.50$: each unit left over costs UV the \$1.50 to ship back and due to the partial credit, it loses an additional \$10 per unit. Table 17.3 provides data on UV's optimal order quantity, expected sales, expected leftover inventory, and expected profit. The table also indicates Zamatia's profit with this partial refund is \$9,528, which is slightly better than its profit without a buy-back at all. Furthermore, the supply chain's total profit has jumped to \$17,600, which is reasonably close to the maximum profit, \$17,830. One way to evaluate the quality of a contract is by its *supply chain efficiency,* which is the fraction of the optimal profit the supply chain achieves. In this case, efficiency is 17,600 / 17,830 = 99 percent; that is, the supply chain earns 99 percent of its potential profit.

Instead of holding the wholesale price fixed and reducing the buy-back price, Zamatia could hold the buy-back price fixed and increase the wholesale price. For example, it could

TABLE 17.3
UV's Order Quantity Q and Performance Measures for Several Possible Wholesale Price Contracts

Wholesale price	\$75	\$75	\$75	\$85
Buy-back price	\$55	\$65	\$75	\$75
C_u	\$40	\$40	\$40	\$30
C_o	\$21.50	\$11.50	\$1.50	\$11.50
Critical ratio	0.6504	0.7767	0.9639	0.7229
z	0.39	0.77	1.80	0.60
Q	299	346	475	325
Expected sales	221	234	248	229
Expected leftover inventory	78	112	227	96
Expected profits:				
Umbra	\$7,163	\$8,072	\$9,580	\$5,766
Zamatia	\$9,737	\$9,528	\$7,990	\$11,594
Supply chain	\$16,900	\$17,600	\$17,570	\$17,360

increase the wholesale price to $85 and still agree to buy back inventory for $75. That contract indeed works well for Zamatia: it earns a whopping $11,594. It even is not a bad deal for UV: its profit is $5,766, which is still better than the original situation without any refund on returned inventory. But overall supply chain performance has slipped a bit: efficiency is now only 17,360 / 17,830 = 97 percent.

While we seem to be making some progress, we also seem to be fishing around without much guidance. There are many possible combinations of wholesale prices and buy-back prices, so what combinations should we be considering? Recall from the previous section that our objective should be to maximize the size of the pie and then worry about how to divide it. Every firm can be given a bigger piece if the pie is made bigger. So let's first look for wholesale/buy-back price combinations that maximize supply chain profit. In other words, we are looking for a wholesale price and a buy-back price such that UV's expected profit-maximizing order quantity given those terms is the supply chain optimal order quantity, 404 Bassanos. If we find such a contract, then we say that contract "coordinates the supply chain" because the supply chain achieves 100 percent efficiency, that is, it earns the maximum supply chain profit.

We could hunt for our desired wholesale/buy-back price combinations in Excel (for every wholesale price, slowly adjust the buy-back price until we find the one that makes UV order 404 Bassanos), or we could take a more direct route by using the following equation:

$$\text{Buy-back price} = \text{Shipping cost} + \text{Price} - (\text{Price} - \text{Wholesale price}) \times \left(\frac{\text{Price} - \text{Salvage value}}{\text{Price} - \text{Cost}} \right) \quad \textbf{(17.1)}$$

In other words, if we have chosen a wholesale price, then equation (17.1) gives us the buy-back price that would cause UV to choose the supply chain optimal order quantity. In that case, the pie would be maximized; that is, we coordinate the supply chain and supply chain efficiency is 100 percent! (If you are curious about how to derive equation (17.1), see Appendix D.)

Let's evaluate equation (17.1) with the wholesale price of $75:

$$\text{Buy-back price} = \$1.50 + \$115 - (\$115 - \$75) \times \left(\frac{\$115 - \$25}{\$115 - \$35} \right) = \$71.50$$

Hence, if the wholesale price is $75 and Zamatia agrees to buy back leftover inventory for $71.50 per pair, then UV orders 404 Bassano sunglasses and the supply chain earns the maximum profit, $17,830.

Table 17.4 provides performance data for several different wholesale prices assuming equation (17.1) is used to choose the buy-back price.

Interestingly, with a wholesale price of $75, the firms split the supply chain's profit, that is, each earns $8,915. In that case, UV does much better than just a wholesale price contract, but Zamatia does worse. However, both firms do significantly better with the wholesale price of $85 and the buy-back price of $82.75 than they do with the original contract we considered (just a $75 wholesale price and no buy-back).

Table 17.4 reveals some remarkable observations:

- There are many different wholesale price/buy-back price pairs that maximize the supply chain's profit. In other words, there are many different contracts that achieve 100 percent supply chain efficiency.

TABLE 17.4
Performance Measures When the Buy-Back Price Is Chosen to Coordinate the Supply Chain—to Ensure 100 percent Supply Chain Efficiency

Wholesale price	$35	$45	$55	$65	$75	$85	$95	$105
Buy-back price	$26.50	$37.75	$49.00	$60.25	$71.50	$82.75	$94.00	$105.25
C_u	$80	$70	$60	$50	$40	$30	$20	$10
C_o	$10.00	$8.75	$7.50	$6.25	$5.00	$3.75	$2.50	$1.25
Critical ratio	0.8889	0.8889	0.8889	0.8889	0.8889	0.8889	0.8889	0.8889
z	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
Q	404	404	404	404	404	404	404	404
Expected sales	243	243	243	243	243	243	243	243
Expected leftover inventory	161	161	161	161	161	161	161	161
Expected profits:								
Umbra	$17,830	$15,601	$13,373	$11,144	$8,915	$6,686	$4,458	$2,229
Zamatia	$0	$2,229	$4,458	$6,686	$8,915	$11,144	$13,373	$15,601
Supply chain	$17,830	$17,830	$17,830	$17,830	$17,830	$17,830	$17,830	$17,830

• Virtually any allocation of the supply chain's profit between the two firms is feasible; that is, there exist contracts that give the lion's share of the profit to the supplier, contracts that equally divide the profit, and contracts that give the lion's share to the retailer.

• The firms now truly do face a zero-sum game; that is, increasing one firm's profit means the other firm's profit decreases. However, at least now the sum that they can fight over is the maximum possible.

Which contracts will the firms ultimately agree upon? We cannot really say. If Zamatia is the better negotiator or if it is perceived to have more bargaining power than UV, then we would expect Zamatia might get UV to agree to a buy-back contract with a high wholesale price. Even though Zamatia's profit can increase substantially, it is important to note that UV's profit also may increase relative to the status quo because buy-back contracts increase the size of the pie. However, if UV has the stronger negotiating skills, then it is possible UV will secure a contract that it favors (a buy-back contract with a low wholesale price).

17.5 More Supply Chain Contracts

The previous section focused on buy-back contracts, but those are not the only type of contracts that are implemented in supply chains. This section briefly describes several other types of contracts and how they may alleviate supply chain incentive conflicts. This is by no means an exhaustive list of the types of contracts that are observed in practice.

Quantity Discounts

Quantity discounts are quite common, but they come in many different forms. For example, with an all-unit quantity discount, a buyer receives a discount on all units if the quantity ordered exceeds a threshold; whereas with an incremental quantity discount, a buyer receives a discount on all units purchased above a threshold. No matter the form, quantity discounts encourage buyers to order additional inventory because the purchase price of the last unit purchased is decreasing with the amount purchased (See Section 7.6.) In the context of the newsvendor model, a quantity discount increases the underage cost, thereby increasing the critical ratio. In contrast, recall that the buy-back contract increases the critical ratio by decreasing the overage cost.

Options Contracts

With an options contract, a buyer pays one price to purchase options, say w_o, and another price to exercise the purchased options, w_e. These contracts are often used when a buyer

wants a supplier to build capacity well in advance of the selling season. At that time, the buyer has only an uncertain forecast of demand. As the selling season approaches, the buyer anticipates that she will have a much better demand forecast, but by then it is too late to build additional capacity if demand is quite high. Without the options contract, the supplier bears all of the supply chain's risk, so the supplier is likely to build too little capacity. The options contract allows the firms to share the risk of demand–supply mismatches: The supplier earns at least something upfront (the option's price) while the buyer doesn't have to pay for all of the unused capacity (the exercise price is paid only on capacity actually exercised). Hence, just as with buy-back contracts, options contracts are able in some settings to achieve 100 percent supply chain efficiency (i.e., the supplier builds the right amount of capacity) and arbitrarily divide the supply chain's profit between the two firms (i.e., there is more than one options contract that achieves supply chain coordination).

Revenue Sharing

With revenue sharing, a retailer pays a wholesale price per unit purchased to a supplier but then also pays a portion of the revenue earned on that unit to the supplier. As with buy-back contracts, revenue sharing allows the firms in the supply chain to share the risk of demand–supply mismatches: The retailer pays something to the supplier upfront (the wholesale price) but only pays an additional amount if the unit actually generates revenue (the revenue share).

The most notable application of revenue sharing occurred in the video-rental industry. Back around 1998, the standard wholesale price contract was predominant in the industry: studios would sell videocassettes to video rental retailers for about $60 to $75 per tape and retailers would keep all rental revenue. At a rental price of about $3, retailers could only break even on a tape if it rented more than 20 times. But because demand for tapes generally starts high upon its release and fades quickly, retailers could not afford to purchase too many tapes. As a result, availability of newly released movies was quite low, driving many consumers to consider other entertainment forms (cable TV, pay-per-view, etc.). Considering that the manufacturing cost of a tape is quite low, it is clear that maximizing supply chain profit requires additional tapes at the retailer.

Around 1998 the industry's biggest player, Blockbuster, negotiated revenue sharing deals with the major studios. With revenue sharing, the retailer pays a far lower wholesale price (about $8) but shares a portion of the rental revenue (about 50 percent). With those terms, the breakeven on a tape reduces to fewer than six rentals, thereby allowing Blockbuster to justify purchasing many more tapes. They used their additional availability to launch their "Guaranteed to be there" and "Go home happy" marketing campaigns.

Quantity Flexibility Contracts

Consider an ongoing relationship between a buyer and a supplier. For example, the buyer is Sun Microsystems, the supplier is Sony, and the product is a monitor. Sun's demand fluctuates over time, but Sun nevertheless wants Sony to build enough capacity to satisfy all of Sun's needs, which could be either higher or lower than forecasted. But since Sun probably doesn't incur the cost of idle capacity, Sun is biased toward giving Sony overly rosy forecasts in the hope that Sony will respond to the forecast by building extra capacity. But Sony is no fool; that is, Sony knows that Sun is biased toward optimistic forecasts and so Sony may view Sun's forecasts with a skeptical eye. Unfortunately, Sun may actually have an optimistic forecast, but due to its lack of credibility with Sony, Sony may not respond with additional capacity.

The problem in this relationship is that Sony bears the entire risk of excess capacity; hence, Sun is biased toward rosy forecasts. One solution is to implement *quantity flexibility (QF) contracts:* with a QF contract, Sun provides an initial forecast but then must purchase some quantity within a certain percentage of that forecast. For example, suppose the firms agree to a 25 percent QF contract. Furthermore, it is the first quarter of the year and Sun forecasts its demand for the fourth quarter will be 2,000 units. By the time the fourth quarter rolls around, Sun is committed to purchasing from Sony at least 1,500 units (75 percent of the forecast) and Sony is committed to delivering up to 2,500 units (125 percent of the forecast) should Sun need more than the forecast. If demand turns out to be low, Sony is somewhat protected by the lower collar, whereas if demand turns out to be high, Sun can take advantage of that upside by knowing that Sony has some additional capacity (up to the upper collar). Hence, via quantity flexibility contracts, it can be shown that both firms are better off; that is, the supply chain pie gets bigger and each firm gets a bigger share.

Price Protection

In the PC industry, distributors are concerned with holding too much inventory because that inventory could become obsolete; that is, they must sell that inventory at deeply discounted prices. But there is another concern with holding too much inventory. Suppose a distributor purchases 1,000 computers today at $2,000 each, but one week later the supplier cuts the price to $1,800. Unless the distributor sells the entire batch of 1,000 computers in the next week, the distributor would be better off to purchase fewer computers at $2,000 and to purchase the remainder one week later at $1,800. In other words, the tendency of suppliers to cut their wholesale prices frequently and without notice creates an incentive among distributors to be cautious in the purchase quantities. If distributors then curtail their purchases below the supply chain optimal amount, it can be beneficial to provide them with an incentive to increase their order quantities.

Allowing distributors to return inventory helps to encourage distributors to order more inventory, but it is not the only way. *Price protection* is another way: with price protection, a supplier compensates the distributor for any price reductions on remaining inventory. For example, suppose at the end of the week the distributor sold 700 computers purchased at $2,000, but has 300 computers remaining. With price protection, the supplier would then send the distributor a check for $300 \times (\$2{,}000 - \$1{,}800) = \$60{,}000$. In other words, the distributor becomes indifferent between purchasing 1,000 computers for $2,000 now and purchasing 700 computers for $2,000 now and 300 computers for $1,800 in one week.

17.6 Summary

Optimal supply chain performance is not guaranteed even if every firm in the supply chain optimizes its own performance. Self-interest and decentralized decision making do not naturally lead to 100 percent supply chain efficiency. As a result, firms in a supply chain can benefit from better coordination of their actions.

The bullwhip effect (the propagation of demand variability up the supply chain) provides a serious challenge to supply chain operations. There are many causes of the bullwhip effect (order synchronization, order batching, trade promotions, overreactive ordering, and shortage gaming) and more than one of them can be present at the same time. Solutions to the bullwhip effect such as sharing demand information, removing pathological incentives, and Vendor-Managed Inventory are designed to combat those root causes.

The bullwhip effect is not the only challenge posed upon supply chains. Given the terms of trade between supply chain members, it is quite possible that supply chain actions will not be taken because of conflicting incentives. For example, with a simple wholesale price contract, it is generally found that the retailer's incentive to order inventory leads it to order less than the supply chain optimal amount of inventory, a phenomenon called double marginalization. Fortunately, incentive conflicts can be alleviated or even eliminated with the use of carefully designed contractual terms such as buy-back contracts.

17.7 Further Reading

For a description of the causes, consequences, and solutions to the bullwhip effect, see Lee, Padmanabhan, and Whang (1997).

Buzzell, Quelch, and Salmon (1990) provide a history of trade promotions and discuss their pros and cons.

For the original research on buy-back contracts, see Pasternack (1985). For a more managerial description of the application of buy-back contracts, see Padmanabhan and Png (1995). For a review of the theoretical literature on supply chain contracting, see Cachon (2004).

17.8 Practice Problems

Q17.1* **(Buying Tissues)** P&G, the maker of Puffs tissues, traditionally sells these tissues for $9.40 per case, where a case contains eight boxes. A retailer's average weekly demand is 25 cases of a particular Puffs SKU (color, scent, etc.). P&G has decided to change its pricing strategy by offering two different plans. With one plan, the retailer can purchase that SKU for the everyday-low-wholesale price of $9.25 per case. With the other plan, P&G charges the regular price of $9.40 per case throughout most of the year, but purchases made for a single delivery at the start of each quarter are given a 5 percent discount. The retailer receives weekly shipments with a one-week lead time between ordering and delivery. Suppose with either plan the retailer manages inventory so that at the end of each week there is on average a one-week supply of inventory. Holding costs are incurred at the rate of 0.4 percent of the value of inventory at the end of each week. Assume 52 weeks per year.

a. Suppose the retailer chose the first plan ($9.25 per case throughout the year). What is the retailer's expected annual purchasing and inventory holding cost?

b. Suppose the retailer chooses the second plan and only buys at the discount price ($9.40 is the regular price and a 5 percent discount for delivery at the start of each quarter). What is the retailer's expected annual purchasing and inventory holding cost?

c. Consider the first plan and propose a new everyday-low wholesale price. Call this the third plan. Design your plan so that both P&G and the retailer prefer it relative to the second plan.

Q17.2* **(Returning books)** Dan McClure is trying to decide on how many copies of a book to purchase at the start of the upcoming selling season for his bookstore. The book retails at $28.00. The publisher sells the book to Dan for $20.00. Dan will dispose of all of the unsold copies of the book at 75 percent off the retail price, at the end of the season. Dan estimates that demand for this book during the season is normal with a mean of 100 and a standard deviation of 42.

a. How many books should Dan order to maximize his expected profit?

b. Given the order quantity in part a what is Dan's expected profit?

c. The publisher's variable cost per book is $7.50. Given the order quantity in part a, what is the publisher's expected profit?

The publisher is thinking of offering the following deal to Dan. At the end of the season, the publisher will buy back unsold copies at a predetermined price of $15.00. However, Dan would have to bear the costs of shipping unsold copies back to the publisher at $1.00 per copy.

(* indicates that the solution is at the end of the book)

d. How many books should Dan order to maximize his expected profits given the buy-back offer?

e. Given the order quantity in part d, what is Dan's expected profit?

f. Assume the publisher is able on average to earn $6 on each returned book net the publisher's handling costs (some books are destroyed while others are sold at a discount and others are sold at full price). Given the order quantity in part d what is the publisher's expected profit?

g. Suppose the publisher continues to charge $20 per book and Dan still incurs a $1 cost to ship each book back to the publisher. What price should the publisher pay Dan for returned books to maximize the supply chain's profit (the sum of the publisher's profit and Dan's profit)?

Q17.3** **(Component options)** Handi Inc., a cell phone manufacturer, procures a standard display from LCD Inc. via an options contract. At the start of quarter 1 (Q1), Handi pays LCD $4.50 per option. At that time, Handi's forecast of demand in Q2 is normally distributed with mean 24,000 and standard deviation 8,000. At the start of Q2, Handi learns exact demand for Q2 and then exercises options at the fee of $3.50 per option, (for every exercised option, LCD delivers one display to Handi). Assume Handi starts Q2 with no display inventory and displays owned at the end of Q2 are worthless. Should Handi's demand in Q2 be larger than the number of options held, Handi purchases additional displays on the spot market for $9 per unit.

For example, suppose Handi purchases 30,000 options at the start of Q1, but at the start of Q2 Handi realizes that demand will be 35,000 units. Then Handi exercises all of its options and purchases 5,000 additional units on the spot market. If, on the other hand, Handi realizes demand is only 27,000 units, then Handi merely exercises 27,000 options.

a. Suppose Handi purchases 30,000 options. What is the expected number of options that Handi will exercise?

b. Suppose Handi purchases 30,000 options. What is the expected number of displays Handi will buy on the spot market?

c. Suppose Handi purchases 30,000 options. What is Handi's expected total procurement cost?

d. How many options should Handi purchase from LCD?

e. What is Handi's expected total procurement cost given the number of purchased options from part d?

Q17.4 **(Selling Grills)** Smith and Jackson Inc. (SJ) sells an outdoor grill to Cusano's Hardware Store. SJ's wholesale price for the grill is $185. (The wholesale price includes the cost of shipping the grill to Cusano). Cusano sells the grill for $250 and SJ's variable cost per grill is $100. Suppose Cusano's forecast for season sales can be described with a Poisson distribution with mean 8.75. Furthermore, Cusano plans to make only one grill buy for the season. Grills left over at the end of the season are sold at a 75 percent discount.

a. How many grills should Cusano order?

b. What is Cusano's expected profit given Cusano's order in part a?

c. What is SJ's expected profit given Cusano's order in part a?

d. To maximize the supply chain's total profit (SJ's profit plus Cusano's profit), how many grills should be shipped to Cusano's Hardware?

Suppose SJ were to accept unsold grills at the end of the season. Cusano would incur a $15 shipping cost per grill returned to SJ. Among the returned grills, 45 percent of them are damaged and SJ cannot resell them the following season, but the remaining 55 percent can be resold to some retailer for the full wholesale price of $185.

e. Given the possibility of returning grills to SJ, how many grills should be sent to Cusano's to maximize the supply chain's total profit?

Suppose SJ gives Cusano a 90 percent credit for each returned grill, that is, SJ pays Cusano $166.50 for each returned grill. Cusano still incurs a $15 cost to ship each grill back to SJ.

f. How many grills should Cusano order to maximize his profit?

g. What is Cusano's expected profit given Cusano's order in part f?

h. What is SJ's expected profit given Cusano's order in part f?

i. To maximize the supply chain's total profit, what should SJ's credit percentage be? (The current credit is 90 percent.)

Dave Luna, the director of marketing and sales at SJ, suggests yet another arrangement. He suggests that SJ offer an advanced purchase discount. His plan works as follows: there is a 10 percent discount on any grill purchased before the season starts (the prebook order), but then retailers are able to purchase additional grills as needed during the season at the regular wholesale price (at-once orders). With this plan, retailers are responsible for selling any excess grills at the end of the season, that is, SJ will not accept returns. Assume SJ makes enough grills to satisfy Cusano's demand during the season and any leftover grills can be sold the next season at full price.

j. Given this advanced purchase discount plan, how many grills should Cusano prebook to maximize his profit?

k. What is Cusano's expected profit given Cusano's prebook order quantity in part j?

l. What is SJ's expected profit from sales to Cusano this season given Cusano's prebook order quantity in part j?

m. As a thought experiment, which one of these contractual arrangements would you recommend to SJ?

You can view a video of how problems marked with a ** are solved by going on www.cachon-terwiesch.net and follow the links under 'Solved Practice Problems'

Chapter 18

Sustainable Operations

Seven billion. That is the estimate of the world's population in October 2011, projected to rise to 9 billion by 2050. It is a lot of people, and it is just one reason that sustainability has become an important topic of discussion. This chapter explores how operations influences sustainability and how sustainable thinking can influence operations management. We start with some background to motivate the topic of sustainability, and then we outline the business case for focusing attention on sustainability. Finally we conclude with a discussion of how the tools of good operations management can be applied to a sustainability initiative.

18.1 Sustainability: Background

Sustainable business practices are said to be those that sustain people and the planet. That is, by implementing these practices we will be as well off in the future as we are now. This is a broad definition, and so sustainable business practices can be divided into many domains. We highlight five of them:

- Energy
- Water
- Materials
- Agriculture, fishing, and forestry
- People

We primarily focus on energy, but the others are discussed briefly as well.

Energy

Sustainability is often associated with "global warming" or, the more preferred term, "climate change." The evidence for climate change comes from many sources. Figure 18.1 displays the steady increase in our atmospheric carbon dioxide. (This is also referred to as the Keeling curve, in recognition of the scientist Charles Keeling, who began collecting these data more than 50 years ago.) Interestingly, we can see the Earth "breathing" in these data—because there is more land in the Northern Hemisphere than the Southern Hemisphere, carbon dioxide levels drop as vegetation grows in the northern summer and then increases again as the vegetation dies off in the winter.

Carbon dioxide is not harmful to humans or plants, but it does cause the Earth to retain heat, as if adding a blanket to our atmosphere: We are already about 1°C warmer than we were at the start of the Industrial Revolution. This temperature change may not seem large, but it has already contributed to melting glaciers and rising sea levels. In fact, it has been estimated that if all of the ice sitting on Greenland were to melt, sea level would rise more

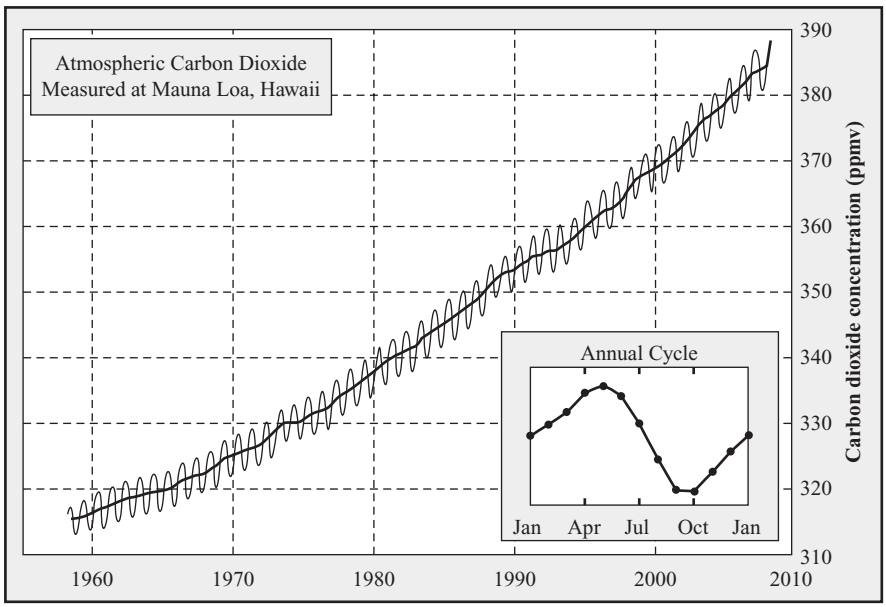


FIGURE 18.1
Atmospheric Carbon Dioxide Measured At Mauna Loa, Hawaii

than 20 feet. Higher average temperatures may also change the severity or frequency of adverse weather and influence rainfall patterns. Finally, a significant amount of the carbon dioxide we emit is absorbed by the oceans, which increases the acidity of the ocean, thus contributing to coral degradation, among other consequences.

Data from ice core samples indicates that our current levels of carbon dioxide are not unprecedented. However, the data also show that the Earth has experienced a rapid increase in carbon dioxide levels as over the last 100 years.

Carbon dioxide is not the only *greenhouse gas (GHG),* nor is it the most prevalent. The most common greenhouse gas is actually water vapor. But human activity is not directly contributing additional water vapor to the atmosphere. Two of the other major GHGs are methane and nitrous oxide. Methane comes from landfills (decomposition of organic matter), natural gas production, and digestion/manure from animals (such as cattle, milking cows, and pigs). Fertilizers are the main source of nitrous oxide. A set of chlorofluorocarbons provide the other greenhouse gases.

Given equal weights, greenhouse gases have different warming potentials. Just like it is useful to express the output of different countries in a common currency (say, the U.S. dollar), it is useful to express total emissions in terms of a common unit. That unit is called a *carbon dioxide equivalent (CO_2e).* The CO_2e of carbon dioxide is 1, whereas it is 21 for methane and 310 for nitrous oxide. That means that 1 kilogram of methane is equivalent (in warming potential) to 21 kilograms of carbon dioxide, and 1 kilogram of nitrous oxide is equivalent to 310 kilograms of carbon dioxide.

In 2009, U.S. emissions of carbon dioxide were 5.2 billion metric tons of CO_2e. Generation of electricity is the largest contributor to emissions (41 percent of the total) followed by transportation (33 percent). With electricity, coal is the major fuel responsible for emissions in the United States, followed by natural gas. For transportation, emissions primarily come from the combustion of gasoline, diesel, and jet fuel.

With transportation it is relatively straightforward to tally up total emissions—just count the amount of fuel burned. For example, the combustion of 1 gallon of gasoline emits 8.8 kilograms of carbon dioxide into the atmosphere.

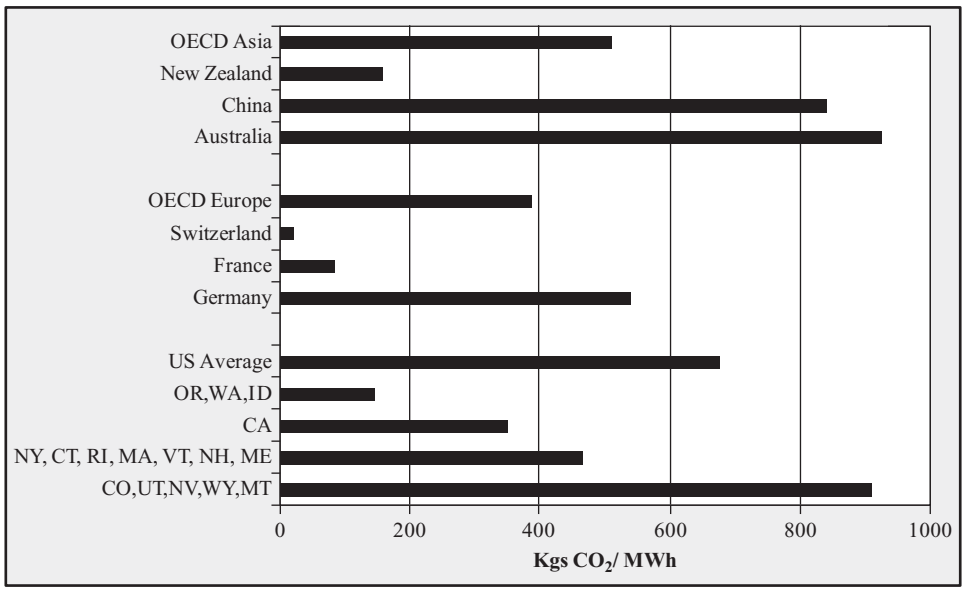


FIGURE 18.2
Emissions from Electricity from Different Countries and Regions

Counting emissions with electricity is more difficult. A company is likely to know how many kilowatt-hours they consumed, but what fuel was used to produce that power? The electricity on a grid comes from many different sources. Therefore, the emissions associated with electricity vary considerably across different countries and regions within countries, as indicated in Figure 18.2. For example, France relies heavily on nuclear power (relatively low CO_2 emissions), whereas Australia and China rely on coal. Within the United States, the Pacific Northwest produces much of its electricity with hydro (low emissions), whereas the mountain states again rely on coal.

Although the energy source is relevant, the efficiency of that energy also matters. For example, in transportation we are interested not only in total emissions but emissions per kilogram per kilometer of product transported. Figure 18.3 shows that there is considerable variation in this measure across different modes.

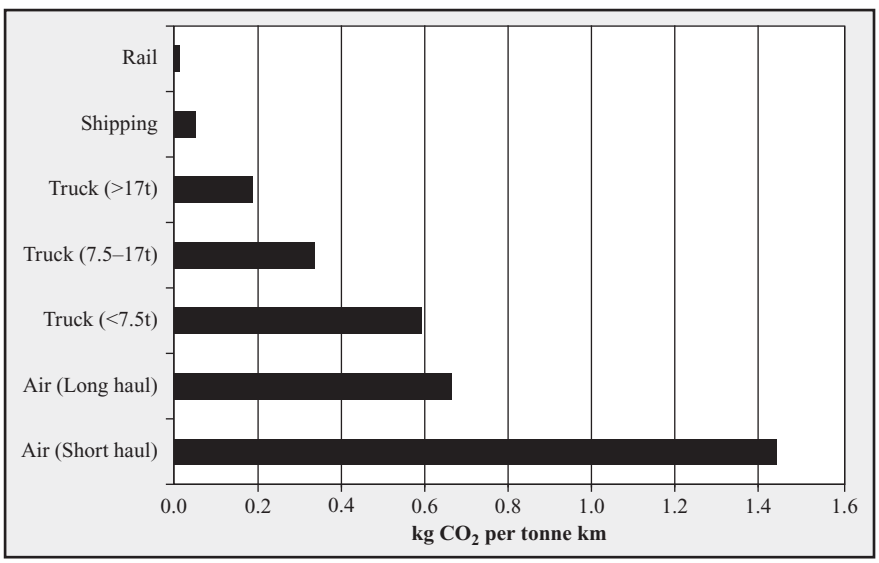


FIGURE 18.3
Emissions per Tonne Hauled per Kilometer Traveled for Various Modes of Transportation

Data like those displayed in these figures are used by companies to evaluate their *carbon footprint.* As this short discussion hopefully indicates, determining the carbon footprint of a product is not exactly easy to do. To help structure the task, emissions are often divided into three categories, or scopes: scopes 1, 2, and 3.

A firm's *scope 1* emissions are all direct GHG emissions, such as fuel burned in their own trucks or oil burned in their own boilers to heat their factory. A firm can usually obtain good data to get an accurate estimate of its scope 1 emissions.

Scope 2 emissions are from the consumption of purchased electricity, heat, or steam. This can be more complicated to assess because the firm needs to know the source of its electricity. Reasonable estimates can be obtained by measuring the electricity at each of the company's locations and then using published average emissions per kilowatt for those locations (like the data in Figure 18.3).

Scope 3 emissions are all emissions not accounted for in the other two scopes. In particular, scope 3 emissions are all indirect emissions associated with the firm (other than purchased electricity, heat, or steam, which is scope 2). Scope 3 emissions are by far the most challenging of the three categories to measure. For example, scope 3 includes the emissions associated with a company's employees driving to work. So are the emissions associated with how customers use a company's product. For example, when Philips sells a lightbulb to a customer, the emissions associated with the use of that lightbulb are part of Philips' scope 3 emissions. Scope 3 also includes all upstream emissions associated with a product. For example, if Apple assembles an iPod in China through a subcontractor, then that subcontractor's electricity usage is part of Apple's scope 3 emissions. So are the emissions of that contractor's suppliers. And so on.

As you can imagine, scope 3 emissions may be substantially larger than the other two scope emissions. Scope 3 is also probably the most difficult to assess. Estimates must be made regarding customer usage of products, employee travel, and supplier emissions. The data collection challenge is substantial. To assist firms with their carbon footprinting, two nonprofit organizations, the Carbon Disclosure Project and the World Resource Initiative, have put together extensive guidelines.

Water

There is plenty of water on the planet, but fresh water is only 2.5 percent of it, and more than two-thirds of that is locked up in glaciers and ice sheets. In addition, the fresh water we do have is not evenly distributed; more importantly, it is not always found in the same places we choose to live. Consequently, water conservation is a critical issue in much of the world.

Consider a pair of jeans. Water is needed to grow the cotton for the jeans (an astonishing 1,800 gallons per pair). Water is needed in the manufacturing process (to wash the fabric, create the stonewash look, and so on), and, of course, water is used by the customer to wash his or her jeans. As this example illustrates, the water usage associated with a product can be substantial and associated with many different phases of the product's life cycle (e.g. production, usage, and disposal).

Material

In addition to water, many companies focus on their materials usage, in both raw materials and packaging. Considerations include switching to more sustainable materials (more abundant or easily recycled) and lighter materials (to reduce the energy needed in transportation).

Agriculture, Fishing, and Forestry

Sustainable agriculture includes issues like soil management and crop selection (in addition to water usage). The objective of sustainable fishing practices is to ensure that fish stocks

remain robust and healthy. Forestry has similar concerns—can timber be harvested in a way that maintains the productivity of the land and the biodiversity of its environment?

People

Although sustainability is often associated with natural resources, many companies now include people in their sustainability objectives—the people involved in the delivery of a company's products and services should be treated with respect and given the opportunity to live a good life. For example, children should not be forced to work, but rather be given access to education and workers should be provided with a safe working environment.

18.2 Sustainability: The Business Case

While the adoption of sustainability goals and practices may be considered the "right thing to do," it is not immediately clear that it is the reason firms should adopt sustainability. Firms also have a responsibility to their investors, so management should have an interest in long-run value. There are at least three arguments for why sustainability can be compatible with maximizing profit:

- Build a brand
- Protect a brand
- Lower costs

Some customers value sustainability and are willing to pay a premium to know that they are purchasing a product or service that they view as good for the environment and good for the people involved in the production process. Hence, sustainability can be used to build a brand. Patagonia, the outdoor apparel company, has taken this approach. It provides high-quality clothing but also emphasizes the sustainability of its clothing. Patagonia customers may be willing to pay a premium for its products because they value Patagonia's commitment to sustainability.

Some customers are not willing to pay a premium, but at the same time they are not willing to participate in practices that they view as inappropriate, distasteful, or downright wrong. Take Nike. It has established a formidable brand based, in part, on well-placed associations with outstanding athletes. It has a strong interest to maintain that brand by ensuring that the company does not become associated with practices that could be viewed as inappropriate by its customers.

Finally, although there is no doubt that some sustainable practices increase costs, there are also many opportunities to reduce costs and simultaneously make your company's offerings more sustainable. As already discussed, carbon emissions are closely linked to energy usage. Reduce energy usage and you naturally become more sustainable. And because energy costs money, reducing energy usage also leads to cost savings. Similarly, if you reduce the amount of material used in your packaging, you need to purchase less of it and need to spend less to move it. Consistent with this view, Figure 18.4 displays a list of projects that McKinsey argues yield net savings from the perspective of sustainability.

To understand how to read Figure 18.4, consider the third project from the left, "Residential buildings—lighting." It has a negative "cost" of about \$90 per CO_2e, which means that while there are out-of-pocket costs associated with making lighting in residential buildings more sustainable (e.g., purchasing compact florescents or LEDs), the net savings (e.g., in terms of reduced electricity usage) is actually positive. In other words, a project like this is a "free lunch"—society saves money and reduces emissions at the same time.

FIGURE 18.4 **Costs and Benefits of Carbon-Reducing Projects**

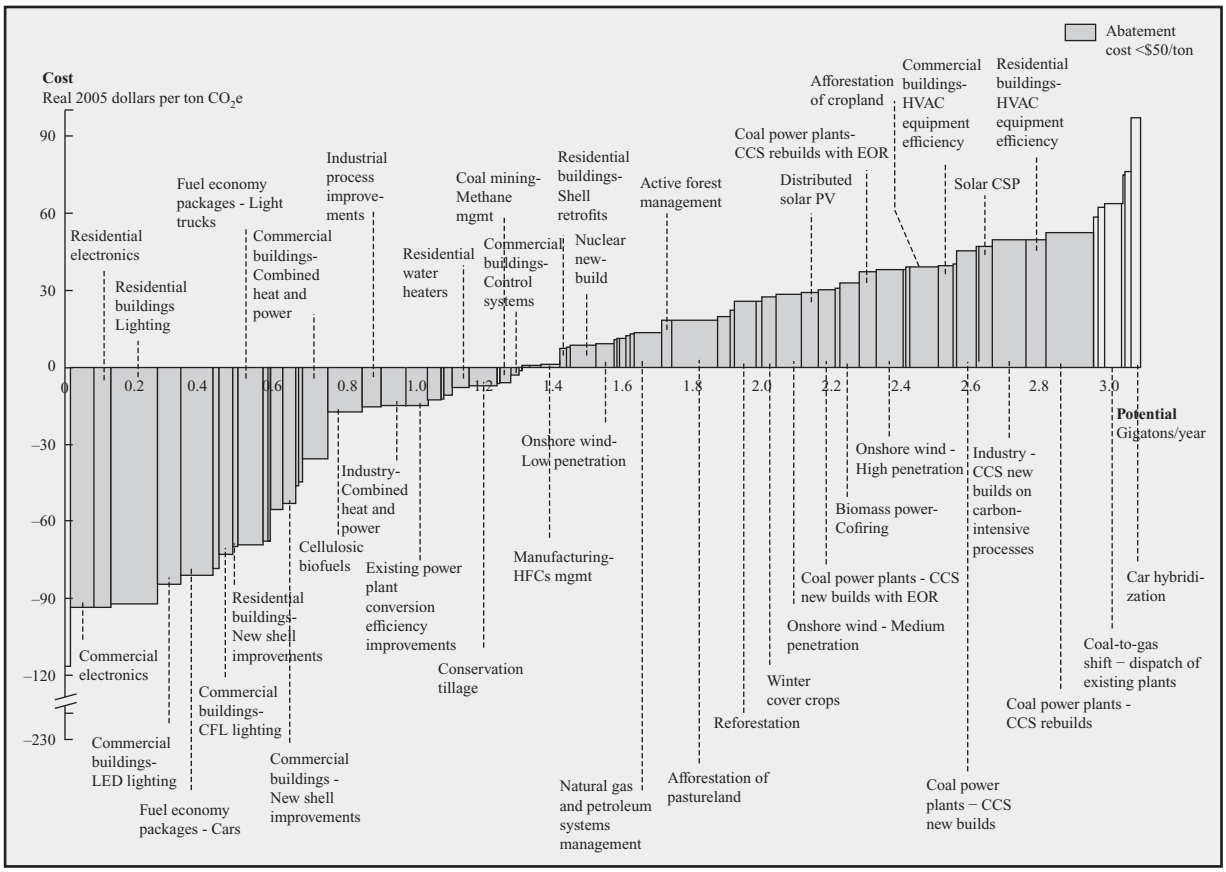


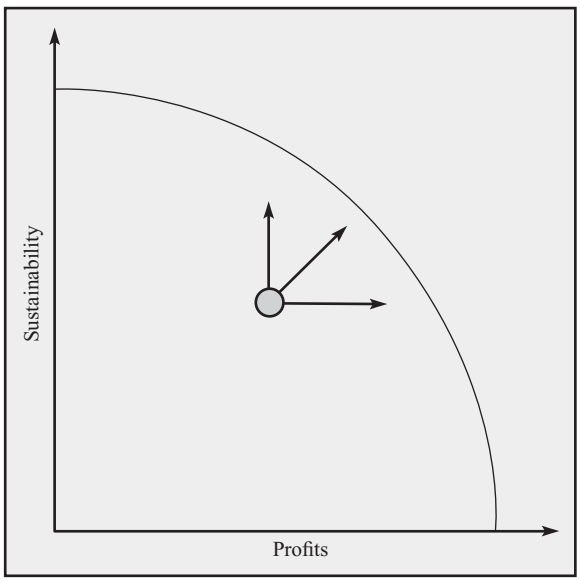
Figure 18.4 also includes projects that reduce emissions with a net cost. For example, "Coal power plants—CCS rebuild" costs about $50 per metric ton of CO_2e. ("CCS" stands for "carbon capture and sequestration," a process that captures the CO_2 from electricity production before it is emitted to the atmosphere).

Data like those in Figure 18.3 can be used to estimate the total cost of achieving CO_2 emission reduction targets; companies should prioritize their projects so that they start with the "free lunch" ones. They are the ones with the highest return on investment.

18.3 Sustainability and Operations Management

Throughout this text, we have articulated many principles to help managers improve their operations. Fortunately, those principles can be applied to sustainability initiatives as well. To begin, recall from Chapter 1 the notion of trade-offs and a production frontier. Figure 18.5 draws a simple frontier curve in which, at some point, it becomes necessary to sacrifice either sustainability or profits. However, as suggested earlier (see Figure 18.4), it is entirely possible, and probably likely, that a firm will not find itself on the frontier when it begins its sustainability initiative. Given that it has not focused on sustainability before, the firm may be able to identify projects in which it can improve sustainability and simultaneously increase profits; that is, it can move from the interior toward the frontier.

FIGURE 18.5
The Sustainability and Profit Frontier Curve



So how should a firm begin the process of identifying promising projects? Like all strategies to improve operations, begin with collecting data (Chapters 3 and 12). This is absolutely critical in the context of sustainability. You cannot improve something that you do not measure. But it is also a challenging task for several reasons. First, many companies will not have systems in place to easily gather data on electricity consumption, fuel purchases, and water usage, in part because these commodities may not be purchased centrally.

Next, even when you find your electricity usage, as already mentioned, it is not immediately obvious how to translate that number into a carbon emission—you need to know how that electricity was produced.

Third, it is not always easy to know how to allocate usage numbers to various product lines. In some sense, this is a classic accounting problem (How do you allocate fixed overhead to various items?), but the problem is no easier to solve in the context of sustainability. For example, suppose you want to determine the carbon footprint of a gallon of milk. Cows belch (among other types of "emissions"), giving off methane. It is possible to estimate the amount of methane emitted per cow, but how should that methane be allocated across the various products produced with that cow (e.g., milk and leather)?

Finally, once you have completed your task for scope 1 and 2 emissions, you need to look both up and down your supply chain, i.e., you need to consider scope 3 activities. If the data challenges are tough within the firm, they only get compounded when you must consider the boundaries outside of the firm. Despite these challenges, it is worthwhile to collect data even when the data are less than perfect.

With data in hand, one of the first tasks could be to use the Pareto diagram tool from quality management (Chapter 10): rank-order different sources of emissions, water usage, or chemical use to prioritize opportunities for improvement. In doing this prioritization, it is also important to collect data on traditional quality defects (like warranty claims). If a defective product is made in China and shipped to the United States, then not only are the materials in the product potentially wasted, the firm incurs wasted emission costs associated with transporting the item.

Just like good quality management is consistent with sustainability, the ideas of process and lean management are consistent with sustainability (Chapter 11). For example, a major goal

of process management is to increase utilization and capacity with the same set of resources (Chapters 4 and 6). Many of these resources have fixed emissions or water usage. For example, the cost of lighting a facility may act like a fixed cost; the building is lit during the eight hours of a shift whether the shift produces at 75 percent capacity or 90 percent capacity. Consequently, any reduction in wasted resources is likely to reduce the carbon or water footprint of an item because more units are produced with the same overhead (such as lighting). The same is true of lean management—by identifying root causes of waste, such as with a Ishikawa diagram (see Chapter 11), more output can be produced with no more or maybe even less energy and water.

It is also important to note that collecting sustainability data can change how a firm focuses on its process design. Consider Levi Strauss, a leading maker of denim jeans. Normally, Levi's might focus on reducing labor content or the amount of fabric for each pair of jeans. Those are surely valuable activities, but when you discover the amount of water used in the production process (as mentioned earlier), the focus may turn to developing new processes that do not require as much water.

Not only can sustainability influence a firm's thinking within its own processes, sustainability can strongly influence its supply chain strategy. Take the issue of location. In the environmental movement, location is often a lightning rod for activists. For example, "localvores" advocate eating food that is grown locally to avoid emissions associated with transportation. Another popular expression is the notion of "food miles"—the distance food has to travel to reach your plate. However, it is important to note that transportation is only one part of the total impact of a product, albeit an important part. The production process also matters, such as the amount of electricity (and the source of the electricity) along with other inputs (such as the amount of fertilizer used). For example, bauxite is mined in Australia, but it is smelted into aluminum in New Zealand. While moving the bauxite to New Zealand is costly (including the emissions aspect), the electricity on the south island of New Zealand is made with hydro power. Given that a significant amount of electricity is needed to make aluminum, doing it in New Zealand is better than doing it in Australia with its coal-powered plants (see Figure 18.2).

In addition to location, the mode of transportation is important for supply chain management. A key lesson over the past 20 to 30 years has been that many good things can come with speed: With faster lead times, you generally need less inventory to hit target service levels, or you can increase service levels with the same amount of inventory (Chapter 14). However, sustainability provides a new perspective on this approach. As we see in Figure 18.3, faster usually means dirtier, at least in terms of transportation emissions. However, if faster means less inventory and less inventory can lead to smaller buildings, then it is possible that total emissions could decrease—smaller buildings require less heating, cooling, and lighting.

While a firm should be careful about evaluating its transportation mode, it appears that delayed differentiation is likely to be a sustainability friendly strategy (Chapter 15). By adding components late in the supply chain, the firm's inventory investment can be reduced. What was not mentioned in Chapter 15 is that this also tends to lighten the product in the transportation stage. A lighter product requires less energy to move around. For example, instead of bottling wine in Argentina and sending the bottles to the United States, some wineries transport their wine from South America in large steel containers. The steel vessel weighs proportionally less than the glass bottles (note, the ratio of surface area to volume decreases as volume increases), so even if there are no risk pooling benefits from this strategy, there is an environmental benefit.

In general, packaging provides significant opportunities for sustainability improvements. Sticking with the spirits industry, French champagne manufacturers have recently focused on the size and shape of their bottles: Can they reduce the amount of glass in the bottle (and therefore its weight), while still maintaining the strength needed to store the champagne inside at high pressure? Like the glass in champagne, nearly all products require some form

of packaging. And unlike most products, which you hope will last for at least a couple of uses, if not several years' worth of use, packaging is almost always immediately discarded after its first "use." Hence, changing to more environmentally friendly materials, reducing the weight of the materials, and redesigning the packaging to allow for recycling for reuse are all worthwhile strategies.

18.4 Summary

Sustainability and the environment are important topics that have grabbed the attention of many CEOs. At its heart, sustainability is about the efficient use of resources, which is precisely the aim of operations management. Hence, the tools of operations management apply naturally to any sustainability initiative.

18.5 Further Reading

For an introduction to the science related to climate change, visit http://www.epa.gov/climatechange/. For more details on climate change, consider the latest report from the The Intergovernmental Panel on Climate Change (http://www.ipcc.ch/). For a discussion on sustainability and corporate strategy, see Porter and Kramer (2011). For guides on how to assess the carbon footprint of a product, visit the Carbon Disclosure project (https://www.cdproject.net) or the World Resource Initiative (www.wri.org).

18.6 Practice Problems

Q18.1* **(Bauxite to New Zealand)** Australian bauxite ore is shipped 3,000 kilometers to New Zealand in a bulk cargo ship. The ship carries 300,000 metric tonnes of ore and consumers 1,400,000 liters of fuel oil on the journey. Fuel oil emits 38.2 kgs CO_2 per liter. For bauxite ore shipped from Australia to New Zealand, what is the emission of CO_2 (in kgs) per tonne kilometer traveled?

Q18.2 A consumer who lives in New York switches from a 60 watt incandescent light bulb to an 8 watt LED. Assume usage remains the same, which is 4 hours per day on average. Electricity costs the consumer $0.12 per kWh. (A kWh is the amount of electricity need to produce 1000 watts of energy for 1 hour.) The incandescent light bulb costs $0.40. The LED costs $12.00. The LED lasts 27,000 hours whereas the incandescent light bulb lasts 1000 hours.

a. Including the cost of replacement bulbs and the cost of electricity, how long does it take for the LED to breakeven? (That is, after how much time will the consumer have spent as much with the LED as with the incandescent light bulb.)

b. The consumer's electricity emits 450 kgs CO_2/MWh. (1 MWh = 1000 kWh.) How many kgs of CO_2 would the consumer emit to operate the 60 watt light bulb for one year?

Chapter 19

Business Model Innovation

Netflix changed the video industry and drove Blockbuster into bankruptcy. Zipcar is emerging as a credible substitute for owning a vehicle. Both of these companies started by offering a service that differed substantially from what was the norm, not only in terms of what service customers were offered, but also in how each company delivered its service. Both are innovators and, in particular, both are examples of *business model innovation*—a term that has become a recent buzzword, used to explain the success of a number of rapidly growing businesses.

To be complete, one should acknowledge that such radical innovations are by no means a recent phenomenon. Dell revolutionized the computer industry over the course of the 1990s, a time period in which Southwest Airlines redefined air travel. One might even argue that Gottlieb Daimler and Henry Ford redefined transportation and forced many producers of horse carts out of business. Nevertheless, modern technology has surely enabled a steady stream of business model innovations in recent times.

The purpose of this chapter is to understand the forces behind such new business models. Instead of compiling a set of buzzwords and anecdotes, we want to present a solid framework that helps you understand and create new business models. Not surprisingly, given the title of our book, our framework is based on the idea that a firm can increase its profitability by identifying new and better ways in which it can match supply with demand. More specifically, in this chapter, we aim to explain:

- The economic forces behind the new business models of Netflix and Zipcar.
- The different ways in which a firm can innovate and which of these innovations classify as business model innovations.
- How a new business model can increase customer utility and often draw a new set of customers into the market.
- The ways in which a firm can leverage its operations to deliver on this utility while maximizing its profitability.

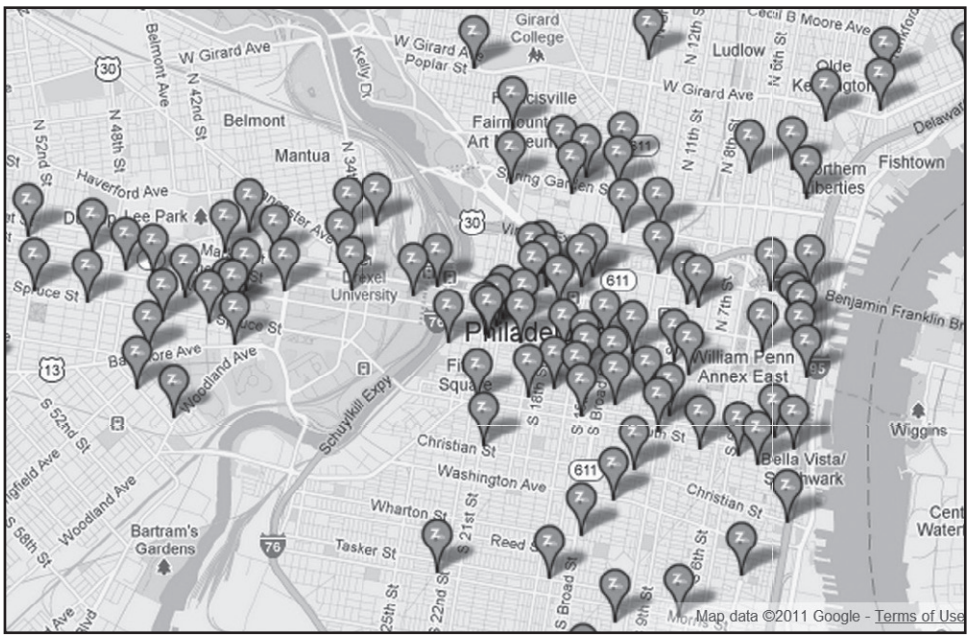
19.1 Zipcar and Netflix

In case you are not familiar with Zipcar and Netflix, this section describes what they offer and how they offer it.

Zipcar is a car-sharing company that was founded in 2000. Within 10 years, the company grew to an 8,000-vehicle fleet that serves a customer base of some 560,000 members. Members

FIGURE 19.1
Zipcar Locations in Philadelphia

Source: Google Maps.



can reserve vehicles online or by phone—they can do so minutes prior to their vehicle usage or several months in advance. There are generally many locations to choose from within a neighborhood (see Figure 19.1). At the time of use, a customer unlocks the vehicle using his or her access card (which is provided when signing up as a customer), or, more recently, an iPhone app. An annual membership costs about $60, and vehicles can be used for hourly rates that are as low as $7.50 (actual rates vary depending on make and model, as well as the time of the day and the day of the week). The hourly rate includes gas, maintenance, and insurance—members refuel the vehicle when needed and get reimbursed for the fuel expenses (many gas stations also accept the access card as a form of payment). After use, members return the vehicle by parking it in a designated space. Members are responsible for leaving the vehicle in a clean condition, ready to be used by the next member. All of this happens without the presence of a Zipcar employee.

Now let's switch to our second business model innovation, Netflix. Given the speed with which the video market has evolved from video rental companies, such as Blockbuster, to today's streaming video solutions of Netflix, Apple, or Amazon, it is easy to forget how Netflix started its success. The company began in 1997 in California. By 2010, it had a collection of more than 100,000 titles, which were available to its more than 10 million subscribers for a monthly flat fee. In 2011, there were 58 shipping locations in the United States, handling an estimated volume of about 2.5 million DVDs per day.

A member manages an ordered list (the Netflix queue) of movies she or he is interested in watching. Movies are sent to the member by U.S. Postal delivery in DVD format. They can be kept by the member as long as desired (no late fees), with the constraint that at any given time the member can only have three DVDs at home (that number varies depending on the subscription plan). To get a new DVD, the subscriber needs to return one of the DVDs through the U.S. Postal Service. Once a movie is returned, another movie from the customer's queue is mailed to the customer. Netflix tries to send the movie at the top of a customer's queue, but if it is not available, possibly because it is a newly released popular title, Netflix may instead send another movie from the customer's queue.

The fact that Netflix now is largely used as a video on-demand service provides an interesting case study on how quickly new business models come and go. With Apple and Amazon

streaming video in return for either rental fees or subscription plans, Netflix's amazing physical supply chain is transitioning to the virtual world. And, once again, a new business model has to be invented.

19.2 Innovation and Value Creation

The Netflix and Zipcar examples illustrate two ways in which firms innovated to supply solutions to the needs of their customers. We define an *innovation* as a novel match between a solution and a need so that value is created (see Terwiesch and Ulrich 2009). Our definition is best explained in a profit maximization paradigm.

Customers have a utility function and purchase a product or service if their utility of consumption exceeds the price. Mind you, a consumer's utility includes many components and certainly can include nonmonetary rewards, such as a preference for environmental conservation or the well-being of a group of workers. Independent of the particular components, consumers care about their net utility:

$$\text{Net utility} = \text{Utility} - \text{Price}$$

where Price is meant to include the total cost of owning the product or receiving the service. Firms, on the other side, have a profit goal. They obtain profits that can be summarized in a simple equation:

$$\text{Profits} = \text{Flow rate} \times (\text{Price} - \text{Average cost})$$

Because price reduces the net utility of the (potential) customer and increases profits, there exists an inherent tension between the interests of the customer and that of the firm. Some may argue that it is possible to produce a substantial innovation in terms of price (e.g., charging a subscription fee for music or bundling the cost of a cell phone into monthly service fees). However, we focus on two other means to generate an innovation:

- Change the way a product or service meets customer needs, thereby generating more utility. For example, we could change the performance of our service along one or more attributes, or create new attributes. It is even possible that an attribute is eliminated all together. In the end, if we create more utility for customers, we can command higher prices and draw more customers to our offerings.
- Change the way we supply the product or service. In other words, deliver the same level of customer utility, but develop a more efficient solution for doing so, thereby lowering our average cost.

As an illustration how these forces play out, consider the airline industry and the data that we discussed in Chapter 6. What matters to airlines is how much they can charge per revenue passenger mile (the yield) and how much it costs them to supply that mile. Because labor is the biggest cost driver in the industry, labor cost relative to revenue passenger miles provides a useful measure of efficiency. Over the course of the 1990s and early 2000s, Southwest was able to grow quickly and gain a significant market share. Relative to legacy carriers, customers paid only 80 percent of what the other carriers were charging, but Southwest was profitable because it produced the same service with double the efficiency.

How was that possible? Some of this is achieved through more efficient operations (see Chapter 6). However, Southwest also offered a different service. Many customers flying Southwest would not previously have been in the market for air travel at all. They might have taken a Greyhound bus or simply stayed at home. Southwest identified that unmet need: no frills air travel for an aggressive price. And, over that time period, Southwest was by far the most profitable—despite being the low-end player in the market.

Interestingly, just a couple of years later, history repeated itself. By 2005, Southwest's labor costs increased substantially, similar to the level of the legacy carriers. This time, JetBlue took the position of the low-cost airline, obtaining a labor productivity that was almost double what Southwest was able to provide (and thus, almost four times of what the legacy carriers offered). This allowed JetBlue to even further expand the market for air travel.

The success of JetBlue is visible in Figure 19.2. On the vertical dimension, the graph shows the amount of money the average passenger was paying for one mile of air travel on the various carriers. This amount is expressed relative to the industry average. On the horizontal dimension, we show how many passenger miles an airline can generate with $1.00 of labor cost, again, relative to the industry average. We observe that JetBlue was able to provide a service that was 60 percent more efficient in labor usage relative to the industry average. That allowed them to charge prices that were 40 percent lower compared to their competitors. Southwest in this time period had fallen behind in labor productivity. While each employee, on average, served more passengers compared to other airlines, Southwest employees were paid substantially above industry average. However, because of lower fuel costs/higher fuel efficiency as well as lower other expenses (such as landing fees, commissions, sales and marketing expenses), Southwest still turned substantial profits.

Note that a low-cost, efficiency-driven strategy might not be the only way to succeed. Imagine a hypothetical airline Golden Air, which caters to the very high end of the market: Limo service, business lounges, and new planes. The decisive question is this: "Are customers willing to pay premium prices for the premium services Golden Air could offer?" For the major carriers, the answer we get out of Figure 19.2 is clear: No airline is able to obtain prices that are substantially above industry average. Some companies, like NetJet, are active in this space, but for the most part, air travel seems to be a commoditized market.

So a firm can create a business model innovation either by improving customer utility or by improving operating efficiency. But firms are constantly coming up with new ways to meet customer needs and new business processes. Should we label all of these innovations as business model innovations? Clearly not. For example, it may be valuable for an airline to develop a baggage handling system that allows its baggage handlers to increase their output by 5 percent per shift, but this is not an innovation that customers would notice. Similarly, a pharmaceutical company may develop a new and useful compound, but this is what pharmaceutical companies do.

We suggest that a business model innovation is something that has the potential to fundamentally shift an industry. Usually, a business model innovation involves a simultaneous

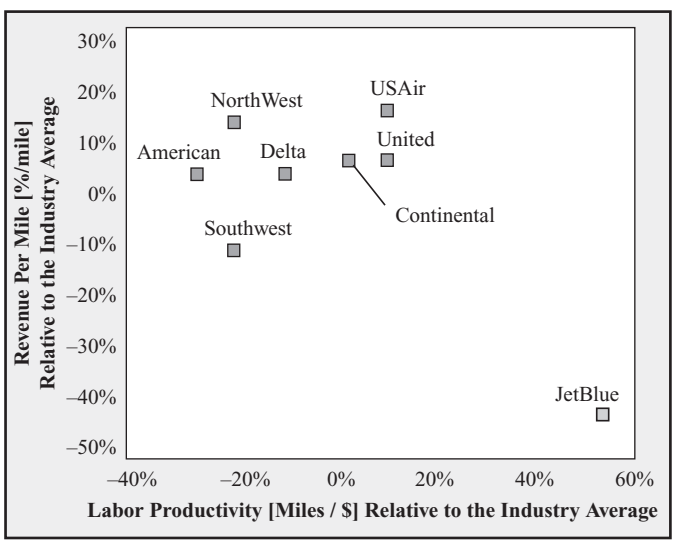
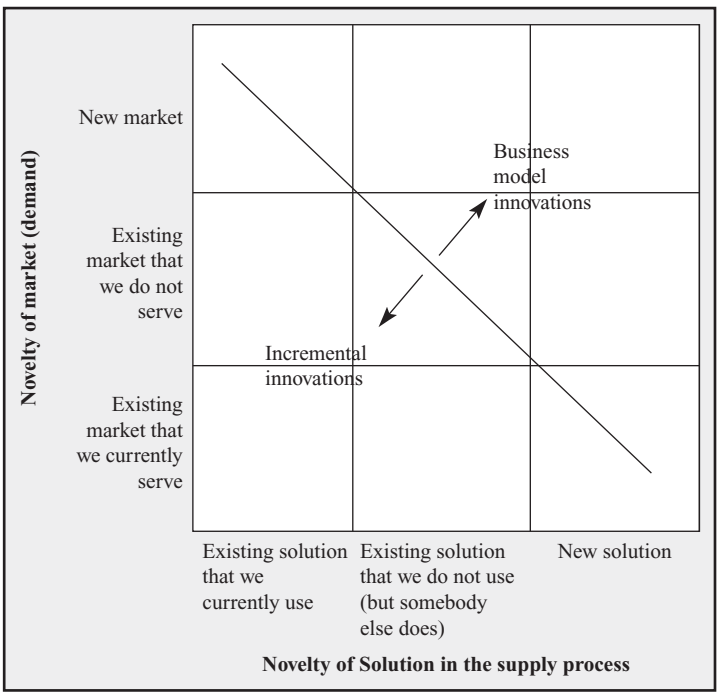


FIGURE 19.2
Benchmarking of the Major U.S. Airlines

Source: Data based on 2005 reports.

FIGURE 19.3
Different Levels of Innovation

Source: Adapted from Terwiesch and Ulrich 2009.



and significant shift in what customer needs are fulfilled and how the firm goes about filling them. Both Netflix and Zipcar meet these criteria—at the risk of not being particularly precise, both Netflix and Zipcar feel like very different ways of doing things. Netflix provides movie watching, just like Blockbuster, but doing it via a queue maintained on the Internet, and shipping DVDs by mail is not even close to how Blockbuster offered its service. At a high level, Zipcar is another car rental service, but its dense network of locations and extensive customer involvement makes it substantially different than the traditional car rental companies like Hertz, Avis, or National.

A useful way to measure the degree to which an innovation is substantial and deserves being labeled a new business model is explained in Figure 19.3. The lower left in the figure corresponds to refinements of existing technologies and solutions, but the company continues to serve its existing customers. The upper left represents an attempt to use existing solutions to reach new customers by entering new markets, either in the form of new geographic markets or in the form of new market segments. Either way, these innovations are rather incremental in nature and do not deserve to be labeled a new business model.

We want to reserve the term *business model innovation* for innovations that are characterized by substantial novelty in needs (and thus new customers) as well as solutions (and thus new operations). Business model innovations are thus in the upper right of Figure 19.3.

19.3 The Customer Value Curve: The Demand Side of Business Model Innovation

Let's start to explore how a firm generates more customer value (or net *utility,* if you prefer). Marketers typically think of products and services in terms of their attributes—cars, for example, possess attributes of fuel economy, style, acceleration, and ride quality. To keep our focus on business model innovation (as opposed to other types of innovation such

as new pricing schemes or new product designs), we look at the following four categories of customer attributes:

- Price
- Preference fit
- Transactional efficiency
- Quality

Price includes not only the total payment to the firm (and possibly other entities, such as sales taxes), but potentially other price-related attributes such as the timing of the payments. For example, Zipcar charges an annual fee and a per-usage fee, as we discussed above. Together, these payments create a total cost for the service of transportation.

Preference fit refers to the firm's ability to provide the consumer with the product or service they want or need. In other words, how well does the firm satisfy the customer's need. Customers are often very heterogeneous in their preferences. They wear different size jeans, like different songs or movies, and enjoy eating different types of food. Often, this category of attributes is labeled "product variety." We prefer the term "preference fit" because customers do not care about "variety" per se. For example, as a customer of a video rental service, the number of titles the service makes available is, by itself, not the attribute you care about. Instead, you want to find a movie that you want to watch. Thus, variety is simply a means to an end. Preference fit is clearly a key strength of the Netflix business model—it offers a tremendous variety of movies.

Transactional efficiency has two major components that influence how easy it is to do business with a firm:

- How much effort does the customer need to exert in the process of communicating and fulfilling her or his needs. For example, one important strength of Netflix is that customers can browse through a huge video selection online, from the comfort of their home. Similarly, the advantage of Zipcar over other car rental services is that (urban) customers only have to walk a couple of blocks before getting to a vehicle.
- How much time elapses between when the customer identifies the need and when the need is fulfilled. This subdimension of transactional efficiency was initially seen as the Achilles heel in the Netflix model. After all, it can take two or three days between making a change in the Netflix queue (and returning a DVD) to receive the next DVD. Interestingly, the assumption that there should be only a few customers who would want to wait that long was proven wrong. Given the other subdimension of transactional efficiency and the strong preference fit, customers apparently are willing to ignore (or at least overlook) this attribute.

One may argue that transactional efficiency is really a part of preference fit. But transactional efficiency is rarely the need that a customer has. For example, with Zipcar, the need is transportation. Customer may like that they have nearby access to a Zipcar, but that just means that they recognize a low transactional cost of satisfying their true need (to be able to use a car). Thus, while transactional efficiency is surely important to customers, it deserves to be considered separately from preference fit.

Quality includes the subdimensions of conformance quality and performance quality. Conformance quality measures consistency and thus captures how closely the firm's offering matches what it claims it offers. This dimension is closely related to our discussion of six sigma in Chapter 6. Performance quality captures the utility that a typical customer derives from the product or service. In the case of Zipcar, conformance quality relates to the cleanliness and functioning of the vehicle. Performance quality relates to the vehicle types that are available.

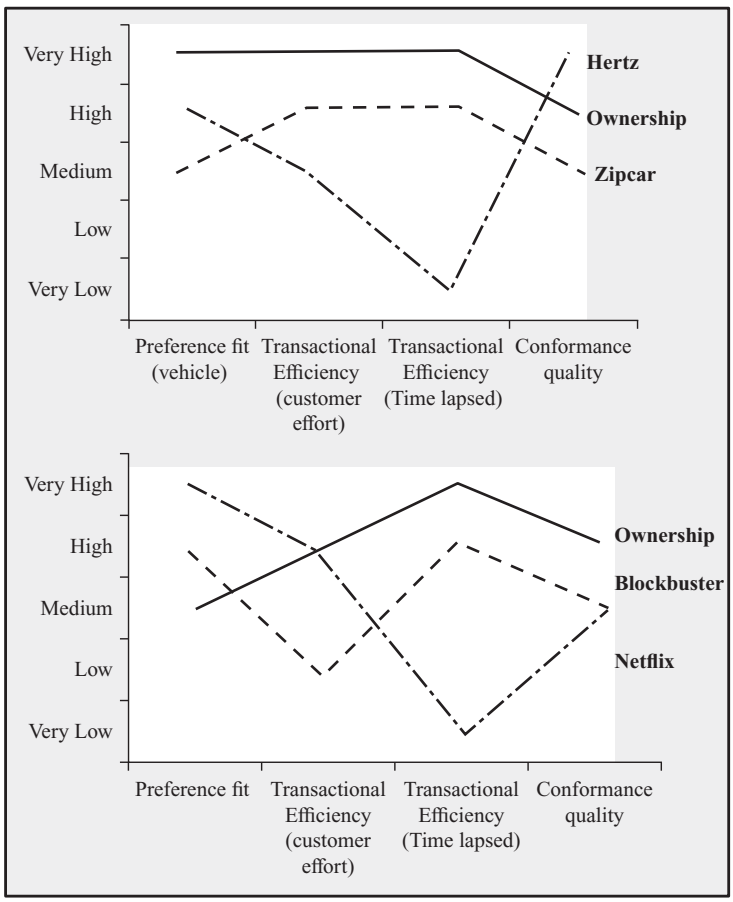
TABLE 19.1 The Four Categories of Attributes Mapped out for Zipcar and Netflix

	Zipcar	Netflix
Reference services	Traditional car rental (Hertz) and car ownership.	Traditional movie rental service (Blockbuster) and DVD ownership.
Price	Cheaper than Hertz, especially when rented by the hour. Cheaper than owning for occasional drivers.	Cheaper than Blockbuster for frequent viewers. Cheaper than buying the DVD (unless a movie is watched many times).
Preference fit	Some selection of vehicles but not as wide a variety of vehicles as Hertz. Relative to owning a vehicle, it is possible to have access to multiple vehicles.	More variety to choose from than Blockbuster or DVD ownership.
Transactional efficiency: effort by the customer	Short walk to the car makes Zipcar easier on the customer compared to Hertz. However, relative to parking a vehicle in your driveway, the effort is greater.	As easy as purchasing a DVD on Amazon; less effort compared to going to a rental store.
Transactional efficiency: time elapsed between demand and fulfillment	Short relative to Hertz (because of the proximity to the vehicles). Long relative to owning a car.	Much longer time to fulfillment relative to rental outlet.
Quality: conformance	Potential concern about cleanliness of a vehicle, fuel level, and availability (does the previous customer return the car on time?).	Potential loss of DVD in the mail.
Quality: performance	Acceptable (unless you are used to a Porsche).	Not relevant.

A firm's offering can be mapped onto these four categories of attributes (See Table 19.1.) The customer *value curve* is a graphic depiction of a company's relative performance across these attributes. Figure 19.4 (upper part) shows the value curves for Zipcar relative to owning a vehicle as well as relative to renting a vehicle. There are a limited number of vehicles at each location, so preference fit scores low with respect to vehicle type. Conformance quality is not as good as the traditional rental car service because there is no Zipcar employee available to clean cars as they arrive or to ensure that the gas tank is full and the previous customer may not return the vehicle on time. However, relative to a traditional rental car company, Zipcar scores high in terms of transactional efficiency—after a few clicks and a short walk, you can be driving a Zipcar where you need to go. The lower part of Figure 19.4 shows the value curves for Netflix relative to going to a Blockbuster store as well as relative to purchasing a DVD. Netflix's approach was different than the one chosen by Zipcar. Netflix decided to sacrifice on the dimension of transactional efficiency (time elapsed). In return, it was able to offer an amazing number of movie titles, moving the industry to a new level of preference fit.

A key observation is that business model innovation often involves a smart sacrifice—dramatically improve one attribute of the value curve at the expense of another, possibly even an attribute that is viewed as a "sacred cow" in the industry. This suggests a strategy for developing a new business model innovation: (1) map out existing attributes, and then (2) consider which ones can be dramatically improved and which ones can be sacrificed. In thinking through how to shift the value curve, it is essential to not be biased by what currently

FIGURE 19.4
Value Curves for Zipcar and Netflix



exists. It is also useful to ignore how those attributes could be delivered (the solution). Once a set of attributes is developed, the next section provides possible solutions for delivering them profitably.

19.4 Solutions: The Supply Side of Business Model Innovation

It is important to know that business model innovation often dramatically shifts the customer value curve. But we also need a solution that will do this profitably. That is the topic of this section.

Among the business model innovations we have observed, we have noticed that they generally involve changes to one or more of the following components of the firm's operations:

- Process timing
- Process location
- Process standardization

In all cases, shifts in these three dimensions can lead to substantial reductions in the cost to deliver the service and/or substantial changes in the customer value curve. They do this basically through one of three mechanisms. For one, they allow a firm to take advantage of specialized high-volume assets (e.g., large, automated sorting equipment). Alternatively, they allow the firm to use the same assets as competitors (e.g., the same type of employees,

the same equipment, the same size buildings, and so on) but with a higher utilization, thereby, in effect, reducing the cost of those assets. Finally, they allow a firm to purchase less expensive assets (e.g., cheaper labor, inexpensive tooling, small buildings, and so on) while achieving the same level and quality of output.

Process Timing

At some moment in time, a customer realizes that she has a need for a product or service. For example, she might decide at 7 p.m. on a Friday night that she wants to watch a movie at home. What are her options? With Blockbuster, she would drive to a local store, choose a DVD, and return home. She might start the movie by 8 p.m. With Netflix, she would have to choose a movie, place it on her queue, and wait for the movie to arrive, hopefully within a couple of days. With Blockbuster, the fulfillment of the need can occur within hours of identifying the need, whereas with Netflix the time to fulfillment is better measured in days. Blockbuster's portion of the process (purchasing the DVD, placing it on a store shelf) takes place before the customer's need occurs, whereas with Netflix, a substantial portion of the process (packing the DVD and mailing it to the customer) occurs only after the customer's need is identified. In short, relative to Blockbuster, Netflix substantially changed the timing of its process.

There are several advantages to changing a process's timing. To start, delaying the process until after a customer reveals her need allows the firm to make better supply choices, which can lead to higher utilization of critical assets. To illustrate this point, consider the personal computer industry over the 20-year period from 1985 to 2005. With the traditional model a supplier, say, Hewlett-Packard (HP), assembles a personal computer (PC), ships the PC to a distributor or retailer, and then it is sold to a customer, about two months *after* it was assembled. That is, most of the process occurred before the customer's need was identified. Dell changed the process timing by starting the assembly of the personal computer only after the customer's need was revealed. Consequently, components are not assembled into a finished product until after the company knows that there is demand for the finished product. In contrast, with the traditional model, HP must guess at how many of each type of PC will be wanted in two months. Of course, Dell has to guess at how many components it needs, but this is a much simpler task than guessing the number of each PC type.

If delaying process timing is so wonderful, why don't all firms do it? Well, there are some disadvantages to the delay as well. For one, if you delay work until after a customer announces his or her desire, then it will generally take longer to fulfill that desire. Like with Netflix, Dell customers cannot just drive home with their new toy. More subtlety, while delaying a process can help manage one asset, it can make it more difficult to manage a different asset. In Dell's case, beyond components, another crucial asset is assembly labor. Dell will naturally have variations in the total number of PCs demanded each day. If it hires enough assembly capacity to cover every peak day, on most days that labor will be partially idle. If it hires only enough labor to cover average demand, then its backlog of PCs could grow substantially. In effect, for Dell, the assembly process acts like a queuing process. And as we saw in Chapter 8, as the utilization in a queuing process approaches 100 percent, waiting times tend to get very long. So Dell faces a trade-off. Hire many workers and have low labor utilization but small delays between customer orders and shipping, or hire only a little bit more than what is needed to cover average demand and have high labor utilization and long delays between customer orders and shipping. HP, on the other hand, does not face nearly the same challenge. Because there is a two-month period between assembly and customer demand, HP can operate with very high labor utilization—some PCs are made three months before they sell, others one month before they sell, but either way they are made well in advance of sales. What Dell demonstrated is that, when component prices were dropping, it was more important to manage the component asset than the assembly labor asset.

While Dell changed the timing of its processes to make better supply choices, process timing can also be used to lower the direct cost of a process. IKEA, the Swedish furniture company, provides an example of this approach. IKEA's innovation is to sell (reasonably) stylish and functional furniture that needs to be assembled by the customer—the company sacrifices performance quality (there is no comparison between IKEA's furniture and Ethan Allen) and transactional efficiency (few people actually enjoy the time needed to assemble furniture), but it significantly improves on the price dimension of the value curve. It did this by changing process timing. With traditional furniture, final assembly occurs before the customer announces his intention to purchase, whereas with IKEA final assembly occurs after the purchase. Consequently, final assembly is done by the consumer. This allows IKEA to reduce transportation costs (shipping knocked down furniture is less expensive than fully assembled furniture) and labor costs (employees are paid explicitly, whereas customers provide only implicit labor).

Returning to the Netflix example, one may ask how process timing works to Netflix's advantage. It is not that Netflix manufactures DVDs after customers place them in their viewing queue or that it allows them to use cheaper materials. Instead, delaying the fulfillment of the service enables Netflix to change another dimension of their process, their process location, as we discuss next.

Process Location

Changing where a process takes place can lead to a significant business model innovation. For example, Blockbuster, at its peak, operated more than 5,000 stores in the United States, each probably drawing customers from a limited geographic area that did not extend much more than 5 to 10 miles beyond each store. In contrast, Netflix operates about 60 fulfillment centers, two orders of magnitude fewer than Blockbuster. Clearly, via the U.S. mail, these centers could serve customers from much further away than a Blockbuster store.

In general, if you operate with fewer locations, then each location potentially has access to a greater pool of demand, but, at the same time, this expands the distance between customers and products. Demand aggregation works to the favor of the company, whereas moving away from customers works against the firm.

Consider the benefits of *demand aggregation,* which are analogous to the benefits of economies of scale. Scale economies arise for many reasons, but the most relevant ones for our context are:

- Trade-off between fixed and variable cost: If you engage in an activity many, many times, you might consider automating the activity or otherwise investing in some resource that can complete the activity at very low variable cost. However, if you only operate the activity occasionally, such an investment would not pay off. For example, an expensive, high-performance lawn mower that reduces the time to cut the lawn by 20 percent is likely to be a worthy investment for a professional gardener, yet most homeowners would hesitate to make this investment and rather spend a little more time for each cut.
- Learning: The more often you perform an activity, the better you will get at it, and the more effort you are likely to spend at analyzing and improving the activity. When employees more quickly learn how to do their job, they become productive more quickly, effectively lowering the firm's labor costs.
- Opportunities for dedicated resources: A Swiss Army knife does many tasks, but none as well as could be done by a dedicated tool. Similarly, while a person can do many tasks, he or she will naturally be better at some tasks than others. Consequently, with high demand it becomes feasible to utilize assets that are particularly good at narrow tasks (i.e., faster or cheaper at those tasks). For example, doctors in a small emergency department have

to be prepared for all types of cases, making their work highly unpredictable and full of variability. In large emergency departments, in contrast, the volume of patients is sufficiently large to put the patients on different tracks. In so-called fast tracks, nurse practitioners take care of runny noses, trauma experts deal with trauma cases, and emergency physicians deal with other medical conditions. In this way, each track experiences less variability in the tasks it faces, thereby improving the efficiency of their work.

- Statistical economies of scale: As demand gets aggregated, it is generally observed that it also becomes less variable (in the sense of its coefficient of variation). Lower variability means that for any given level of service, the firm can utilize its assets more efficiently—the asset (e.g., inventory, people or equipment) spends less time waiting for customers, and there are fewer cases in which customers wait around for the asset to become available. This is essentially the idea of *demand pooling.* See Chapter 15.

Netflix took advantage of economies of scale in several ways. Its fulfillment centers implemented specialized sorting equipment to ensure a fast and efficient turnaround of DVDs from one customer to the next (high fixed cost, low variable cost equipment). But Netflix really exploited statistical economies of scale to dramatically increase the number of movie titles in its selection. For example, a Blockbuster store may offer 5,000 movie titles at any one time. If it were to offer more obscure titles, most of those titles would sit on the shelf for a long time before a customer request comes along, all the while incurring capital costs for the DVD and space costs for the shelf it sits on. It should be clear that Blockbuster may not be able to make money stocking obscure titles. Netflix, on the other hand, can carry those titles because each of its fulfillment centers serves much greater demand. Consequently, even a couple of copies may turn over fast enough to justify buying the DVD and the space it occupies (which would be cheaper on a per-square-foot basis that prime store-front real estate that Blockbuster would use). So, by operating with far fewer locations, Netflix is able to dramatically expand the variety it offers customers—and make a profit doing so.

Of course, there are two downsides to Netflix's model. First, customers must wait to receive their selection, lowering transactional efficiency. Second, Netflix has to explicitly pay the U.S. Postal Service to deliver the product. As always, the business model only works if the extra utility customers obtain makes them pay prices that are high enough to cover the firm's costs and to create a profit.

Zipcar also takes advantage of a process location change, but in a different direction than Netflix—instead of moving away from the customer, Zipcar moves closer to customers. Demand variability increases as we reduce the amount of demand aggregation; hence, variety will have to be compromised. Each location will not have the same selection of vehicles as a Hertz operation located at an airport. But, here, customers are willing to give up variety for the convenience of a car that is potentially within walking distance from their home. By moving closer to customers relative to Hertz, we expect that the utilization of Zipcar's cars is lower than Hertz's utilization. While this may increase Zipcar's cost, it is important to keep in mind that Zipcar also provides improved transactional efficiency—the convenience of a nearby car—which generates additional customer value.

Although Hertz provides one reference point for Zipcar, the other reference point is car ownership. And because Zipcar's cars are further from customers than their own vehicle, one would expect that Zipcar's utilization is higher than individual ownership. Consider the following, back-of-the-envelope calculations. Net of its subscription fee, Zipcar obtained annual revenues of about $200 million from its 8,000 vehicles (see Zipcar annual report in 2010). This translates to a $200 million/8,000 vehicles = $25,000 per year per vehicle. If we assume an average hourly rental fee of some $10 per hour, we see that the average Zipcar vehicle is likely to be rented out for 2,500 hours per year (more than 6 hours per day). Given

that there are 365 days/year × 24 hours/day = 8,760 hours per year, we obtain a vehicle utilization of 2,500 hours used/8,760 hours available = 28.53 percent. Most consumers, especially those who either own multiple vehicles or use their vehicle only lightly, have a vehicle utilization that is substantially below this (if you use your vehicle 2 hours per day, your utilization is 2 hours/24 hours = 8.33 percent). Thus, by aggregating the demand across multiple consumers, Zipcar enables a threefold increase in asset utilization. This is a source of value.

While one might assume there is a single sweet spot in the process location spectrum, it seems that there can be multiple approaches that work. For example, take Redbox, which operates vending machines that act as a DVD rental store and are found in convenient locations such as supermarkets. In contrast to Netflix, Redbox moved closer to customers than even a Blockbuster store. As expected, variety is sacrificed—each Redbox can only stock a hundred or so titles. But there is a gain in convenience to the customers and cost—a Redbox is much cheaper than a Blockbuster store (less square footage, no employees needed on a constant basis). Both Netflix and Redbox work. The fact that each occupies a position on either end of Blockbuster (one with more variety, further away from customers, and the other with less variety, closer to customers) has certainly contributed to Blockbuster's struggles.

Of course, moving a process away from customers can also allow the firm to move the process to a location with cheaper labor or land or equipment. Outsourcing and offshoring are two strategies closely linked to this idea. For example, Nike was one of the first companies in the athletic shoe industry to move its production from the United States to Asia. Now this is viewed as the "traditional business model," but at the time it was indeed a significant departure from standard practice. It worked—in large part due to process standardization, as we discuss next.

Process Standardization

Higher education is a relatively unstandardized process. Two professors teaching the same topic, even if they plan to give the same exam, generally do not teach in exactly the same way. Even the same professor is unlikely to deliver the exact same lecture twice. Contrast this with how a McDonald's hamburger is made—although by the laws of statistics, no two hamburgers are exactly identical (see Chapter 10), nor is the process of making them, that process is surely more standardized relative to higher education.

A standardized process is one that has been defined so that it can be easily repeated; consequently, its output is relatively consistent. For example, before McDonald's, hamburgers were served at diners that made their hamburgers their own way. Each employee probably used a different amount of meat to construct the patty, and there was no standard process for cooking them (e.g., flip the burger once on the grill or several times). The owner of the diner probably gave the cook no more instruction than "cook hamburgers when customers order them." In contrast, McDonald's created uniform hamburger patties (size and shape), cooked them in a consistent manner, and even applied the toppings in a particular way. McDonald's standardized the process of making hamburgers.

There are some significant implications of process standardization, some of which can lead to business model innovations or be key enablers of business model innovation. For one, standardizing a process generally means that less skill is needed to complete the process, which means that less expensive labor or capital can be used in the process. Returning to our restaurant example, the cook in a diner probably demands hire wages than the employees in a McDonald's because that person is responsible for making more decisions that influence the quality of the output. In a more standardized process, employees do not need to make as many decisions, and they command lower wages.

For a more modern example, consider the role of process standardization in the case of Zipcar's business model. Access to vehicles is standardized by eliminating the traditional (nonstandardized) keys. Instead, a universal access card lets customers use any Zipcar vehicle—there is no need to exchange physical keys that are specific to individual vehicles. Consequently, there is no need to have a physical person present at each location to maintain proper control of these keys.

Process standardization is also a key enabler of the contract manufacturing industry discussed in Chapter 15. A contract manufacturer can use its manufacturing facilities to build products for multiple clients precisely because the process of making these components has been standardized (e.g., stuffing electronic circuit boards with integrated circuits). As hinted earlier, Nike was able to successfully send production of its shoes overseas because it was able to standardize the process of describing how to build the shoe and the actual manufacturing process. As a result, two different factories could make the same shoe, and they could be indistinguishable to a customer.

Of course, there are negative implications to process standardization. The most common one is a loss of variety (a potential reduction in preference fit). A customer does not tell McDonald's how they want their hamburger cooked. Nor does the selection of hamburgers change from month to month. In contrast, the menu at Le Bec Fin (a high-end restaurant in Philadelphia serving French cuisine) rarely stays constant from month to month; the chefs at Le Bec Fin are highly paid, are well trained, and certainly insist on changing the menu (so that their customers are unlikely to see the same menu on subsequent visits, each of which costs $150 or more per person). But as with all business model innovation, process standardization may lead to a smart sacrifice. For example, is it possible to standardize higher education so that you can deliver a valuable product to customers at much lower cost? This is an open question.

19.5 Unsuccessful Business Model Innovation

As with most innovation, business model innovation is not always successful. It is possible that a firm tries to exploit a change in process location but ends up with a product that just does not deliver enough incremental value to be profitable. In fact, there are surely more unsuccessful business model innovations than successful ones—we just do not hear about the unsuccessful ones as often.

Webvan provides a nice example of a reasonably well-known and surely unsuccessful business model innovation. Webvan tried to be an Internet grocer: Customers would order their groceries on the Web and then Webvan would deliver them to their homes.

Webvan provides a clear example of a process location change. With a traditional grocer, customers drive to a store to select among items that the company has stocked there. Webvan eliminated the store, following a demand aggregation strategy. Customers no longer drove to their groceries. Instead, Webvan drove groceries to customers. At first sight, this looks like a brilliant business model innovation—very much like a Netflix for groceries.

Unfortunately for Webvan, this did not lead to a higher utilization of its assets. Most of its warehouses were poorly utilized, and many of its vans were driven around partially full. Furthermore, instead of using "cheap" self-serve labor from customers (customers provide the service of picking their groceries and bringing them to the checkout counter), they utilized "expensive" employees paid explicitly by the company.

The Webvan example does not prove that that there will be no successful business model innovation in groceries. It merely illustrates that it can be a challenge to develop a successful business model, including a supply process that profitably matches supply with demand.

19.6 Summary

Innovation is a novel match between a solution and a need. We identified four key needs as they relate to business model innovation, which together make the customer value curve: price, preference fit, transactional efficiency and quality. A successful business model innovation generally involves some smart sacrifice—dramatically improving along one dimension while sacrificing some other dimension. For example, preference fit may be enhanced by increasing the variety offered while reducing some dimension of transactional efficiency (such as the time to fulfill the need). To achieve a substantial shift in the customer value curve, a firm can change the timing of a process, the location process, and/or the level of standardization of a process. These approaches are illustrated for Netflix and Zipcar, along with several other examples.

19.7 Further Reading

For a strategic discussion of how a firm can radically change its positioning in the market to make "the competition irrelevant", see Chan Kim and Mauborgne (2005). See Terwiesch and Ulrich (2009) for more on the innovation process.

Appendix A

Statistics Tutorial

This appendix provides a brief tutorial to the statistics needed for the material in this book.

Statistics is about understanding and quantifying uncertainty (or, if you prefer, variability). So suppose we are interested in an event that is stochastic, that is, it has an uncertain outcome. For example, it could be the demand for a product, the number of people that call us between 10:00 a.m. and 10:15 a.m., the amount of time until the arrival of the next patient to the emergency room, and so forth. In each case, the outcome of this stochastic event is some number (units of demand, minutes between arrival, etc.). This stochastic event can also be called a *random variable.* Because our random variable could represent a wide variety of situations, for the purpose of this tutorial, let's give our random variable a generic name, *X.*

All random variables have an *expected value,* which is also called the *mean.* Depending on the context, we use different symbols to represent the mean. For example, we generally use the Greek symbol μ to represent the mean of our stochastic demand whereas we use a to represent the mean of the interarrival time of customers to a queuing system. A random variable is also characterized by its *standard deviation,* which roughly describes the amount of uncertainty in the distribution, or how "spread out" the distribution is. The Greek symbol σ is often used to describe the standard deviation of a random variable. Uncertainty also can be measured with the *variance* of a random variable. The variance of a random variable is closely related to its standard deviation: it is the square of the standard deviation:

$$\text{Variance} = (\text{Standard deviation})^2 = \sigma^2$$

Hence, it is sufficient to just work with the standard deviation because the variance can always be evaluated quickly once you know the standard deviation.

The standard deviation measures the absolute amount of uncertainty in a distribution, but it is often useful to think about the relative amount of uncertainty. For example, suppose we have two random variables, one with mean 20 and the other with mean 200. Suppose further they both have standard deviations equal to 10, that is, they have the same absolute amount of uncertainty. A standard deviation of 10 means there is about a two-thirds chance the outcome of the random variable will be within 10 units of the mean. Being within 10 units of a mean of 20 is much more variable in a relative sense than being within 10 units of a mean of 200: in the first case we have a two-thirds chance of being within 50 percent of the mean, whereas in the second case we have a two-thirds chance of being within 5 percent of the mean. Hence, we need a relative measure of uncertainty. We'll use the *coefficient of variation,* which is the

standard deviation of a distribution divided by its mean, for example, σ/μ. In some cases we will use explicit variables to represent the coefficient of variation. For example, in our work with queuing systems, we will let CV_a be the coefficient of variation of the arrival times to the queue and CV_p be the coefficient of variation of the service times in the queue.

Every random variable is defined by its *distribution function* and its *density function.* (Actually, only one of those functions is sufficient to define the random variable, but that is a picky point.) Let's say $F(Q)$ is the distribution function of X and $f(Q)$ is the density function. The density function returns the probability our stochastic event will be exactly Q, while the distribution function returns the probability our stochastic event will be Q or lower:

$$F(Q) = \text{Prob}\{X \text{ will be less than or equal to } Q\}$$
$$f(q) = \text{Prob}\{X \text{ will be exactly } Q\}$$

There are an infinite number of possible distribution and density functions, but a few of the more useful ones have been given names. The *normal distribution* is probably the most well-known distribution: the density function of the normal distribution is shaped like a bell. The normal distribution is defined with two parameters, its mean and its standard deviation, that is, a μ and a σ. The distribution and density functions of a normal distribution with mean 1,000 and standard deviation 300 are displayed in Figure A.1.

Distribution functions are always increasing from 0 to 1 and often have an S shape. Density functions do not have a typical pattern: some have the bell shape like the normal; others are downward curving.

While there are an infinite number of normal distributions (essentially any mean and standard deviation combination), there is one normal distribution that is particularly useful, the *standard normal.* The standard normal distribution has mean 0 and standard deviation 1. Because the standard normal is a special distribution, its distribution function is given special notation: the distribution function of the standard normal is $\Phi(z)$; that is, $\Phi(z)$ is the

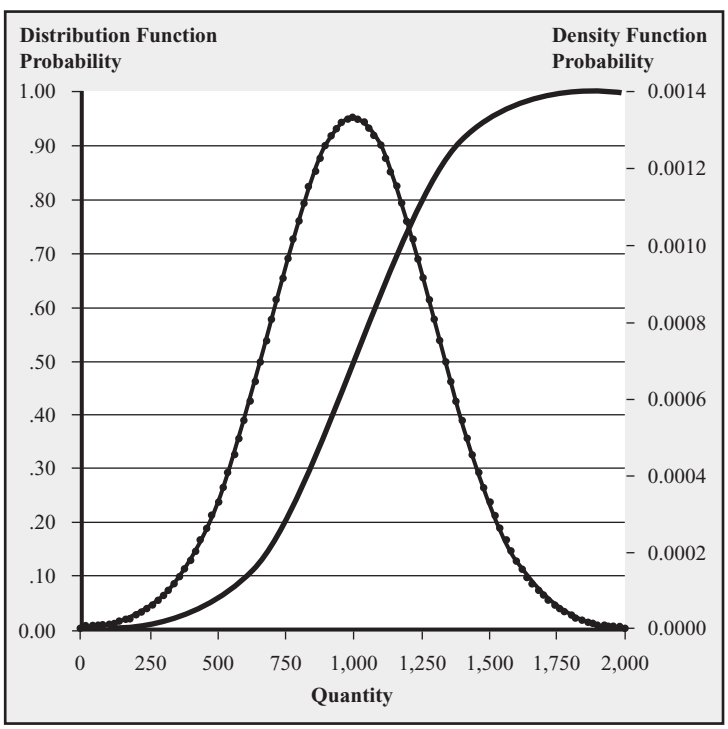


FIGURE A.1
Distribution (solid line) and Density (circles) Functions of a Normal Distribution with Mean 1,000 and Standard Deviation 300

TABLE A.1
The Density Function *f*(*Q*) and Distribution Function *F*(*Q*) of a Poisson Distribution with Mean 1.25

Q	*f*(*Q*)	*F*(*Q*)
0	0.28650	0.28650
1	0.35813	0.64464
2	0.22383	0.86847
3	0.09326	0.96173
4	0.02914	0.99088
5	0.00729	0.99816
6	0.00152	0.99968
7	0.00027	0.99995
8	0.00004	0.99999
9	0.00001	1.00000

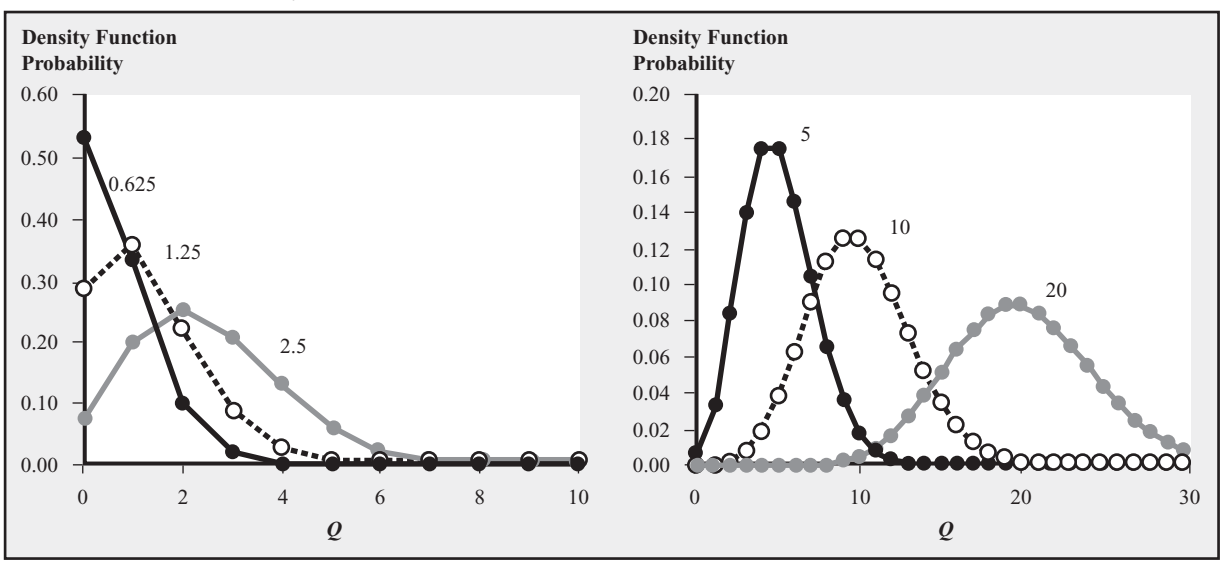
probability the outcome of a standard normal distribution is z or lower. The density function of the standard normal is $\phi(z)$. (Φ and ϕ are the upper- and lowercase, respectively, of the Greek letter phi.)

The normal distribution is a *continuous distribution* because all outcomes are possible, even fractional quantities such as 989.56. The *Poisson distribution* is also common, but it is a *discrete distribution* because the outcome of a Poisson random variable is always an integer value (i.e., 0, 1, 2, . . .). The Poisson distribution is characterized by a single parameter, its mean. The standard deviation of a Poisson distribution equals the square root of its mean:

$$\text{Standard deviation of a Poisson distribution} = \sqrt{\text{Mean of the Poisson distribution}}$$

While the outcome of a Poisson distribution is always an integer, the mean of the Poisson does not need to be an integer. The distribution and density functions of a Poisson distribution with mean 1.25 are displayed in Table A.1. Figure A.2 displays the density function of six different Poisson distributions. Unlike the familiar bell shape of the normal distribution, we can see that there is no standard shape for the Poisson: with a very low mean, the Poisson is a downward-sloping curve, but then as the mean increases, the Poisson begins to adopt a bell-like shape.

FIGURE A.2 **The Density Function of Six Different Poisson Distributions with Means 0.625, 1.25, 2.5, 5,10, and 20**



Because the outcome of a Poisson distribution is never negative and always integer, the Poisson generally better fits data with a low mean, say less than 20. For large means (say more than 20), the Poisson generally does not fit data as well as the normal for two reasons: (1) the Poisson adopts a bell-like shape, so it does not provide a shape advantage, and (2) the Poisson's standard deviation *must* equal the square root of the mean, so it does not allow the flexibility to expand or contract the width of the bell like the normal does (i.e., the normal allows for different bell shapes with the same mean but the Poisson only allows one bell shape for a given mean).

We also make extensive use of the exponential distribution in this text because it provides a good representation of the interarrival time of customers (i.e., the time between customer arrivals). The exponential distribution is characterized by a single parameter, its mean. We'll use a as the mean of the interarrival time. So if X is the interarrival time of customers and it is exponentially distributed with mean a, then the distribution function of X is

$$\text{Prob}\{X \text{ is less than or equal to } t\} = F(X) = 1 - e^{-t/a}$$

where e in the above equation is the natural constant that approximately equals 2.718282. In Excel you would write the exponential distribution function with the Exp function: $1 - \text{Exp}(-t/a)$. Notice that the exponential distribution function is a continuous distribution, which makes sense given that we are talking about time. Figure A.3 displays the distribution and density functions of an exponential distribution with mean 0.8.

The exponential distribution and the Poisson distribution are actually closely related. If the interarrival time of customers is exponentially distributed with mean a, then the

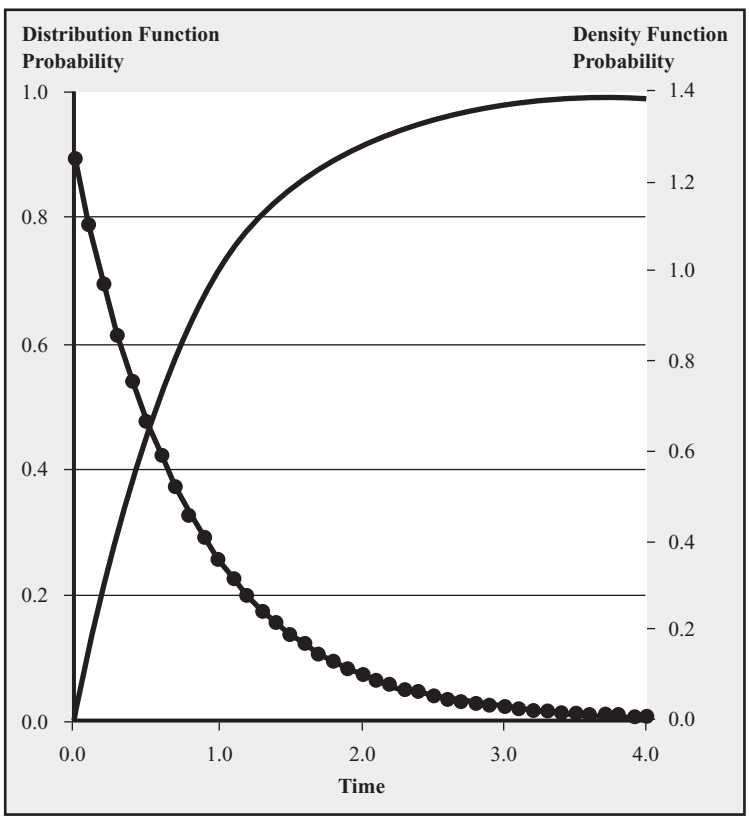


FIGURE A.3
Distribution (solid line) and Density (circles) Functions of an Exponential Distribution with Mean 0.8

number of customers that arrive over an interval of a unit of time is Poisson distributed with mean $1/a$. For example, if the interarrival time of customers is exponentially distributed with a mean of 0.8 (as in Figure A.3), then the number of customers that arrive in one unit of time has a Poisson distribution with mean $1/0.8 = 1.25$ (as in Table A.1).

Other commonly used distributions include the negative binomial and the gamma, but we will not make much use of them in this text.

Finding the Probability *X* Will Be Less Than *Q* or Greater Than *Q*

When working with a random variable, we often need to find the probability the outcome of the random variable will be less than a particular quantity or more than the particular quantity. For example, suppose X has a Poisson distribution with mean 1.25. What is the probability X will be four units or fewer? That can be answered with the distribution function: from Table A.1, $F(4) = 99.088$ percent. What is the probability X will be greater than four units, that is, that it is five or more units? X is either Q or fewer units or it is more than Q units, so

$$\text{Prob}\{X \text{ is } Q \text{ or fewer units}\} + \text{Prob}\{X \text{ is more than } Q \text{ units}\} = 1$$

If we rearrange terms in the above equation, we get

$$\text{Prob}\{X \text{ is more than } Q \text{ units}\} = 1 - \text{Prob}\{X \text{ is } Q \text{ or fewer units}\} = 1 - F(Q)$$

Hence, X will be greater than four units with probability $1 - F(4) = 0.00912$.

A tricky issue in these evaluations is the difference between the "probability X is fewer than Q" and the "probability X is Q or fewer." The first case does not include the outcome that X exactly equals Q, whereas the second case does. For example, when we evaluate the "probability X is more than Q units," we are not including the outcome that X equals Q units. Therefore, be aware of this issue and remember that $F(Q)$ is the probability X is Q or fewer; that is, it includes the probability that X exactly equals Q units.

We also need to find the probability X is more or less than Q when X is normally distributed. Working with the normal distribution is not too hard because all normal distributions, no matter their mean or standard deviation, are related to the standard normal distribution, which is why the standard normal is special and important. Hence, we can find out the probability X will be more or less than Q by working with the standard normal distribution.

Suppose X is normally distributed with mean 1,000 and standard deviation 300 ($\mu = 1{,}000$, $\sigma = 300$) and we want to find the probability X will be less than $Q = 1{,}600$ units. First convert Q into the equivalent order quantity if X followed the standard normal distribution. That equivalent order quantity is z, which is called the *z-statistic:*

$$z = \frac{Q - \mu}{\sigma} = \frac{1{,}600 - 1{,}000}{300} = 2.0$$

Hence, the quantity 1,600 relative to a normal distribution with mean 1,000 and standard deviation 300 is equivalent to the quantity 2.0 relative to a standard normal distribution. The probability we are looking for is then $\Phi(2.0)$, which we can find in the Standard Normal Distribution Function Table in Appendix B: $\Phi(2.0) = 0.9772$. In other words, there is a 97.72 percent chance X is less than 1,600 units if X follows a normal distribution with mean 1,000 and standard deviation 300.

What is the probability X will be greater than 1,600 units? That is just $1 - \Phi(2.0) = 0.0228$; that is, the probability X will be greater than 1,600 units is just 1 minus the probability X will be less than 1,600 units.

With the normal distribution, unlike the Poisson distribution, we do not need to worry too much about the distinction between the "probability X is fewer than Q" and the "probability X is Q or fewer." With the Poisson distribution, there can be a significant probability that the outcome is exactly Q units because the Poisson distribution is a discrete distribution and usually has a low mean, which implies that there are relatively few possible outcomes. The normal distribution is continuous, so there essentially is no distinction between "X being exactly Q units" and "X being just a tiny fraction below Q units."

Expected Value

We often need to know the expected value of something happening. For example, suppose we make a decision and there are two possible outcomes, G for good and B for bad; that is, $X = \text{G}$ or $X = \text{B}$. If the outcome is G, then we earn \$100, but if the outcome is B, we lose \$40. Furthermore, we know the following probabilities: $\text{Prob}\{X = \text{G}\} = 0.25$ and $\text{Prob}\{X = \text{B}\} = 0.75$. (Note, these probabilities must sum to 1 because they are the only two possible outcomes.) The expected value of this decision is

$$\begin{aligned} &\$100 \times \text{Prob}\{X = \text{G}\} + (-\$40 \times \text{Prob}\{X = \text{B}\}) \\ &= \$100 \times 0.25 + (-\$40 \times 0.75) \\ &= -\$5 \end{aligned}$$

In words, to evaluate the expected value, we multiply the probability of each outcome with the value of each outcome and then sum up all of those calculations.

The Loss Function

In statistics the distribution and density functions are well known and used often. Less well known in statistics is the *loss function,* but we make extensive use of it in this text. The loss function $L(Q)$ is the expected amount X is greater than Q. In other words, the expected loss is the expected amount a random variable X exceeds a chosen threshold Q.

To explain further, let X be a Poisson distribution with mean 1.25 and say our chosen threshold is $Q = 2$. (Table A.1 has the distribution function.) If $X = 3$, then X exceeds Q by one unit. If $X = 4$, then X exceeds Q by two units and if $X = 5$, then X exceeds Q by three units, and so on. Furthermore, if X is 2 or fewer, then X exceeds Q by 0 units. The loss function is the expected value of all of those events; that is, $L(2)$ is the expected amount by which X exceeds Q. Table A.2 provides those calculations for $L(2)$.

TABLE A.2
Calculation of the Loss Function for $Q = 2$ and a Poisson Distribution with Mean 1.25

Q	$f(Q)$ (a)	Amount X Exceeds 2 (b)	(a × b)
0	0.286505	0	0.00000
1	0.358131	0	0.00000
2	0.223832	0	0.00000
3	0.093263	1	0.09326
4	0.029145	2	0.05829
5	0.007286	3	0.02186
6	0.001518	4	0.00607
7	0.000271	5	0.00136
8	0.000042	6	0.00025
9	0.000006	7	0.00004
10	0.000001	8	0.00001
$L(2)$ = Total of last column =			0.18114

FIGURE A.4 **Calculation of the Loss Function for a Bell-like Distribution Function That Has Discrete Outcomes 0, 10, . . . , 200.**

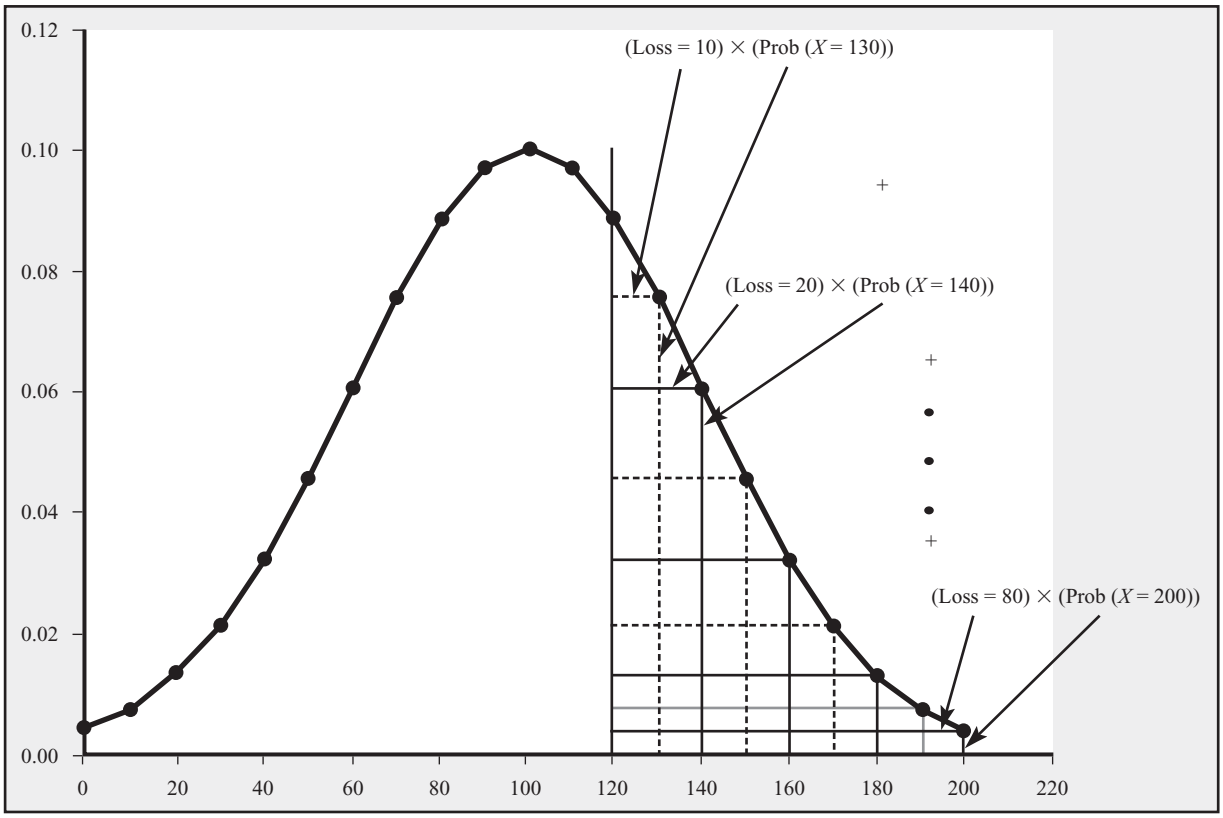


Figure A.4 gives a graphical perspective on the loss function. Depicted is a density function of a random variable X that has a bell shape like a normal distribution, but the only possible outcomes are 0, 10, 20, . . . , 190, and 200. Suppose we are interested in $L(120)$, the expected loss function evaluated at the threshold of $Q = 120$. If $X <= 120$, there is no loss; that is, the random variable does not exceed the threshold Q. If $X = 130$, then the loss is $130 - 120 = 10$, so we take that loss and multiply it by the probability it occurs. We repeat that procedure for the remaining possible outcomes that generate a loss (140 through 200) and sum those values to yield $L(120) = 7.486$. In other words, the random variable X exceeds the fixed threshold $Q = 120$ on average by 7.486. This might strike you as too low given that the losses ranged in our calculations from 10 to 80, but remember that for most outcomes there is actually no loss, that is, X is less than or equal to Q.

We displayed the calculation of the loss function with a discrete random variable in Figure A.4, but conceptually we can do the same calculation with a continuous random variable such as the normal. The only difference is that there is a lot more work to do with a continuous random variable because we need to multiply every possible loss by its probability and sum all of those calculations.

At this point you (hopefully) understand that the loss function is not conceptually difficult, but it is a "pain in the neck" to evaluate. Fortunately, Appendix C provides an easier way to evaluate the loss function of a discrete random variable. But even that easier way requires a decent amount of work, more work than should be done by hand. In other words, either you have a spreadsheet to help you evaluate the loss function or you should have a table that has already been evaluated for you, as in the following for the Poisson with mean 1.25:

Q	f(Q)	F(Q)	L(Q)
0	0.286505	0.286505	1.25000
1	0.358131	0.644636	0.53650
2	0.223832	0.868468	0.18114
3	0.093263	0.961731	0.04961
4	0.029145	0.990876	0.01134
5	0.007286	0.998162	0.00221
6	0.001518	0.999680	0.00038
7	0.000271	0.999951	0.00006
8	0.000042	0.999993	0.00001
9	0.000006	0.999999	0.00000
10	0.000001	1.000000	0.00000

If X is normally distributed, then our loss function is already provided to us in Appendix B. Actually, the loss function of the standard normal distribution is provided, that is, the Standard Normal Loss Function Table gives us $L(z)$, the expected loss function if X is a standard normal distribution. Because we often work with a different normal distribution, we need to learn how to convert the answer we get from that table into the answer that is appropriate for the normal distribution we are working with.

Suppose X is normally distributed with mean 1,000 and standard deviation 300. We are interested in the loss function with $Q = 1{,}600$. Just as we did when we were looking for the probability X will be greater than Q, first convert Q into the corresponding z value for the standard normal distribution:

$$z = \frac{Q - \mu}{\sigma} = \frac{1{,}600 - 1{,}000}{300} = 2.0$$

In other words, $Q = 1{,}600$ and a normal distribution with mean 1,000 and standard deviation 300 is equivalent to $z = 2.0$ and a standard normal distribution. Next, look up $L(z)$ in the Standard Normal Loss Function Table: $L(z) = 0.0085$. In other words, 0.0085 unit is the expected amount a standard normal will exceed the threshold of $z = 2.0$. Finally, we need to convert that value in the loss function to the value for the actual normal distribution. We use the following equation to do that:

$$L(Q) = \sigma \times L(z)$$

which in this case means

$$L(1{,}600) = 300 \times 0.0085 = 2.55$$

Hence, if $Q = 1{,}600$ and X is normally distributed with mean 1,000 and standard deviation 300, then the expected amount X will exceed Q is only 2.55 units. Why is the loss function so small? We evaluated the probability X exceeds Q to be only 2.28 percent, so most of the time X exceeds Q by 0 units.

Independence, Correlation, and Combining (or Dividing) Random Variables

We often need to combine several random variables or to divide a random variable. For example, if we have five random variables, each one representing demand on a particular day of the week, we might want to combine them into a single random variable that

represents weekly demand. Or we might have a random variable that represents monthly demand and we might want to divide it into random variables that represent weekly demand. In addition to combining and dividing random variables across time, we may wish to combine or divide random variables across products or categories.

Suppose you wish to combine n random variables, labeled $X_1, X_2, \ldots, X_n$, into a single random variable X; that is, you want $X = X_1 + X_2 + \cdots + X_n$. Furthermore, we assume each of the n original random variables comes from the same "family," for example, they are all normal or all Poisson. Hence, the combined random variable X is also part of the same family: the sum of two normally random variables is normally distributed; the sum of two Poisson random variables is Poisson; and so forth. So we need a mean to describe X and maybe a standard deviation. The mean of X is easy to evaluate:

$$\mu = \mu_1 + \mu_2 + \cdots + \mu_n$$

In other words, the mean of X is just the sum of the means of the n individual random variables.

If we need a standard deviation for X and the n random variables are independent, then the standard deviation of X is

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}$$

In words, the standard deviation of X is the square root of the sum of the variances of the n random variables. If the n random variables have the same standard deviation (i.e., $\sigma_1 = \sigma_2 = \ldots = \sigma_n$), then the above simplifies to $\sigma = \sqrt{n} \times \sigma_1$.

The key condition in our evaluation of the standard deviation of X is that the n individual random variables are independent. Roughly speaking, two random variables are *independent* if the outcome of one random variable has no influence on the outcome of the other random variable. For example, if one has a rather high demand outcome, then that provides no information as to whether the other random variable will have a high or low outcome.

Two random variables are *correlated* if the outcome of one random variable provides information about the outcome of the other random variable. Two random variables are *positively correlated* if their outcomes tend to move in lock step: if one is high, then the other tends to be high, and if one is low, the other tends to be low. Two random variables are *negatively correlated* if their outcomes tend to move in opposite step: if one is high, then the other tends to be low, and if one is low, the other tends to be high.

The correlation between two random variables can range from −1 to 1. A correlation of −1 means the two are perfectly negatively correlated: as one random variable's outcome increases, the other one's outcome surely decreases. The other extreme is perfectly positively correlated, which means a correlation of 1: as one random variable's outcome increases, the other one's outcome surely increases as well. In the middle is independence: if two random variables are independent, then their correlation is 0.

So how do we evaluate the standard deviation of X when X is the sum of two random variables that may not be independent? Use the following equation:

$$\text{Standard deviation of } X = \sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + 2 \times \sigma_1 \times \sigma_2 \times \text{Correlation}}$$

where *Correlation* in the above equation is the correlation between X_1 and X_2.

Appendix B

Tables

This appendix contains the Erlang Loss Function Table and the distribution and loss function tables for the standard normal distribution and several Poisson distributions.

Erlang Loss Function Table

The Erlang Loss Function Table contains the probability that a process step consisting of m parallel resources contains m flow units, that is, all m resources are utilized. Interarrival times of flow units (e.g., customers or data packets, etc.) are exponentially distributed with mean a and service times have a mean p (service times do not have to follow an exponential distribution).

Because there is no buffer space, if a flow unit arrives and all m servers are busy, then that arriving flow unit leaves the system unserved (i.e., the flow unit is lost). The columns in the table correspond to the number of resources m and the rows in the table correspond to $r = p/a$; that is, the ratio between the service time and the interarrival time. The following two pages include two tables, one for small values of r and one for larger values of r.

Example: Find the probability $P_m(r)$ that a process step consisting of three parallel resources must deny access to newly arriving units. Flow units arrive one every $a = 3$ minutes with exponential interarrival times and take $p = 2$ minutes to serve. First, define $r = p/a = 2/3 = 0.67$ and find the corresponding row heading. Second, find the column heading for $m = 3$. The intersection of that row with that column is $P_m(r) = 0.0255$.

Note that $P_m(r)$ can be computed directly based on the following formula

$$\text{Probability}\{\text{all } m \text{ servers busy}\} = P_m(r) = \frac{\dfrac{r^m}{m!}}{1 + \dfrac{r^1}{1!} + \dfrac{r^2}{2!} + \cdots + \dfrac{r^m}{m!}} \quad \text{(Erlang loss formula)}$$

The exclamation mark (!) in the equation refers to the factorial of an integer number. To compute the factorial of an integer number x, write down all numbers from 1 to x and then multiply them with each other. For example, $4! = 1 \times 2 \times 3 \times 4 = 24$. This calculation can be done with the Excel function FACT(x).

Erlang Loss Table

	m									
$r = p / a$	1	2	3	4	5	6	7	8	9	10
0.10	0.0909	0.0045	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.20	0.1667	0.0164	0.0011	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.2000	0.0244	0.0020	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.30	0.2308	0.0335	0.0033	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.33	0.2500	0.0400	0.0044	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.40	0.2857	0.0541	0.0072	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.50	0.3333	0.0769	0.0127	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
0.60	0.3750	0.1011	0.0198	0.0030	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000
0.67	0.4000	0.1176	0.0255	0.0042	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
0.70	0.4118	0.1260	0.0286	0.0050	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
0.75	0.4286	0.1385	0.0335	0.0062	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
0.80	0.4444	0.1509	0.0387	0.0077	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
0.90	0.4737	0.1757	0.0501	0.0111	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000
1.00	0.5000	0.2000	0.0625	0.0154	0.0031	0.0005	0.0001	0.0000	0.0000	0.0000
1.10	0.5238	0.2237	0.0758	0.0204	0.0045	0.0008	0.0001	0.0000	0.0000	0.0000
1.20	0.5455	0.2466	0.0898	0.0262	0.0063	0.0012	0.0002	0.0000	0.0000	0.0000
1.25	0.5556	0.2577	0.0970	0.0294	0.0073	0.0015	0.0003	0.0000	0.0000	0.0000
1.30	0.5652	0.2687	0.1043	0.0328	0.0085	0.0018	0.0003	0.0001	0.0000	0.0000
1.33	0.5714	0.2759	0.1092	0.0351	0.0093	0.0021	0.0004	0.0001	0.0000	0.0000
1.40	0.5833	0.2899	0.1192	0.0400	0.0111	0.0026	0.0005	0.0001	0.0000	0.0000
1.50	0.6000	0.3103	0.1343	0.0480	0.0142	0.0035	0.0008	0.0001	0.0000	0.0000
1.60	0.6154	0.3299	0.1496	0.0565	0.0177	0.0047	0.0011	0.0002	0.0000	0.0000
1.67	0.6250	0.3425	0.1598	0.0624	0.0204	0.0056	0.0013	0.0003	0.0001	0.0000
1.70	0.6296	0.3486	0.1650	0.0655	0.0218	0.0061	0.0015	0.0003	0.0001	0.0000
1.75	0.6364	0.3577	0.1726	0.0702	0.0240	0.0069	0.0017	0.0004	0.0001	0.0000
1.80	0.6429	0.3665	0.1803	0.0750	0.0263	0.0078	0.0020	0.0005	0.0001	0.0000
1.90	0.6552	0.3836	0.1955	0.0850	0.0313	0.0098	0.0027	0.0006	0.0001	0.0000
2.00	0.6667	0.4000	0.2105	0.0952	0.0367	0.0121	0.0034	0.0009	0.0002	0.0000
2.10	0.6774	0.4156	0.2254	0.1058	0.0425	0.0147	0.0044	0.0011	0.0003	0.0001
2.20	0.6875	0.4306	0.2400	0.1166	0.0488	0.0176	0.0055	0.0015	0.0004	0.0001
2.25	0.6923	0.4378	0.2472	0.1221	0.0521	0.0192	0.0061	0.0017	0.0004	0.0001
2.30	0.6970	0.4449	0.2543	0.1276	0.0554	0.0208	0.0068	0.0019	0.0005	0.0001
2.33	0.7000	0.4495	0.2591	0.1313	0.0577	0.0220	0.0073	0.0021	0.0005	0.0001
2.40	0.7059	0.4586	0.2684	0.1387	0.0624	0.0244	0.0083	0.0025	0.0007	0.0002
2.50	0.7143	0.4717	0.2822	0.1499	0.0697	0.0282	0.0100	0.0031	0.0009	0.0002
2.60	0.7222	0.4842	0.2956	0.1612	0.0773	0.0324	0.0119	0.0039	0.0011	0.0003
2.67	0.7273	0.4923	0.3044	0.1687	0.0825	0.0354	0.0133	0.0044	0.0013	0.0003
2.70	0.7297	0.4963	0.3087	0.1725	0.0852	0.0369	0.0140	0.0047	0.0014	0.0004
2.75	0.7333	0.5021	0.3152	0.1781	0.0892	0.0393	0.0152	0.0052	0.0016	0.0004
2.80	0.7368	0.5078	0.3215	0.1837	0.0933	0.0417	0.0164	0.0057	0.0018	0.0005
2.90	0.7436	0.5188	0.3340	0.1949	0.1016	0.0468	0.0190	0.0068	0.0022	0.0006
3.00	0.7500	0.5294	0.3462	0.2061	0.1101	0.0522	0.0219	0.0081	0.0027	0.0008
3.10	0.7561	0.5396	0.3580	0.2172	0.1187	0.0578	0.0249	0.0096	0.0033	0.0010
3.20	0.7619	0.5494	0.3695	0.2281	0.1274	0.0636	0.0283	0.0112	0.0040	0.0013
3.25	0.7647	0.5541	0.3751	0.2336	0.1318	0.0666	0.0300	0.0120	0.0043	0.0014
3.30	0.7674	0.5587	0.3807	0.2390	0.1362	0.0697	0.0318	0.0130	0.0047	0.0016
3.33	0.7692	0.5618	0.3843	0.2426	0.1392	0.0718	0.0331	0.0136	0.0050	0.0017
3.40	0.7727	0.5678	0.3915	0.2497	0.1452	0.0760	0.0356	0.0149	0.0056	0.0019
3.50	0.7778	0.5765	0.4021	0.2603	0.1541	0.0825	0.0396	0.0170	0.0066	0.0023
3.60	0.7826	0.5848	0.4124	0.2707	0.1631	0.0891	0.0438	0.0193	0.0077	0.0028
3.67	0.7857	0.5902	0.4191	0.2775	0.1691	0.0937	0.0468	0.0210	0.0085	0.0031
3.70	0.7872	0.5929	0.4224	0.2809	0.1721	0.0960	0.0483	0.0218	0.0089	0.0033
3.75	0.7895	0.5968	0.4273	0.2860	0.1766	0.0994	0.0506	0.0232	0.0096	0.0036
3.80	0.7917	0.6007	0.4321	0.2910	0.1811	0.1029	0.0529	0.0245	0.0102	0.0039
3.90	0.7959	0.6082	0.4415	0.3009	0.1901	0.1100	0.0577	0.0274	0.0117	0.0046
4.00	0.8000	0.6154	0.4507	0.3107	0.1991	0.1172	0.0627	0.0304	0.0133	0.0053

Erlang Loss Table

	m									
$r = p / a$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1.0	0.5000	0.2000	0.0625	0.0154	0.0031	0.0005	0.0001	0.0000	0.0000	0.0000
1.5	0.6000	0.3103	0.1343	0.0480	0.0142	0.0035	0.0008	0.0001	0.0000	0.0000
2.0	0.6667	0.4000	0.2105	0.0952	0.0367	0.0121	0.0034	0.0009	0.0002	0.0000
2.5	0.7143	0.4717	0.2822	0.1499	0.0697	0.0282	0.0100	0.0031	0.0009	0.0002
3.0	0.7500	0.5294	0.3462	0.2061	0.1101	0.0522	0.0219	0.0081	0.0027	0.0008
3.5	0.7778	0.5765	0.4021	0.2603	0.1541	0.0825	0.0396	0.0170	0.0066	0.0023
4.0	0.8000	0.6154	0.4507	0.3107	0.1991	0.1172	0.0627	0.0304	0.0133	0.0053
4.5	0.8182	0.6480	0.4929	0.3567	0.2430	0.1542	0.0902	0.0483	0.0236	0.0105
5.0	0.8333	0.6757	0.5297	0.3983	0.2849	0.1918	0.1205	0.0700	0.0375	0.0184
5.5	0.8462	0.6994	0.5618	0.4358	0.3241	0.2290	0.1525	0.0949	0.0548	0.0293
6.0	0.8571	0.7200	0.5902	0.4696	0.3604	0.2649	0.1851	0.1219	0.0751	0.0431
6.5	0.8667	0.7380	0.6152	0.4999	0.3939	0.2991	0.2174	0.1501	0.0978	0.0598
7.0	0.8750	0.7538	0.6375	0.5273	0.4247	0.3313	0.2489	0.1788	0.1221	0.0787
7.5	0.8824	0.7679	0.6575	0.5521	0.4530	0.3615	0.2792	0.2075	0.1474	0.0995
8.0	0.8889	0.7805	0.6755	0.5746	0.4790	0.3898	0.3082	0.2356	0.1731	0.1217
8.5	0.8947	0.7918	0.6917	0.5951	0.5029	0.4160	0.3356	0.2629	0.1989	0.1446
9.0	0.9000	0.8020	0.7064	0.6138	0.5249	0.4405	0.3616	0.2892	0.2243	0.1680
9.5	0.9048	0.8112	0.7198	0.6309	0.5452	0.4633	0.3860	0.3143	0.2491	0.1914
10.0	0.9091	0.8197	0.7321	0.6467	0.5640	0.4845	0.4090	0.3383	0.2732	0.2146
10.5	0.9130	0.8274	0.7433	0.6612	0.5813	0.5043	0.4307	0.3611	0.2964	0.2374
11.0	0.9167	0.8345	0.7537	0.6745	0.5974	0.5227	0.4510	0.3828	0.3187	0.2596
11.5	0.9200	0.8410	0.7633	0.6869	0.6124	0.5400	0.4701	0.4033	0.3400	0.2811
12.0	0.9231	0.8471	0.7721	0.6985	0.6264	0.5561	0.4880	0.4227	0.3604	0.3019
12.5	0.9259	0.8527	0.7804	0.7092	0.6394	0.5712	0.5049	0.4410	0.3799	0.3220
13.0	0.9286	0.8579	0.7880	0.7192	0.6516	0.5854	0.5209	0.4584	0.3984	0.3412
13.5	0.9310	0.8627	0.7952	0.7285	0.6630	0.5987	0.5359	0.4749	0.4160	0.3596
14.0	0.9333	0.8673	0.8019	0.7373	0.6737	0.6112	0.5500	0.4905	0.4328	0.3773
14.5	0.9355	0.8715	0.8081	0.7455	0.6837	0.6230	0.5634	0.5052	0.4487	0.3942
15.0	0.9375	0.8755	0.8140	0.7532	0.6932	0.6341	0.5761	0.5193	0.4639	0.4103
15.5	0.9394	0.8792	0.8196	0.7605	0.7022	0.6446	0.5880	0.5326	0.4784	0.4258
16.0	0.9412	0.8828	0.8248	0.7674	0.7106	0.6546	0.5994	0.5452	0.4922	0.4406
16.5	0.9429	0.8861	0.8297	0.7739	0.7186	0.6640	0.6102	0.5572	0.5053	0.4547
17.0	0.9444	0.8892	0.8344	0.7800	0.7262	0.6729	0.6204	0.5687	0.5179	0.4682
17.5	0.9459	0.8922	0.8388	0.7859	0.7334	0.6814	0.6301	0.5795	0.5298	0.4811
18.0	0.9474	0.8950	0.8430	0.7914	0.7402	0.6895	0.6394	0.5899	0.5413	0.4935
18.5	0.9487	0.8977	0.8470	0.7966	0.7467	0.6972	0.6482	0.5998	0.5522	0.5053
19.0	0.9500	0.9002	0.8508	0.8016	0.7529	0.7045	0.6566	0.6093	0.5626	0.5167
19.5	0.9512	0.9027	0.8544	0.8064	0.7587	0.7115	0.6647	0.6183	0.5726	0.5275
20.0	0.9524	0.9050	0.8578	0.8109	0.7644	0.7181	0.6723	0.6270	0.5822	0.5380
20.5	0.9535	0.9072	0.8611	0.8153	0.7697	0.7245	0.6797	0.6353	0.5913	0.5480
21.0	0.9545	0.9093	0.8642	0.8194	0.7749	0.7306	0.6867	0.6432	0.6001	0.5576
21.5	0.9556	0.9113	0.8672	0.8234	0.7798	0.7364	0.6934	0.6508	0.6086	0.5668
22.0	0.9565	0.9132	0.8701	0.8272	0.7845	0.7420	0.6999	0.6581	0.6167	0.5757
22.5	0.9574	0.9150	0.8728	0.8308	0.7890	0.7474	0.7061	0.6651	0.6244	0.5842
23.0	0.9583	0.9168	0.8754	0.8343	0.7933	0.7525	0.7120	0.6718	0.6319	0.5924
23.5	0.9592	0.9185	0.8780	0.8376	0.7974	0.7575	0.7177	0.6783	0.6391	0.6003
24.0	0.9600	0.9201	0.8804	0.8408	0.8014	0.7622	0.7232	0.6845	0.6461	0.6079
24.5	0.9608	0.9217	0.8827	0.8439	0.8053	0.7668	0.7285	0.6905	0.6527	0.6153
25.0	0.9615	0.9232	0.8850	0.8469	0.8090	0.7712	0.7336	0.6963	0.6592	0.6224
25.5	0.9623	0.9246	0.8871	0.8497	0.8125	0.7754	0.7385	0.7019	0.6654	0.6292
26.0	0.9630	0.9260	0.8892	0.8525	0.8159	0.7795	0.7433	0.7072	0.6714	0.6358
26.5	0.9636	0.9274	0.8912	0.8552	0.8192	0.7835	0.7479	0.7124	0.6772	0.6422
27.0	0.9643	0.9287	0.8931	0.8577	0.8224	0.7873	0.7523	0.7174	0.6828	0.6483
27.5	0.9649	0.9299	0.8950	0.8602	0.8255	0.7910	0.7565	0.7223	0.6882	0.6543
28.0	0.9655	0.9311	0.8968	0.8626	0.8285	0.7945	0.7607	0.7269	0.6934	0.6600
28.5	0.9661	0.9323	0.8985	0.8649	0.8314	0.7979	0.7646	0.7315	0.6985	0.6656
29.0	0.9667	0.9334	0.9002	0.8671	0.8341	0.8013	0.7685	0.7359	0.7034	0.6710
29.5	0.9672	0.9345	0.9019	0.8693	0.8368	0.8045	0.7722	0.7401	0.7081	0.6763
30.0	0.9677	0.9356	0.9034	0.8714	0.8394	0.8076	0.7758	0.7442	0.7127	0.6813
30.5	0.9683	0.9366	0.9050	0.8734	0.8420	0.8106	0.7793	0.7482	0.7172	0.6863
31.0	0.9688	0.9376	0.9064	0.8754	0.8444	0.8135	0.7827	0.7521	0.7215	0.6910
31.5	0.9692	0.9385	0.9079	0.8773	0.8468	0.8164	0.7860	0.7558	0.7257	0.6957
32.0	0.9697	0.9394	0.9093	0.8791	0.8491	0.8191	0.7892	0.7594	0.7297	0.7002

Distribution and Loss Function Tables

The Standard Normal Distribution Function Table contains the probability that the outcome of a standard normal random variable is z or smaller. The table provides z values up to two significant digits. Find the row and column headings that add up to the z value you are looking for. The intersection of that row and column contains the probability you seek, $\Phi(z)$.

Example (1): Find the probability that a standard normal random variable generates an outcome that is $z = -1.54$ or lower. First, find the row heading -1.5. Second, find the column heading -0.04 because $(-1.5) + (-0.04) = -1.54$. The intersection of that row with that column is $\Phi(-1.54) = 0.0618$.

Example (2): Find the probability that a standard normal random variable generates an outcome that is $z = 0.52$ or lower. First, find the row heading 0.5. Second, find the column heading 0.02 because $(0.5) + (0.02) = 0.52$. The intersection of that row with that column is $\Phi(0.52) = 0.6985$.

The Standard Normal Loss Function Table is organized in the same way as the Standard Normal Distribution Function Table.

The Poisson Distribution Function Table provides the probability a Poisson distribution with a given mean (column heading) is S or fewer.

The Poisson Loss Function Table provides the expected amount the outcome of a Poisson distribution with a given mean (column heading) exceeds S.

Example (3): With mean 2.25 and $S = 2$, the loss function of a Poisson distribution is 0.69795: look in the column heading for the mean 2.25 and the row with $S = 2$.

Standard Normal Distribution Function Table, $\Phi(z)$

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.6	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
−3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
−3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
−3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
−3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
−2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
−2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
−2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
−2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
−0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
−0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
−0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
−0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
−0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
−0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
−0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
−0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

(*continued*)

Standard Normal Distribution Function Table, $\Phi(z)$ (Concluded)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Standard Normal Loss Function Table, $L(z)$

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00
−4.0	4.0900	4.0800	4.0700	4.0600	4.0500	4.0400	4.0300	4.0200	4.0100	4.0000
−3.9	3.9900	3.9800	3.9700	3.9600	3.9500	3.9400	3.9300	3.9200	3.9100	3.9000
−3.8	3.8900	3.8800	3.8700	3.8600	3.8500	3.8400	3.8300	3.8200	3.8100	3.8000
−3.7	3.7900	3.7800	3.7700	3.7600	3.7500	3.7400	3.7300	3.7200	3.7100	3.7000
−3.6	3.6900	3.6800	3.6700	3.6600	3.6500	3.6400	3.6300	3.6200	3.6100	3.6000
−3.5	3.5900	3.5800	3.5700	3.5600	3.5500	3.5400	3.5301	3.5201	3.5101	3.5001
−3.4	3.4901	3.4801	3.4701	3.4601	3.4501	3.4401	3.4301	3.4201	3.4101	3.4001
−3.3	3.3901	3.3801	3.3701	3.3601	3.3501	3.3401	3.3301	3.3201	3.3101	3.3001
−3.2	3.2901	3.2801	3.2701	3.2601	3.2502	3.2402	3.2302	3.2202	3.2102	3.2002
−3.1	3.1902	3.1802	3.1702	3.1602	3.1502	3.1402	3.1302	3.1202	3.1103	3.1003
−3.0	3.0903	3.0803	3.0703	3.0603	3.0503	3.0403	3.0303	3.0204	3.0104	3.0004
−2.9	2.9904	2.9804	2.9704	2.9604	2.9505	2.9405	2.9305	2.9205	2.9105	2.9005
−2.8	2.8906	2.8806	2.8706	2.8606	2.8506	2.8407	2.8307	2.8207	2.8107	2.8008
−2.7	2.7908	2.7808	2.7708	2.7609	2.7509	2.7409	2.7310	2.7210	2.7110	2.7011
−2.6	2.6911	2.6811	2.6712	2.6612	2.6512	2.6413	2.6313	2.6214	2.6114	2.6015
−2.5	2.5915	2.5816	2.5716	2.5617	2.5517	2.5418	2.5318	2.5219	2.5119	2.5020
−2.4	2.4921	2.4821	2.4722	2.4623	2.4523	2.4424	2.4325	2.4226	2.4126	2.4027
−2.3	2.3928	2.3829	2.3730	2.3631	2.3532	2.3433	2.3334	2.3235	2.3136	2.3037
−2.2	2.2938	2.2839	2.2740	2.2641	2.2542	2.2444	2.2345	2.2246	2.2147	2.2049
−2.1	2.1950	2.1852	2.1753	2.1655	2.1556	2.1458	2.1360	2.1261	2.1163	2.1065
−2.0	2.0966	2.0868	2.0770	2.0672	2.0574	2.0476	2.0378	2.0280	2.0183	2.0085
−1.9	1.9987	1.9890	1.9792	1.9694	1.9597	1.9500	1.9402	1.9305	1.9208	1.9111
−1.8	1.9013	1.8916	1.8819	1.8723	1.8626	1.8529	1.8432	1.8336	1.8239	1.8143
−1.7	1.8046	1.7950	1.7854	1.7758	1.7662	1.7566	1.7470	1.7374	1.7278	1.7183
−1.6	1.7087	1.6992	1.6897	1.6801	1.6706	1.6611	1.6516	1.6422	1.6327	1.6232
−1.5	1.6138	1.6044	1.5949	1.5855	1.5761	1.5667	1.5574	1.5480	1.5386	1.5293
−1.4	1.5200	1.5107	1.5014	1.4921	1.4828	1.4736	1.4643	1.4551	1.4459	1.4367
−1.3	1.4275	1.4183	1.4092	1.4000	1.3909	1.3818	1.3727	1.3636	1.3546	1.3455
−1.2	1.3365	1.3275	1.3185	1.3095	1.3006	1.2917	1.2827	1.2738	1.2650	1.2561
−1.1	1.2473	1.2384	1.2296	1.2209	1.2121	1.2034	1.1946	1.1859	1.1773	1.1686
−1.0	1.1600	1.1514	1.1428	1.1342	1.1257	1.1172	1.1087	1.1002	1.0917	1.0833
−0.9	1.0749	1.0665	1.0582	1.0499	1.0416	1.0333	1.0250	1.0168	1.0086	1.0004
−0.8	0.9923	0.9842	0.9761	0.9680	0.9600	0.9520	0.9440	0.9360	0.9281	0.9202
−0.7	0.9123	0.9045	0.8967	0.8889	0.8812	0.8734	0.8658	0.8581	0.8505	0.8429
−0.6	0.8353	0.8278	0.8203	0.8128	0.8054	0.7980	0.7906	0.7833	0.7759	0.7687
−0.5	0.7614	0.7542	0.7471	0.7399	0.7328	0.7257	0.7187	0.7117	0.7047	0.6978
−0.4	0.6909	0.6840	0.6772	0.6704	0.6637	0.6569	0.6503	0.6436	0.6370	0.6304
−0.3	0.6239	0.6174	0.6109	0.6045	0.5981	0.5918	0.5855	0.5792	0.5730	0.5668
−0.2	0.5606	0.5545	0.5484	0.5424	0.5363	0.5304	0.5244	0.5186	0.5127	0.5069
−0.1	0.5011	0.4954	0.4897	0.4840	0.4784	0.4728	0.4673	0.4618	0.4564	0.4509
0.0	0.4456	0.4402	0.4349	0.4297	0.4244	0.4193	0.4141	0.4090	0.4040	0.3989

(*continued*)

Standard Normal Loss Function Table, $L(z)$ (Concluded)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.3989	0.3940	0.3890	0.3841	0.3793	0.3744	0.3697	0.3649	0.3602	0.3556
0.1	0.3509	0.3464	0.3418	0.3373	0.3328	0.3284	0.3240	0.3197	0.3154	0.3111
0.2	0.3069	0.3027	0.2986	0.2944	0.2904	0.2863	0.2824	0.2784	0.2745	0.2706
0.3	0.2668	0.2630	0.2592	0.2555	0.2518	0.2481	0.2445	0.2409	0.2374	0.2339
0.4	0.2304	0.2270	0.2236	0.2203	0.2169	0.2137	0.2104	0.2072	0.2040	0.2009
0.5	0.1978	0.1947	0.1917	0.1887	0.1857	0.1828	0.1799	0.1771	0.1742	0.1714
0.6	0.1687	0.1659	0.1633	0.1606	0.1580	0.1554	0.1528	0.1503	0.1478	0.1453
0.7	0.1429	0.1405	0.1381	0.1358	0.1334	0.1312	0.1289	0.1267	0.1245	0.1223
0.8	0.1202	0.1181	0.1160	0.1140	0.1120	0.1100	0.1080	0.1061	0.1042	0.1023
0.9	0.1004	0.0986	0.0968	0.0950	0.0933	0.0916	0.0899	0.0882	0.0865	0.0849
1.0	0.0833	0.0817	0.0802	0.0787	0.0772	0.0757	0.0742	0.0728	0.0714	0.0700
1.1	0.0686	0.0673	0.0659	0.0646	0.0634	0.0621	0.0609	0.0596	0.0584	0.0573
1.2	0.0561	0.0550	0.0538	0.0527	0.0517	0.0506	0.0495	0.0485	0.0475	0.0465
1.3	0.0455	0.0446	0.0436	0.0427	0.0418	0.0409	0.0400	0.0392	0.0383	0.0375
1.4	0.0367	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321	0.0314	0.0307	0.0300
1.5	0.0293	0.0286	0.0280	0.0274	0.0267	0.0261	0.0255	0.0249	0.0244	0.0238
1.6	0.0232	0.0227	0.0222	0.0216	0.0211	0.0206	0.0201	0.0197	0.0192	0.0187
1.7	0.0183	0.0178	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146
1.8	0.0143	0.0139	0.0136	0.0132	0.0129	0.0126	0.0123	0.0119	0.0116	0.0113
1.9	0.0111	0.0108	0.0105	0.0102	0.0100	0.0097	0.0094	0.0092	0.0090	0.0087
2.0	0.0085	0.0083	0.0080	0.0078	0.0076	0.0074	0.0072	0.0070	0.0068	0.0066
2.1	0.0065	0.0063	0.0061	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050
2.2	0.0049	0.0047	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039	0.0038
2.3	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
2.4	0.0027	0.0026	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
2.5	0.0020	0.0019	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015
2.6	0.0015	0.0014	0.0014	0.0013	0.0013	0.0012	0.0012	0.0012	0.0011	0.0011
2.7	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
2.8	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
2.9	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004
3.0	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
3.1	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.2	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
3.3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.4	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.5	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Poisson Distribution Function Table

	Mean									
S	**0.05**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**
0	0.95123	0.90484	0.86071	0.81873	0.77880	0.74082	0.70469	0.67032	0.63763	0.60653
1	0.99879	0.99532	0.98981	0.98248	0.97350	0.96306	0.95133	0.93845	0.92456	0.90980
2	0.99998	0.99985	0.99950	0.99885	0.99784	0.99640	0.99449	0.99207	0.98912	0.98561
3	1.00000	1.00000	0.99998	0.99994	0.99987	0.99973	0.99953	0.99922	0.99880	0.99825
4	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99997	0.99994	0.99989	0.99983
5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999
6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

	Mean									
S	**0.55**	**0.60**	**0.65**	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**	**0.95**	**1.00**
0	0.57695	0.54881	0.52205	0.49659	0.47237	0.44933	0.42741	0.40657	0.38674	0.36788
1	0.89427	0.87810	0.86138	0.84420	0.82664	0.80879	0.79072	0.77248	0.75414	0.73576
2	0.98154	0.97688	0.97166	0.96586	0.95949	0.95258	0.94512	0.93714	0.92866	0.91970
3	0.99753	0.99664	0.99555	0.99425	0.99271	0.99092	0.98887	0.98654	0.98393	0.98101
4	0.99973	0.99961	0.99944	0.99921	0.99894	0.99859	0.99817	0.99766	0.99705	0.99634
5	0.99998	0.99996	0.99994	0.99991	0.99987	0.99982	0.99975	0.99966	0.99954	0.99941
6	1.00000	1.00000	0.99999	0.99999	0.99999	0.99998	0.99997	0.99996	0.99994	0.99992
7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999
8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

	Mean									
S	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**	**2.50**	**2.75**	**3.00**	**3.25**	**3.50**
0	0.28650	0.22313	0.17377	0.13534	0.10540	0.08208	0.06393	0.04979	0.03877	0.03020
1	0.64464	0.55783	0.47788	0.40601	0.34255	0.28730	0.23973	0.19915	0.16479	0.13589
2	0.86847	0.80885	0.74397	0.67668	0.60934	0.54381	0.48146	0.42319	0.36957	0.32085
3	0.96173	0.93436	0.89919	0.85712	0.80943	0.75758	0.70304	0.64723	0.59141	0.53663
4	0.99088	0.98142	0.96710	0.94735	0.92199	0.89118	0.85538	0.81526	0.77165	0.72544
5	0.99816	0.99554	0.99087	0.98344	0.97263	0.95798	0.93916	0.91608	0.88881	0.85761
6	0.99968	0.99907	0.99780	0.99547	0.99163	0.98581	0.97757	0.96649	0.95227	0.93471
7	0.99995	0.99983	0.99953	0.99890	0.99773	0.99575	0.99265	0.98810	0.98174	0.97326
8	0.99999	0.99997	0.99991	0.99976	0.99945	0.99886	0.99784	0.99620	0.99371	0.99013
9	1.00000	1.00000	0.99998	0.99995	0.99988	0.99972	0.99942	0.99890	0.99803	0.99669
10	1.00000	1.00000	1.00000	0.99999	0.99998	0.99994	0.99986	0.99971	0.99944	0.99898
11	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99997	0.99993	0.99985	0.99971
12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998	0.99996	0.99992
13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99998
14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

(*continued*)

Poisson Distribution Function Table (Concluded)

	Mean											
S	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50
0	0.02352	0.01832	0.01426	0.01111	0.00865	0.00674	0.00525	0.00409	0.00318	0.00248	0.00193	0.00150
1	0.11171	0.09158	0.07489	0.06110	0.04975	0.04043	0.03280	0.02656	0.02148	0.01735	0.01400	0.01128
2	0.27707	0.23810	0.20371	0.17358	0.14735	0.12465	0.10511	0.08838	0.07410	0.06197	0.05170	0.04304
3	0.48377	0.43347	0.38621	0.34230	0.30189	0.26503	0.23167	0.20170	0.17495	0.15120	0.13025	0.11185
4	0.67755	0.62884	0.58012	0.53210	0.48540	0.44049	0.39777	0.35752	0.31991	0.28506	0.25299	0.22367
5	0.82288	0.78513	0.74494	0.70293	0.65973	0.61596	0.57218	0.52892	0.48662	0.44568	0.40640	0.36904
6	0.91372	0.88933	0.86169	0.83105	0.79775	0.76218	0.72479	0.68604	0.64639	0.60630	0.56622	0.52652
7	0.96238	0.94887	0.93257	0.91341	0.89140	0.86663	0.83925	0.80949	0.77762	0.74398	0.70890	0.67276
8	0.98519	0.97864	0.97023	0.95974	0.94701	0.93191	0.91436	0.89436	0.87195	0.84724	0.82038	0.79157
9	0.99469	0.99187	0.98801	0.98291	0.97636	0.96817	0.95817	0.94622	0.93221	0.91608	0.89779	0.87738
10	0.99826	0.99716	0.99557	0.99333	0.99030	0.98630	0.98118	0.97475	0.96686	0.95738	0.94618	0.93316
11	0.99947	0.99908	0.99849	0.99760	0.99632	0.99455	0.99216	0.98901	0.98498	0.97991	0.97367	0.96612
12	0.99985	0.99973	0.99952	0.99919	0.99870	0.99798	0.99696	0.99555	0.99366	0.99117	0.98798	0.98397
13	0.99996	0.99992	0.99986	0.99975	0.99957	0.99930	0.99890	0.99831	0.99749	0.99637	0.99487	0.99290
14	0.99999	0.99998	0.99996	0.99993	0.99987	0.99977	0.99963	0.99940	0.99907	0.99860	0.99794	0.99704
15	1.00000	1.00000	0.99999	0.99998	0.99996	0.99993	0.99988	0.99980	0.99968	0.99949	0.99922	0.99884
16	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99996	0.99994	0.99989	0.99983	0.99972	0.99957
17	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99994	0.99991	0.99985
18	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99995
19	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998

	Mean											
S	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50
0	0.00117	0.00091	0.00071	0.00055	0.00043	0.00034	0.00026	0.00020	0.00016	0.00012	0.00010	0.00007
1	0.00907	0.00730	0.00586	0.00470	0.00377	0.00302	0.00242	0.00193	0.00154	0.00123	0.00099	0.00079
2	0.03575	0.02964	0.02452	0.02026	0.01670	0.01375	0.01131	0.00928	0.00761	0.00623	0.00510	0.00416
3	0.09577	0.08177	0.06963	0.05915	0.05012	0.04238	0.03576	0.03011	0.02530	0.02123	0.01777	0.01486
4	0.19704	0.17299	0.15138	0.13206	0.11487	0.09963	0.08619	0.07436	0.06401	0.05496	0.04709	0.04026
5	0.33377	0.30071	0.26992	0.24144	0.21522	0.19124	0.16939	0.14960	0.13174	0.11569	0.10133	0.08853
6	0.48759	0.44971	0.41316	0.37815	0.34485	0.31337	0.28380	0.25618	0.23051	0.20678	0.18495	0.16495
7	0.63591	0.59871	0.56152	0.52464	0.48837	0.45296	0.41864	0.38560	0.35398	0.32390	0.29544	0.26866
8	0.76106	0.72909	0.69596	0.66197	0.62740	0.59255	0.55770	0.52311	0.48902	0.45565	0.42320	0.39182
9	0.85492	0.83050	0.80427	0.77641	0.74712	0.71662	0.68516	0.65297	0.62031	0.58741	0.55451	0.52183
10	0.91827	0.90148	0.88279	0.86224	0.83990	0.81589	0.79032	0.76336	0.73519	0.70599	0.67597	0.64533
11	0.95715	0.94665	0.93454	0.92076	0.90527	0.88808	0.86919	0.84866	0.82657	0.80301	0.77810	0.75199
12	0.97902	0.97300	0.96581	0.95733	0.94749	0.93620	0.92341	0.90908	0.89320	0.87577	0.85683	0.83643
13	0.99037	0.98719	0.98324	0.97844	0.97266	0.96582	0.95782	0.94859	0.93805	0.92615	0.91285	0.89814
14	0.99585	0.99428	0.99227	0.98974	0.98659	0.98274	0.97810	0.97257	0.96608	0.95853	0.94986	0.94001
15	0.99831	0.99759	0.99664	0.99539	0.99379	0.99177	0.98925	0.98617	0.98243	0.97796	0.97269	0.96653
16	0.99935	0.99904	0.99862	0.99804	0.99728	0.99628	0.99500	0.99339	0.99137	0.98889	0.98588	0.98227
17	0.99976	0.99964	0.99946	0.99921	0.99887	0.99841	0.99779	0.99700	0.99597	0.99468	0.99306	0.99107
18	0.99992	0.99987	0.99980	0.99970	0.99955	0.99935	0.99907	0.99870	0.99821	0.99757	0.99675	0.99572
19	0.99997	0.99996	0.99993	0.99989	0.99983	0.99975	0.99963	0.99947	0.99924	0.99894	0.99855	0.99804
20	0.99999	0.99999	0.99998	0.99996	0.99994	0.99991	0.99986	0.99979	0.99969	0.99956	0.99938	0.99914
21	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99995	0.99992	0.99988	0.99983	0.99975	0.99964
22	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99997	0.99996	0.99993	0.99990	0.99985
23	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99998	0.99998	0.99996	0.99994
24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.99999	0.99999	0.99999	0.99998

Poisson Loss Function Table

	Mean									
S	**0.05**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**
0	0.05000	0.10000	0.15000	0.20000	0.25000	0.30000	0.35000	0.40000	0.45000	0.50000
1	0.00123	0.00484	0.01071	0.01873	0.02880	0.04082	0.05469	0.07032	0.08763	0.10653
2	0.00002	0.00016	0.00052	0.00121	0.00230	0.00388	0.00602	0.00877	0.01219	0.01633
3	0.00000	0.00000	0.00002	0.00006	0.00014	0.00028	0.00051	0.00084	0.00131	0.00194
4	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002	0.00003	0.00007	0.00011	0.00019
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

	Mean									
S	**0.55**	**0.60**	**0.65**	**0.70**	**0.75**	**0.80**	**0.85**	**0.90**	**0.95**	**1.00**
0	0.55000	0.60000	0.65000	0.70000	0.75000	0.80000	0.85000	0.90000	0.95000	1.00000
1	0.12695	0.14881	0.17205	0.19659	0.22237	0.24933	0.27741	0.30657	0.33674	0.36788
2	0.02122	0.02691	0.03342	0.04078	0.04901	0.05812	0.06813	0.07905	0.09089	0.10364
3	0.00276	0.00379	0.00508	0.00664	0.00850	0.01070	0.01325	0.01620	0.01955	0.02334
4	0.00029	0.00044	0.00063	0.00089	0.00121	0.00162	0.00212	0.00274	0.00347	0.00435
5	0.00003	0.00004	0.00007	0.00010	0.00015	0.00021	0.00029	0.00039	0.00052	0.00069
6	0.00000	0.00000	0.00001	0.00001	0.00002	0.00002	0.00003	0.00005	0.00007	0.00009
7	0.00000	0.00000	0.00001	0.00001	0.00002	0.00002	0.00003	0.00005	0.00007	0.00009
8	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00003	0.00004	0.00006	0.00008

	Mean									
S	**1.25**	**1.50**	**1.75**	**2.00**	**2.25**	**2.50**	**2.75**	**3.00**	**3.25**	**3.50**
0	1.25000	1.50000	1.75000	2.00000	2.25000	2.50000	2.75000	3.00000	3.25000	3.50000
1	0.53650	0.72313	0.92377	1.13534	1.35540	1.58208	1.81393	2.04979	2.28877	2.53020
2	0.18114	0.28096	0.40165	0.54134	0.69795	0.86938	1.05366	1.24894	1.45356	1.66609
3	0.04961	0.08980	0.14562	0.21802	0.30729	0.41320	0.53511	0.67213	0.82313	0.98693
4	0.01134	0.02416	0.04481	0.07514	0.11672	0.17077	0.23815	0.31936	0.41454	0.52357
5	0.00221	0.00558	0.01191	0.02249	0.03870	0.06195	0.09353	0.13462	0.18619	0.24901
6	0.00038	0.00113	0.00278	0.00592	0.01134	0.01993	0.03270	0.05070	0.07501	0.10662
7	0.00006	0.00020	0.00058	0.00139	0.00297	0.00574	0.01026	0.01719	0.02728	0.04134
8	0.00001	0.00003	0.00011	0.00029	0.00070	0.00149	0.00292	0.00529	0.00902	0.01460
9	0.00000	0.00000	0.00002	0.00006	0.00015	0.00035	0.00076	0.00149	0.00273	0.00472
10	0.00000	0.00000	0.00000	0.00001	0.00003	0.00008	0.00018	0.00038	0.00076	0.00141
11	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002	0.00004	0.00009	0.00020	0.00039
12	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002	0.00005	0.00010
13	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002
14	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001
15	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

(*continued*)

Poisson Loss Function Table (Concluded)

	Mean											
S	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50
0	3.75000	4.00000	4.25000	4.50000	4.75000	5.00000	5.25000	5.50000	5.75000	6.00000	6.25000	6.50000
1	2.77352	3.01832	3.26426	3.51111	3.75865	4.00674	4.25525	4.50409	4.75318	5.00248	5.25193	5.50150
2	1.88523	2.10989	2.33915	2.57221	2.80840	3.04717	3.28804	3.53065	3.77467	4.01983	4.26593	4.51278
3	1.16230	1.34800	1.54286	1.74579	1.95575	2.17182	2.39316	2.61903	2.84877	3.08180	3.31763	3.55582
4	0.64606	0.78147	0.92907	1.08808	1.25763	1.43684	1.62483	1.82073	2.02371	2.23300	2.44788	2.66766
5	0.32361	0.41030	0.50919	0.62019	0.74303	0.87734	1.02260	1.17824	1.34362	1.51806	1.70086	1.89134
6	0.14649	0.19543	0.25413	0.32312	0.40277	0.49330	0.59479	0.70716	0.83024	0.96374	1.10727	1.26038
7	0.06021	0.08476	0.11582	0.15417	0.20052	0.25548	0.31958	0.39320	0.47663	0.57004	0.67348	0.78690
8	0.02259	0.03363	0.04839	0.06758	0.09192	0.12211	0.15882	0.20268	0.25426	0.31402	0.38238	0.45966
9	0.00778	0.01226	0.01861	0.02732	0.03893	0.05402	0.07318	0.09704	0.12620	0.16126	0.20276	0.25123
10	0.00247	0.00413	0.00662	0.01023	0.01529	0.02219	0.03136	0.04326	0.05842	0.07733	0.10056	0.12862
11	0.00073	0.00129	0.00219	0.00356	0.00559	0.00849	0.01253	0.01801	0.02528	0.03471	0.04673	0.06178
12	0.00020	0.00038	0.00067	0.00116	0.00191	0.00304	0.00469	0.00702	0.01026	0.01462	0.02040	0.02790
13	0.00005	0.00010	0.00019	0.00035	0.00061	0.00102	0.00165	0.00257	0.00391	0.00579	0.00838	0.01187
14	0.00001	0.00003	0.00005	0.00010	0.00018	0.00032	0.00054	0.00089	0.00141	0.00217	0.00325	0.00477
15	0.00000	0.00001	0.00001	0.00003	0.00005	0.00010	0.00017	0.00029	0.00048	0.00077	0.00119	0.00181
16	0.00000	0.00000	0.00000	0.00001	0.00001	0.00003	0.00005	0.00009	0.00015	0.00026	0.00042	0.00066
17	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00003	0.00005	0.00008	0.00014	0.00022
18	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00004	0.00007
19	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002

	Mean											
S	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50
0	6.75000	7.00000	7.25000	7.50000	7.75000	8.00000	8.25000	8.50000	8.75000	9.00000	9.25000	9.50000
1	5.75117	6.00091	6.25071	6.50055	6.75043	7.00034	7.25026	7.50020	7.75016	8.00012	8.25010	8.50007
2	4.76025	5.00821	5.25657	5.50525	5.75420	6.00335	6.25268	6.50214	6.75170	7.00136	7.25108	7.50086
3	3.79599	4.03784	4.28109	4.52551	4.77090	5.01711	5.26399	5.51142	5.75931	6.00759	6.25618	6.50502
4	2.89176	3.11961	3.35072	3.58466	3.82103	4.05949	4.29974	4.54153	4.78462	5.02882	5.27395	5.51988
5	2.08880	2.29260	2.50210	2.71672	2.93589	3.15912	3.38593	3.61589	3.84863	4.08378	4.32105	4.56015
6	1.42257	1.59331	1.77203	1.95815	2.15112	2.35036	2.55532	2.76549	2.98036	3.19947	3.42238	3.64868
7	0.91016	1.04302	1.18519	1.33631	1.49597	1.66373	1.83912	2.02167	2.21087	2.40625	2.60732	2.81362
8	0.54606	0.64173	0.74671	0.86095	0.98434	1.11669	1.25777	1.40726	1.56485	1.73015	1.90277	2.08229
9	0.30712	0.37082	0.44267	0.52292	0.61174	0.70924	0.81546	0.93037	1.05387	1.18580	1.32597	1.47411
10	0.16204	0.20132	0.24694	0.29932	0.35885	0.42586	0.50062	0.58334	0.67418	0.77321	0.88047	0.99594
11	0.08031	0.10280	0.12973	0.16156	0.19876	0.24175	0.29094	0.34671	0.40936	0.47920	0.55644	0.64127
12	0.03746	0.04945	0.06427	0.08232	0.10403	0.12983	0.16013	0.19537	0.23593	0.28221	0.33454	0.39326
13	0.01648	0.02245	0.03007	0.03965	0.05152	0.06603	0.08354	0.10445	0.12913	0.15798	0.19137	0.22968
14	0.00685	0.00964	0.01332	0.01809	0.02418	0.03185	0.04137	0.05304	0.06718	0.08413	0.10422	0.12782
15	0.00270	0.00392	0.00559	0.00783	0.01077	0.01459	0.01947	0.02561	0.03326	0.04266	0.05409	0.06783
16	0.00101	0.00152	0.00223	0.00322	0.00456	0.00636	0.00872	0.01178	0.01569	0.02063	0.02678	0.03436
17	0.00036	0.00056	0.00085	0.00126	0.00184	0.00264	0.00372	0.00517	0.00706	0.00952	0.01266	0.01663
18	0.00012	0.00020	0.00031	0.00047	0.00071	0.00105	0.00152	0.00217	0.00304	0.00420	0.00573	0.00770
19	0.00004	0.00007	0.00011	0.00017	0.00026	0.00040	0.00059	0.00087	0.00125	0.00177	0.00248	0.00342
20	0.00001	0.00002	0.00004	0.00006	0.00009	0.00014	0.00022	0.00033	0.00049	0.00072	0.00103	0.00145
21	0.00000	0.00001	0.00001	0.00002	0.00003	0.00005	0.00008	0.00012	0.00019	0.00028	0.00041	0.00059
22	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00003	0.00004	0.00007	0.00010	0.00016	0.00023
23	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00004	0.00006	0.00009
24	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00003

Appendix C

Evaluation of the Loss Function

The loss function $L(Q)$ is the expected amount a random variable exceeds a fixed value. For example, if the random variable is demand, then $L(Q)$ is the expected amount demand is greater than Q. See Appendix A, Statistics Tutorial, for a more extensive description of the loss function.

This appendix describes how the loss function of a discrete distribution function can be efficiently evaluated. (Appendix A gives one solution method, but it is inefficient.) If you need to evaluate the loss function of a continuous distribution, then convert the continuous distribution into a discrete distribution by "chopping it up" into many pieces. For example, the standard normal table is the discrete (i.e., "chopped up") version of the continuous standard normal distribution function.

Let N be the number of quantities in the distribution function and let $Q_1, Q_2, Q_3, \ldots, Q_N$ be those quantities. For example, take the empirical distribution function in Chapter 12, repeated here for convenience:

Q	$F(Q)$	Q	$F(Q)$	Q	$F(Q)$
800	0.0303	2,592	0.3636	3,936	0.6970
1,184	0.0606	2,624	0.3939	4,000	0.7273
1,792	0.0909	2,752	0.4242	4,064	0.7576
1,792	0.1212	3,040	0.4545	4,160	0.7879
1,824	0.1515	3,104	0.4848	4,352	0.8182
1,888	0.1818	3,136	0.5152	4,544	0.8485
2,048	0.2121	3,264	0.5455	4,672	0.8788
2,144	0.2424	3,456	0.5758	4,800	0.9091
2,208	0.2727	3,680	0.6061	4,928	0.9394
2,304	0.3030	3,744	0.6364	4,992	0.9697
2,560	0.3333	3,808	0.6667	5,120	1.0000

$F(Q)$ = Probability demand is less than or equal to the quantity Q

With this distribution function, there are 33 quantities, so $N = 33$ and $Q_1 = 800$, $Q_2 = 1{,}184, \ldots,$ and $Q_{33} = 5{,}120$. Furthermore, recall that we use μ to represent expected demand, which in this case is $\mu = 3{,}192$.

We can recursively evaluate the loss function, which means we start with $L(Q_1)$ and then use $L(Q_1)$ to evaluate $L(Q_2)$, and then use $L(Q_2)$ to evaluate $L(Q_3)$, and so forth.

The expected lost sales if we order Q_1 (which in this case is 800 units) are

$$L(Q_1) = \mu - Q_1 = 3{,}192 - 800 = 2{,}392$$

Expected lost sales if we order Q_2 are

$$\begin{aligned} L(Q_2) &= L(Q_1) - (Q_2 - Q_1) \times (1 - F(Q_1)) \\ &= 2{,}392 - (1{,}184 - 800) \times (1 - 0.0303) \\ &= 2{,}020 \end{aligned}$$

Expected lost sales if we order Q_3 are

$$\begin{aligned} L(Q_3) &= L(Q_2) - (Q_3 - Q_2) \times (1 - F(Q_2)) \\ &= 2{,}020 - (1{,}792 - 1{,}184) \times (1 - 0.0606) \\ &= 1{,}448 \end{aligned}$$

In general, the ith expected lost sales are

$$L(Q_i) = L(Q_{i-1}) - (Q_i - Q_{i-1}) \times (1 - F(Q_{i-1}))$$

So you start with $L(Q_1) = \mu - Q_1$ and then you evaluate $L(Q_2)$, and then $L(Q_3)$, up to $L(Q_N)$. The resulting table is

Q	$F(Q)$	$L(Q)$	Q	$F(Q)$	$L(Q)$	Q	$F(Q)$	$L(Q)$
800	0.0303	2,392	2,592	0.3636	841	3,936	0.6970	191
1,184	0.0606	2,020	2,624	0.3939	821	4,000	0.7273	171
1,792	0.0909	1,448	2,752	0.4242	744	4,064	0.7576	154
1,792	0.1212	1,448	3,040	0.4545	578	4,160	0.7879	131
1,824	0.1515	1,420	3,104	0.4848	543	4,352	0.8182	90
1,888	0.1818	1,366	3,136	0.5152	526	4,544	0.8485	55
2,048	0.2121	1,235	3,264	0.5455	464	4,672	0.8788	36
2,144	0.2424	1,160	3,456	0.5758	377	4,800	0.9091	20
2,208	0.2727	1,111	3,680	0.6061	282	4,928	0.9394	8
2,304	0.3030	1,041	3,744	0.6364	257	4,992	0.9697	5
2,560	0.3333	863	3,808	0.6667	233	5,120	1.0000	1

Q = Order quantity
$F(Q)$ = Probability demand is less than or equal to the order quantity
$L(Q)$ = Loss function (the expected amount demand exceeds Q)

With this empirical distribution example, the quantities differ by more than one unit, for example, $Q_2 - Q_1 = 384$. Now suppose the demand forecast is the Poisson distribution with mean 1.25. The distribution function is given in Table A.1 but is repeated here for convenience:

Q	$f(Q)$	$F(Q)$
0	0.28650	0.28650
1	0.35813	0.64464
2	0.22383	0.86847
3	0.09326	0.96173
4	0.02914	0.99088
5	0.00729	0.99816
6	0.00152	0.99968
7	0.00027	0.99995
8	0.00004	0.99999
9	0.00001	1.00000

Now we have $Q_1 = 0$, $Q_2 = 1$, and so forth. We find the expected lost sales with the same process: $L(Q_1) = 1.25 - 0 = 1.25$ and

$$\begin{aligned} L(Q_2) &= L(Q_1) - (Q_2 - Q_1) \times (1 - F(Q_1)) \\ &= 0.53650 - (2 - 1) \times (1 - 0.64469) \\ &= 0.18114 \end{aligned}$$

Completing the table yields

Q	$f(Q)$	$F(Q)$	$L(Q)$
0	0.28650	0.28650	1.25000
1	0.35813	0.64464	0.53650
2	0.22383	0.86847	0.18114
3	0.09326	0.96173	0.04961
4	0.02914	0.99088	0.01134
5	0.00729	0.99816	0.00221
6	0.00152	0.99968	0.00038
7	0.00027	0.99995	0.00006
8	0.00004	0.99999	0.00001
9	0.00001	1.00000	0.00000

Appendix D

Equations and Approximations

This appendix derives in detail some equations and explains several approximations.

Derivation, via Calculus, of the Order Quantity That Maximizes Expected Profit for the Newsvendor (Chapter 12)

Let the selling price be p, the purchase cost per unit be c, and the salvage revenue from leftover inventory be v. The expected profit function is

$$\pi(Q) = -cQ + p\left(\int_0^Q xf(x)dx + (1 - F(Q))Q\right) + v\int_0^Q (Q - x)f(x)dx$$

$$= (p - c)Q + \int_0^Q (p - v)xf(x)dx - (p - v)F(Q)Q$$

where $f(x)$ is the density function and $F(x)$ is the distribution function ($Prob(D = x)$ and $Prob(D \leq x)$, respectively, where D is the random variable representing demand).

Via integration by parts, the profit function can be written as

$$\pi(Q) = (p - c)Q + (p - v)\left(QF(Q) - \int_0^Q F(x)dx\right) - (p - v)F(Q)Q$$

Differentiate the profit function and remember that the derivative of the distribution function equals the density function, that is, $dF(x)/dx = f(x)$

$$\frac{d\pi(Q)}{dQ} = (p - c) + (p - v)(F(Q) + Qf(Q) - F(Q)) - (p - v)(F(Q) + f(Q)Q)$$

$$= (p - c) - (p - v)F(Q)$$

and

$$\frac{d^2\pi(Q)}{dQ^2} = -(p - v)f(Q)$$

Because the second derivative is negative, the profit function is concave, so the solution to the first-order condition provides the optimal order quantity:

$$\frac{d\pi(Q)}{dQ} = (p - c) - (p - v)F(Q) = 0$$

Rearrange terms in the above equation and you get

$$F(Q) = \frac{p - c}{p - v}$$

Note that $C_o = c - v$ and $C_u = p - c$, so the above can be written as

$$F(Q) = \frac{C_u}{C_u + C_o}$$

The Round-up Rule (Chapter 12)

To understand why the round-up rule is correct, we need to derive the optimal order quantity with a discrete distribution function. Suppose demand will be one of a finite set of outcomes, $D \in \{d_1, d_2, \ldots, d_n\}$. For example, with the empirical distribution function for the Hammer 3/2, the possible demand outcomes included {800, 1,184, . . . , 5,120}. Clearly, the optimal order quantity will equal one of these possible demand outcomes. Suppose we have decided to order d_i units and we are deciding whether to order d_{i+1} units. This is prudent if the expected gain from this larger order quantity is at least as large as the expected cost. The expected gain is

$$C_u(d_{i+1} - d_i)\big(1 - F(d_i)\big)$$

because we sell an additional $(d_{i+1} - d_i)$ units if demand is greater than d_i, which occurs with probability $1 - F(d_i)$. The expected loss is

$$C_o(d_{i+1} - d_i)F(d_i)$$

because we need to salvage an additional $(d_{i+1} - d_i)$ units if demand is d_i or fewer, which occurs with probability $F(d_i)$. So we should increase our order from d_i to d_{i+1} when

$$C_u(d_{i+1} - d_i)\big(1 - F(d_i)\big) \geq C_o(d_{i+1} - d_i)F(d_i)$$

which simplifies to

$$\frac{C_u}{C_o + C_u} \geq F(d_i)$$

Thus, if the critical ratio is greater than $F(d_i)$, then we should increase our order from d_i to d_{i+1}. When the critical ratio is greater than $F(d_i)$ but less than $F(d_{i+1})$, in other words, between the

two entries in the table, we should order d_{i+1} units and not increase our order quantity further. Put another way, we choose the larger order quantity when the critical ratio falls between two entries in the table. That is the round-up rule.

The common error is to want to choose the order quantity that yields $F()$ closest to the critical ratio. But that can lead to a suboptimal action. To illustrate, suppose demand was Poisson with mean 1.0, $C_u = 1$, and $C_o = 0.21$. The critical ratio is 0.83, which is about in the middle between $F(1) = 0.74$ and $F(2) = 0.92$. However, expected profit with an order quantity of two units is about 20 percent higher than the profit with an order quantity of one unit. That said, if $F(d_i)$ and $F(d_{i+1})$ are reasonably close together, then choosing the lower order quantity is not going to cause a significant profit loss.

Derivation of the Standard Normal Loss Function (Chapter 12)

We wish to derive the following equation for the standard normal loss function:

$$L(z) = \phi(z) - z(1 - \Phi(z))$$

Begin with the density function of the standard normal distribution,

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

and differentiate

$$\frac{d\phi(z)}{dz} = -z\frac{1}{\sqrt{2\pi}} e^{-z^2/2} = -z\phi(z)$$

Let $L(z)$ be the expected loss function:v

$$\begin{aligned} L(z) &= \int_z^\infty (x - z)\phi(x)dx \\ &= \int_z^\infty x\phi(x)dx - \int_z^\infty z\phi(x)dx \end{aligned}$$

The first integral is

$$\int_z^\infty x\phi(x)dx = -\phi(x) \Big|_z^\infty = \phi(z)$$

because $d\phi(x)/dx = -x\phi(x)$ and the second integral is

$$\int_z^\infty z\phi(x)dx = z(1 - \Phi(z))$$

Thus, $L(z) = \phi(z) - z(1 - \Phi(z))$.

Evaluation of the Fill Rate (Chapter 12)

The fill rate is the probability a customer finds an item available for purchase. This is not the same as the in-stock probability, which is the probability that all demand is satisfied. (To see why, suppose 9 units are available, but 10 customers arrive to make a purchase. The firm is not in-stock, because there will be one person who is unable to purchase a unit. However, each customer has a 9 out of 10 chance to be one of the lucky customers that can purchase an item.)

The fill rate can be evaluated with the following formula:

$$\text{Fill rate} = \frac{\text{Expected sales}}{\text{Expected demand}} = \frac{\text{Expected sales}}{\mu}$$

For example, if O'Neill orders 3,500 Hammer 3/2 wetsuits, then we evaluated in the Chapter 12 that their Expected sales = 2,858. Expected demand is 3,192, so the fill rate would be

$$\text{Fill rate} = \frac{2{,}858}{3{,}192} = 89.5\%$$

Mismatch Cost as a Percentage of the Maximum Profit (Chapter 13)

We will use the following notation:

μ	= Expected demand
σ	= Standard deviation of demand
Q	= Expected profit-maximizing order quantity
$z = (Q-\mu)/\sigma$	= Normalized order quantity
$\phi(z)$	= Density function of the standard normal distribution
$\Phi(z)$	= Distribution function of the standard normal

The easiest way to evaluate $\phi(z)$ is to use the Excel function Normdist(z,0,1,0), but it also can be evaluated by hand with the following function:

$$\phi(z) = e^{-(1/2)\times z^2}/\sqrt{2 \times \pi}$$

Begin with the mismatch cost as a percentage of the maximum profit

$$\begin{aligned}\text{Mismatch cost as a \% of the maximum profit} &= (C_o \times \text{Expected leftover inventory})/(\mu \times C_u) \\ &\quad + (C_u \times \text{Expected lost sales}) \;/\; (\mu \times C_u)\end{aligned} \qquad \textbf{(D.1)}$$

We also know the following:

$$\begin{aligned}\text{Expected leftover inventory} &= (Q - \text{Expected sales}) \\ &= (Q - \mu + \text{Expected lost sales})\end{aligned} \qquad \textbf{(D.2)}$$

and we can rearrange $Q = \mu + z \times \sigma$ into

$$z \times \sigma = (Q - \mu) \qquad \textbf{(D.3)}$$

Substitute equation (D.3) into equation (D.2), then substitute that equation into equation (D.1) and simplify:

$$\begin{array}{c}\text{Mismatch cost as a \% of the} \\ \text{maximum profit}\end{array} = ((C_o \times z \times \sigma) + (C_o + C_u) \times \text{Expected lost sales})/(\mu \times C_u \quad \textbf{(D.4)}$$

Recall that

$$\begin{aligned}\text{Expected lost sales} &= \sigma \times (\phi(z) - z \times (1 - \Phi(z))) \\ &= \sigma \times \left(\phi(z) - z \times \frac{C_o}{C_o + C_u}\right) \qquad \textbf{(D.5)}\end{aligned}$$

where the second line in that equation follows from the critical ratio, $\Phi(z) = C_u/(C_o + C_u)$. Substitute equation (D.5) into equation (D.4) and simplify to obtain equation (13.2):

$$\begin{array}{c}\text{Mismatch cost as a \% of the} \\ \text{maximum profit}\end{array} = \left(\frac{\phi(z)}{\Phi(z)}\right) \times \left(\frac{\sigma}{\mu}\right)$$

The above equation is composed of two terms, $\phi(z)/\Phi(z)$ and σ/μ, so the mismatch cost is high when the product of those two terms is high. The second term is the coefficient of variation, which we discussed in the text. The first term is the ratio of the standard normal density function to the standard normal distribution function evaluated at the normalized order quantity. It depends on z and z depends on the critical ratio (the higher the critical ratio, the higher the optimal z-statistic). In fact, a simple plot reveals that as the critical ratio increases, $\phi(z)/\Phi(z)$ decreases. Thus, the mismatch cost becomes smaller as the critical ratio increases. In other words, all else being equal, between two products, the product with the lower critical ratio has the higher mismatch cost.

Exact Stockout Probability for the Order-up-to Model (Chapter 14)

Recall our main result from Section 14.3 that the inventory level at the end of the period equals S minus demand over $l + 1$ periods. If the inventory level is negative at the end of that interval, then one or more units are back-ordered. A stockout occurs in the last period of that interval if there is at least one unit back-ordered and the most recent back order occurred in that last period. Equation (14.1) in Chapter 14 acknowledges the first part of that statement (at least one unit is back-ordered), but it ignores that second part (the most recent back order must occur in the last period).

For example, suppose $l = 1$ and $S = 2$. If demand over two periods is three units, then there is one unit back-ordered at the end of the second period. As long as one of those three units of demand occurred in the second period, then a stockout occurred in the second period. A stockout does not occur in the second period only if all three units of demand occurred in the first period. Hence, the exact equation for the stockout probability is

$$\begin{aligned}\text{Stockout probability} &= \text{Prob}\{\text{Demand over } l + 1 \text{ periods} > S\} \\ &\quad - \text{Prob}\{\text{Demand over } l \text{ periods} > S\} \\ &\quad \times \text{Prob}\{\text{Demand in one period} = 0\}\end{aligned}$$

Equation (14.1) is an approximation because it ignores the second term in the exact equation above. The second term is the probability that the demand over $l + 1$ periods occurs only in the first l periods; that is, there is no demand in the $(l + 1)$th period. If the service level is high, then the second term should be small. Notice that the approximation overestimates the true stockout probability because it does not subtract the second term. Hence, the approximation is conservative.

If each period's demand is a Poisson distribution with mean 0.29 and there is a two-period lead time, then the approximate and exact stockout probabilities are

	Stockout Probability	
S	Approximation	Exact
0	44.010%	25.174%
1	11.536	8.937
2	2.119	1.873
3	0.298	0.280
4	0.034	0.033
5	0.003	0.003
6	0.000	0.000

Fill Rate for the Order-up-to Model (Chapter 14)

The fill rate is the probability that a customer is able to purchase a unit immediately (i.e., the customer is not backordered). The fill rate can be evaluated with the following equation:

$$\text{Fill rate} = 1 - \frac{\text{Expected back order}}{\text{Expected demand in one period}}$$

The logic behind the above equation is as follows: The number of customers in a period is the expected demand in one period, and the number of customers who are not served in a period is the expected back order, so the ratio of the expected back order to the expected demand is the fraction of customers who are not served. One minus the fraction of customers who are not served is the fraction of customers who are served, which is the fill rate. Note that this logic does not depend on the particular demand distribution (but the evaluation of the expected back order does depend on the demand distribution).

You also might wonder why the denominator of the fraction in the fill rate equation is the expected demand over a single period and not the expected demand over $l + 1$ periods. We are interested in the fraction of customers who are not served immediately from stock (one minus that fraction is the expected fill rate). The lead time influences the fraction of customers in a period who are not served (the expected back order), but it does not influence the number of customers we have. Therefore, the lead time influences the numerator of that ratio (the number of customers who are not served) but not the denominator (the number of customers who arrive).

The above equation for the fill rate is actually an approximation of the fill rate. It happens to be an excellent approximation if the fill rate is reasonably high (say, 90 percent or higher). The advantage of that formula is that it is reasonably easy to work with. However, the remainder of this section derives the exact formula.

The fill rate is one minus the probability of not being served in a period, which is the following:

$$\text{Probability of not being served} = \frac{\text{Expected back orders that occur in a period}}{\text{Expected demand in one period}}$$

We know the denominator of that fraction, the expected demand in one period. We need to determine the numerator. The expected back orders that occur in a period are not quite the same as the expected back order in a period. The difference is that some of the back order might not have occurred in the period. (This is the same issue with the evaluation of the stockout probability.) For example, if the back order in a period is four units and demand in the period was three units, then only three of the four back orders actually occurred in that period; the remaining back-ordered unit was a carryover from a previous period.

Let's define some new notation. Let

$$B(l) = \text{Expected back orders if the lead time is } l$$

Hence, $B(l)$ is what we have been calling the *expected back order*.

The expected back order at the end of the $(l + 1)$th period of an interval of $l + 1$ periods is $B(l)$. If we subtract from those back orders the ones that were back-ordered at the end of the lth period in that interval, then we have the number of back orders that occurred in that last period of the interval. Hence,

$$\text{Probability of not being served} = \frac{B(l) - B(l-1)}{\text{Expected demand in one period}}$$

The numerator of the above fraction, in words, is the expected back order minus what the expected back order would be if the lead time were one period faster. Our exact fill rate equation is thus

$$\text{Expected fill rate} = 1 - \frac{\text{Expected back order} - B(l-1)}{\text{Expected demand in one period}}$$

The first fill rate equation presented in this section is an approximation because it does not subtract $B(l - 1)$ from the expected back order in the numerator. If the service level is very high, then $B(l - 1)$ will be very small, which is why the equation in the chapter is a good approximation.

If demand is Poisson with mean 0.29 per period and the lead time is one period, then

	Expected Fill Rate	
S	Approximation	Exact
0	−100.000%	0.000%
1	51.759	64.954
2	91.539	92.754
3	98.844	98.930
4	99.871	99.876
5	99.988	99.988
6	99.999	99.999

The approximation underestimates the fill rate, especially when the fill rate is low. However, the approximation is accurate for high fill rates.

Coordinating Buy-Back Price (Chapter 17)

If the wholesale price has been chosen, then we want to find the buy-back price that will lead the retailer to order the supply chain profit-maximizing quantity. This can be achieved if the retailer's critical ratio equals the supply chain's critical ratio because it is the critical ratio that determines the optimal order quantity.

Let's define some notation:

p = Retail price

c = Production cost

v = Retailer's salvage value

t = Shipping cost

w = wholesale price

b = buy-back price

The supply chain's critical ratio is $(p - c)/(p - v)$ because $C_u = p - c$ and $C_o = c - v$. The retailer's underage cost with the buy-back contract is $C_u = p - w$ and its overage cost is $C_o = t + w - b$ (i.e., the shipping cost plus the amount not credited by the supplier on returned inventory, $w - b$). Hence, the retailer's critical ratio equals the supply chain's critical ratio when

$$\frac{p - c}{p - v} = \frac{p - w}{(t + w - b) + p - w}$$

If we take the above equation and rearrange terms, we get equation (17.1).

Appendix E

Solutions to Selected Practice Problems

This appendix provides solutions to marked (*) practice problems.

Chapter 2

Q2.1 (Dell)

The following steps refer directly to Exhibit 2.1.

Step 1. For 2001, we find in Dell's 10-k: Inventory = \$400 (in millions)

Step 2. For 2001, we find in Dell's 10-k: COGS = \$26,442 (in millions)

Step 3. $\text{Inventory turns} = \frac{\$26,442/\text{Year}}{\$400} = 66.105 \text{ turns per year}$

Step 4. $\text{Per-unit inventory cost} = \frac{40\% \text{ per year}}{66.105 \text{ per year}} = 0.605 \text{ percent per unit}$

Chapter 3

Q3.1 (Single Flow Unit)

The following steps refer directly to Exhibit 3.1.

Step 1. We first compute the capacity of the three resources:

$$\text{Resource 1}: \frac{2}{10} \text{ unit per minute} = 0.2 \text{ unit per minute}$$

$$\text{Resource 2}: \frac{1}{6} \text{ unit per minute} = 0.1666 \text{ unit per minute}$$

$$\text{Resource 3}: \frac{3}{16} \text{ unit per minute} = 0.1875 \text{ unit per minute}$$

Step 2. Resource 2 has the lowest capacity; process capacity therefore is 0.1666 unit per minute, which is equal to 10 units per hour.

Step 3.Flow rate = Min{Process capacity, Demand}
= Min{8 units per hour, 10 units per hour} = 8 units per hour

This is equal to 0.1333 unit per minute.

Step 4. We find the utilizations of the three resources as

Resource 1: 0.1333 unit per minute/0.2 unit per minute = 66.66 percent
Resource 2: 0.1333 unit per minute/0.1666 unit per minute = 80 percent
Resource 3: 0.1333 unit per minute/0.1875 unit per minute = 71.11 percent

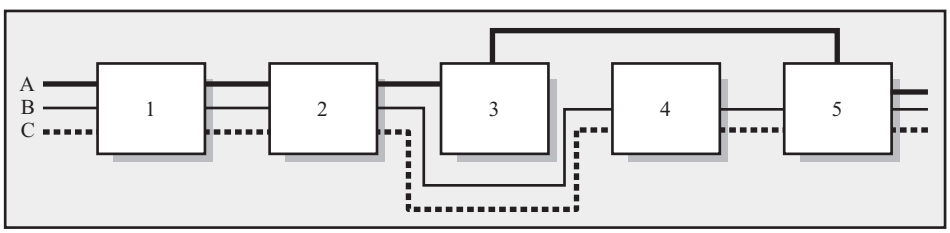
Q3.2 (Multiple Flow Units)

The following steps refer directly to Exhibit 3.2.

Step 1. Each resource can contribute the following capacity (in minutes of work per day):

Resource	Number of Workers	Minutes per Day
1	2	$2 \times 8 \times 60 = 960$
2	2	$2 \times 8 \times 60 = 960$
3	1	$1 \times 8 \times 60 = 480$
4	1	$1 \times 8 \times 60 = 480$
5	2	$2 \times 8 \times 60 = 960$

Step 2. Process flow diagram:



Step 3. We create a table indicating how much capacity will be consumed by the three products at the resources.

Resource	Capacity Requirement from A	Capacity Requirement from B	Capacity Requirement from C
1	$5 \times 40 = 200$	$5 \times 50 = 250$	$5 \times 60 = 300$
2	$3 \times 40 = 120$	$4 \times 50 = 200$	$5 \times 60 = 300$
3	$15 \times 40 = 600$	$0 \times 50 = 0$	$0 \times 60 = 0$
4	$0 \times 40 = 0$	$3 \times 50 = 150$	$3 \times 60 = 180$
5	$6 \times 40 = 240$	$6 \times 50 = 300$	$6 \times 60 = 360$

Step 4. Add up the rows to get the workload for each resource:

Workload for resource 1: $200 + 250 + 300 = 750$
Workload for resource 2: $120 + 200 + 300 = 620$
Workload for resource 3: $600 + 0 + 0 = 600$
Workload for resource 4: $0 + 150 + 180 = 330$
Workload for resource 5: $240 + 300 + 360 = 900$

Resource	Minutes per Day (see Step 1)	Workload per Day (see Step 4)	Implied Utilization (Step 4/Step 1)
1	960	750	0.78
2	960	620	0.65
3	480	600	1.25
4	480	330	0.69
5	960	900	0.94

Step 5. Compute implied utilization levels. Hence, resource 3 is the bottleneck. Thus, we cannot produce units A at a rate of 40 units per day. Since we are overutilized by 25 percent, we can produce units A at a rate of 32 units per day (four units per hour). Assuming the ratio between A, B, and C is constant (40:50:60), we will produce B at five units per hour and C at six units per hour. If the ratio between A, B, and C is *not* constant, this answer changes. In this case, we would produce 32 units of A and produce products B and C at the rate of demand (50 and 60 units per day respectively).

Chapter 4

Q4.1 (Empty System, Labor Utilization)

Part a

The following computations are based on Exhibit 4.1 in the book. Time to complete 100 units:

Step 1. The process will take 10 + 6 + 16 minutes = 32 minutes to produce the first unit.

Step 2. Resource 2 is the bottleneck and the process capacity is 0.1666 unit per minute.

Step 3. Time to finish 100 units $= 32 \text{ minutes} + \dfrac{99 \text{ units}}{0.166 \text{ unit/minute}} = 626 \text{ minutes}$

Parts b, c, and d

We answer these three questions together by using Exhibit 4.2 in the book.

Step 1. Capacities are

$$\text{Resource 1}: \frac{2}{10}\text{unit/minute} = 0.2 \text{ unit/minute}$$

$$\text{Resource 2}: \frac{1}{6}\text{unit/minute} = 0.1666 \text{ unit/minute}$$

$$\text{Resource 3}: \frac{3}{16}\text{unit/minute} = 0.1875 \text{ unit/minute}$$

Resource 2 is the bottleneck and the process capacity is 0.1666 unit/minute.

Step 2. Since there is unlimited demand, the flow rate is determined by the capacity and therefore is 0.1666 unit/minute; this corresponds to a cycle time of 6 minutes/unit.

Step 3. Cost of direct labor $= \dfrac{6 \times \$10/\text{hour}}{60 \text{ minutes/hour} \times 0.1666 \text{ unit/minute}} = \$6/\text{unit}$

Step 4. Compute the idle time of each worker for each unit:

$$\begin{aligned}\text{Idle time for workers at resource 1} &= 6 \text{ minutes/unit} \times 2 - 10 \text{ minutes/unit} \\ &= 2 \text{ minutes/unit}\end{aligned}$$

$$\begin{aligned}\text{Idle time for worker at resource 2} &= 6 \text{ minutes/unit} \times 1 - 6 \text{ minutes/unit} \\ &= 0 \text{ minute/unit}\end{aligned}$$

$$\begin{aligned}\text{Idle time for workers at resource 3} &= 6 \text{ minutes/unit} \times 3 - 16 \text{ minutes/unit} \\ &= 2 \text{ minutes/unit}\end{aligned}$$

Step 5. Labor content $= 10 + 6 + 16$ minutes/unit $= 32$ minutes/unit

Step 6. Average labor utilization $= \dfrac{32}{32 + 4} = 0.8888$

Chapter 5

Q5.1 (Venture Fair)

Part a

Dependency Matrix:

			Information-Providing Activity (Upstream)										
			1	2	3	4	5	6	7	8	9	10	11
	1	Ideation	■										
	2	Interview Customers	X	■									
	3	Analyze Competing Products	X		■								
Information-Receiving Activity (Downstream)	4	User/Customer Observation		X		■							
	5	Send E-Mail Surveys		X			■						
	6	Target Specifications			X	X	X	■					
	7	Product Design						X	■				
	8	Get Price Quotes							X	■			
	9	Build Prototype							X		■		
	10	Test Prototype with Customers									X	■	
	11	Prepare Info for Venture Fair								X		X	■
		Activity / Days	1 / 3	2 / 6	3 / 12	4 / 10	5 / 4	6 / 5	7 / 10	8 / 6	9 / 4	10 / 5	11 / 3

Part b

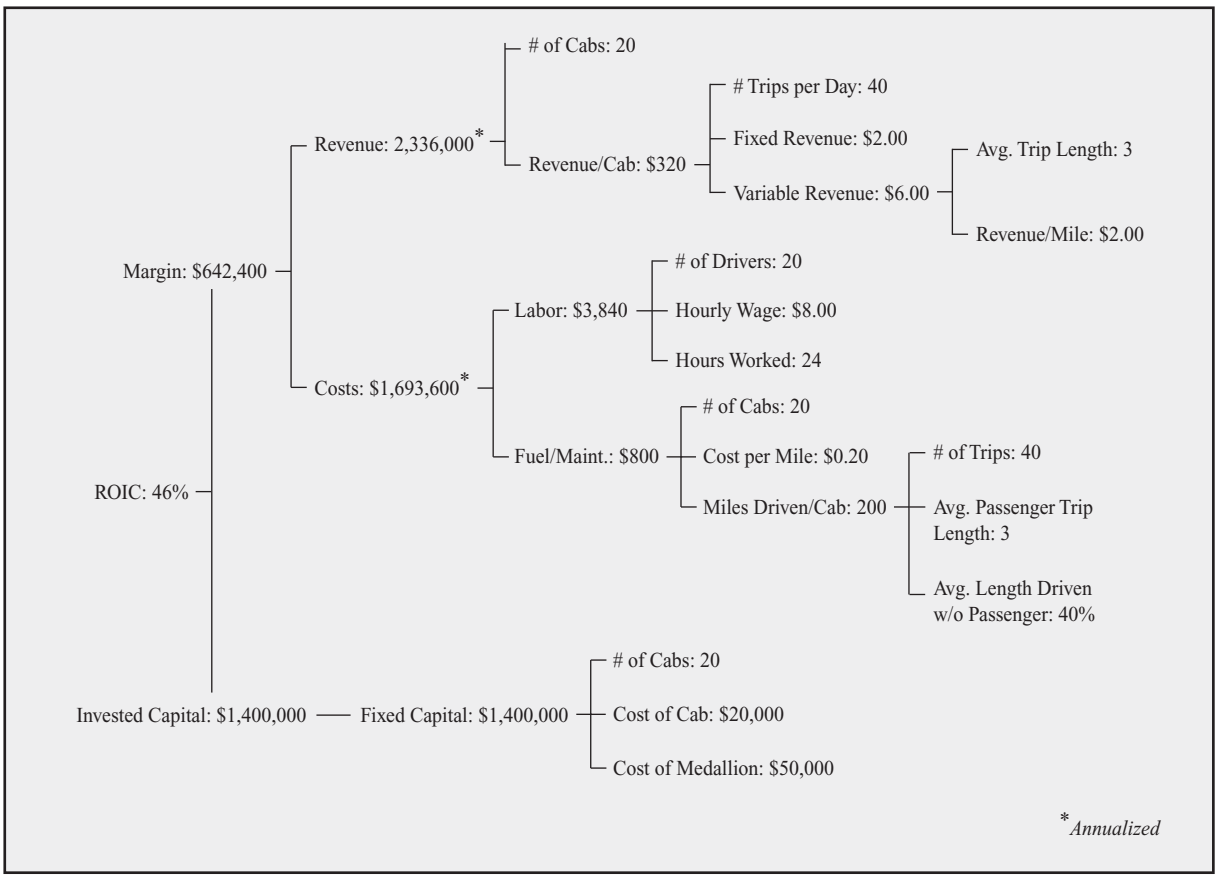
The critical path is A1→A2→A4→A6→A7→A9→A10→A11, which has a total duration of $3 + 6 + 10 + 5 + 10 + 4 + 5 + 3 = 46$. If the project team must have the materials finished by the day before the project fair (April 17th), then they must begin no later than March 3rd (29 days of work in March and 17 days in April).

Chapter 6

Q6.1 (Crazy Cab)

Part a/b

ROIC Tree:



Part c

There are several variables that could be classified as operational value drivers including the number of trips per day, the average trip length, the drivers' hourly wage, and the average distance driven without passengers. Other variables such as the revenue per passenger mile, the fixed fees and the maintenance/fuel cost per mile driven are harder for management to influence because they are either regulated through the cab medallions or are strongly influenced by fuel prices (management could, however, invest in more fuel-efficient cars to reduce this cost).

Given the high capital investments associated with purchasing a cab and medallion, as well as the fixed labor requirements it is important that each cab maximizes its revenue. An additional trip is almost pure profit, particularly if it replaces idle driving time between passengers.

Part d

$$\begin{aligned}\text{Labor Efficiency} &= \text{Revenue/Labor Costs}\\ &= \text{Revenue/Mile} \times \text{Mile/Trip} \times \text{Trips/Day} \times \text{Day/Labor Costs}\end{aligned}$$

In this equation, the first ratio measures the company's operational yield, which is largely a reflection of the company's pricing power. The next two ratios are measures of efficiency: the length of each trip and the number of daily trips, respectively. The final ratio is a measure of the cost of a resource, in this instance the company's labor costs.

A similar equation can be evaluated to determine the efficiency of each cab within the fleet:

$$\begin{aligned}\text{Cab Efficiency} &= \text{Revenue/Cab}\\ &= \text{Revenue/Mile} \times \text{Mile/Trip} \times \text{Trips/Cab}\end{aligned}$$

Chapter 7

Q7.1 (Window Boxes)

The following computations are based on Exhibit 7.1.

Part a

Step 1. Since there is sufficient demand, the step (other than the stamping machine) that determines flow rate is assembly. Capacity at assembly is $\frac{12}{27}$ unit/minute.

Step 2. The production cycle consists of the following parts:

- Setup for A (120 minutes).
- Produce parts A (360 × 1 minute).
- Setup for B (120 minutes).
- Produce parts B (720 × 0.5 minute).

Step 3. There are two setups in the production cycle, so the setup time is 240 minutes.

Step 4. Every completed window box requires one part A (one minute per unit) and two parts B (2 × 0.5 minute per unit). Thus, the per-unit activity time is two minutes per unit.

Step 5. Use formula

$$\begin{aligned}\text{Capacity given batch size} &= \frac{360 \text{ units}}{240 \text{ minutes} + 360 \text{ units} \times 2 \text{ minutes/unit}}\\ &= 0.375 \text{ unit/minute}\end{aligned}$$

Step 6. Capacity at stamping for a general batch size is

$$\frac{\text{Batch size}}{240 \text{ minutes} + \text{Batch size} \times 2 \text{ minutes/unit}}$$

We need to solve the equation

$$\frac{\text{Batch size}}{240 \text{ minutes} + \text{Batch size} \times 2 \text{ minutes/unit}} = \frac{12}{27}$$

for the batch size. The batch size solving this equation is Batch size = 960. We can obtain the same number directly by using

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Time per unit}} = \frac{\frac{12}{27} \times 240}{1 - \frac{12}{27} \times 2} = 960$$

Q7.10 (Cat Food)

$$\frac{7 \times 500}{EOQ} = 1.62$$

Part a

Holding costs are \$0.50 × 15%/50 = 0.0015 per can per week. Note, each can is purchased for \$0.50, so that is the value tied up in inventory and therefore determines the holding cost. The EOQ is then

Part b

The ordering cost is \$7 per order. The number of orders per year is 500/EOQ. Thus, order cost = \$/week = 81\$/year

Part c

The average inventory level is EOQ/2. Inventory costs per week are thus 0.5 × EOQ × 0.0015 = \$1.62. Given 50 weeks per year, the inventory cost per year is \$81

Part d

Inventory turns 5 Flow rate/Inventory

Flow Rate 5 500 cans per week

Inventory 5 0.5 3 EOQ

Thus, Inventory Turns = R/(0.5×EOQ) = 0.462 turns per week = 23.14 turns per year

Q7.11 (Beer Distributor)

The holding costs are 25% per year = 0.5% per week = 8*0.005 = \$0.04 per week

$$\frac{7 \times 500}{EOQ} = 1.62$$

Part a

EOQ=

Part b

Inventory turns = Flow Rate/Inventory = 100 × 50/(0.5 × EOQ) = 5000/EOQ = 44.7 turns per year

Part c

Per unit inventory cost =

Part d

You would never order more than Q = 600

For Q = 600, we would get the following costs: 0.5 × 600 × 0.04 × 0.95 + 10 × 100/600 = 13.1

The cost per unit would be 13.1/100 = \$0.131

The quantity discount would save us 5%, which is \$0.40 per case. However, our operating costs increase by \$0.131 − 0.089 = \$0.042. Hence, the savings outweigh the cost increase and it is better to order 600 units at a time.

Chapter 8

Q8.1 (Online Retailer)

Part a

We use Exhibit 8.1 for our computations.

Step 1. We collect the basic ingredients for the waiting time formula:

Activity time = 4 minutes

$$CV_p = \frac{2}{4}$$

Interarrival time = 2 minutes

$CV_a = 1$

Number of resources = 3

Step 2. This allows us to compute utilization as

$$p/am = 4/(2 \times 3) = 0.6666$$

Step 3. We then use the waiting time formula

$$T_q \approx \left(\frac{4}{3}\right) \times \left(\frac{0.666^{\sqrt{2(3+1)}-1}}{1-0.6666}\right) \times \left(\frac{1^2 + 0.5^2}{2}\right) = 1.19 \text{ minutes}$$

Step 4. We find the

Inventory in service: $I_p = m \times u = 3 \times 0.666 = 2$

Inventory in the queue: $I_q = T_q/a = 1.19/2 = 0.596$

Inventory in the system: $I = I_p + I_q = 2.596$

Part b

The number of e-mails that have been received but not yet answered corresponds to the total inventory of e-mails. We find this to be 2.596 e-mails (see Step 4 above).

Chapter 9

Q9.1 (Loss System)

We use Exhibit 9.1 to answer parts a through c.

Step 1. The interarrival time is 60 minutes per hour divided by 55 units arriving per hour, which is an interarrival time of $a = 1.0909$ minutes/unit. The processing time is $p = 6$ minutes/unit; this allows us to compute $r = p/a = 6/1.0909 = 5.5$.

Step 2. With $r = 5.5$ and $m = 7$, we can use the Erlang Loss Formula Table to look up $P_7(5.5)$ as 0.1525. Alternatively, we can use the actual loss formula (see Appendix C) to compute the probability that all seven servers are utilized:

$$\text{Prob \{all 7 servers are busy\}} = P_7(5.5) = \frac{\frac{5.5^7}{7!}}{1 + \frac{5.5^1}{1!} + \frac{5.5^2}{2!} + \cdots + \frac{5.5^7}{7!}} = 0.1525$$

Step 3. Compute the flow rate: $R = 1/a \times (1 - P_m) = 1/1.0909 \times (1 - 0.153) = 0.77$ unit per minute or 46.585 units per hour.

Step 4. Compute lost customers:

$$\text{Customers lost} = 1/a \times P_m = 1/1.0909 \times 0.153 = 0.14 \text{ unit per minute}$$

which corresponds to 8.415 units per hour.

Thus, from the 55 units that arrive every hour, 46.585 will be served and 8.415 will be lost.

Chapter 12

Q12.1 (McClure Books)

Part a

We first find the z-statistic for 400 (Dan's blockbuster threshold): $z = (400 - 200)/80 = 2.50$. From the Standard Normal Distribution Function Table, we see that $\Phi(2.50) = 0.9938$. So there is a 99.38 percent chance demand is 400 or fewer. Demand is greater than 400 with probability $1 - \Phi(2.50) = 0.0062$; that is, there is only a 0.62 percent chance this is a blockbuster.

Part b

We first find the z-statistic for 100 units (Dan's dog threshold): $z = (100 - 200)/80 = -1.25$. From the Standard Normal Distribution Function Table, we see that $\Phi(-1.25) = 0.1056$. So there is a 10.56 percent chance demand is 100 or fewer; that is, there is a 10.56 percent chance this book is a dog.

Part c

Demand is within 20 percent of the mean if it is between $1.2 \times 200 = 240$ and $0.8 \times 200 = 160$. Using Exhibit 12.2, we first find the z-statistic for 240 units (the upper limit on that range): $z = (240 - 200)/80 = 0.5$. From the Standard Normal Distribution Function Table, we see that $\Phi(0.5) = 0.6915$. Repeat the process for the lower limit on the range: $z = (160 - 200)/80 = -0.5$ and $\Phi(-0.5) = 0.3085$. The probability demand is between 160 and 240 is $\Phi(0.5) - \Phi(-0.50) = 0.6915 - 0.3085 = 0.3830$; that is, 38.3 percent.

Part d

The underage cost is $C_u = 20 - 12 = 8$. The salvage value is $12 - 4 = 8$ because Dan can return leftover books for a full refund ($12) but incurs a $4 cost of shipping and handling. Thus, the overage cost is cost minus salvage value: $C_o = 12 - 8 = 4$. The critical ratio is $C_u/(C_o + C_u) = 8/12 = 0.6667$. In the Standard Normal Distribution Function Table, we see that $\Phi(0.43) = 0.6664$ and $\Phi(0.44) = 0.6700$, so use the round-up rule and choose $z = 0.44$. Now convert z into the order quantity for the actual demand distribution: $Q = \mu + z \times \sigma = 200 + 0.44 \times 80 = 235.2$.

Part e

We want to find a z such that $\Phi(z) = 0.95$. In the Standard Normal Distribution Function Table, we see that $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$, so use actual $200 + 1.65 \times 80 = 332$.

Part f

If the in-stock probability is 95 percent, then the stockout probability (which is what we are looking for) is 1 minus the in-stock, that is, $1 - 95\% = 5$ percent.

Part g

The z-statistic for 300 units is $z = (300 - 200)/80 = 1.25$. From the Standard Normal Loss Function Table, we see that $L(1.25) = 0.0506$. Expected lost sales are $\sigma \times L(1.25) = 4.05$. Expected sales are $200 - 4.05 = 195.95$, expected leftover inventory is $300 - 195.95 = 104.05$, and

$$\begin{aligned}\text{Expected profit} &= (\text{Price} - \text{Cost}) \times \text{Expected sales} \\ &\quad - (\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory} \\ &= (20 - 12) \times 195.95 - (12 - 8) \times 104.05 \\ &= 1151.4\end{aligned}$$

Q12.2 (EcoTable Tea)

Part a

We need to evaluate the stockout probability with $Q = 3$. From the Poisson Distribution Function Table, $F(3) = 0.34230$. The stockout probability is $1 - F(3) = 65.8$ percent.

Part b

They will need to mark down three or more baskets if demand is seven or fewer. From the Poisson Distribution Function Table, $F(7) = 0.91341$, so there is a 91.3 percent probability this will occur.

Part c

First evaluate their critical ratio. The underage cost (or cost of a lost sale) is $\$55 - \$32 = \$23$. The overage cost (or the cost of having a unit left in inventory) is $\$32 - \$20 = \$12$. The critical ratio is $C_u/(C_o + C_u) = 0.6571$. From the Poisson Distribution Function Table, with a mean of 4.5, we see that $F(4) = 0.53210$ and $F(5) = 0.70293$, so we apply the round-up rule and order five baskets.

Part d

With four baskets, expected lost sales is 1.08808, according to the Poisson Loss Function Table. Expected sales is then $4.5 - 1.08808 = 3.4$.

Part e

With six baskets, expected lost sales is 0.32312, according to the Poisson Loss Function Table. Expected sales is then $4.5 - 0.32312 = 4.17688$. Expected leftover inventory is then $6 - 4.17688 = 1.72312 \approx 1.8$.

Part f

From the Poisson Distribution Function Table, $F(6) = 0.83105$ and $F(7) = 0.91314$. Hence, order seven baskets to achieve at least a 90 percent in-stock probability (in fact, the in-stock probability will be 91.3 percent).

Part g

If they order eight baskets, then expected lost sales is 0.06758. Expected sales is $4.5 - 0.06758 = 4.43242$. Expected leftover inventory is $8 - 4.43242 = 3.56758$. Profit is then $\$23 \times 4.43242 - \$12 \times 3.56758 = \$59.13$.

Q12.3 (Pony Express Creations)

Part a

If they purchase 40,000 units, then they need to liquidate 10,000 or more units if demand is 30,000 units or lower. From the table provided, $F(30{,}000) = 0.7852$, so there is a 78.52 percent chance they need to liquidate 10,000 or more units.

Part b

The underage cost is $C_u = 12 - 6 = 6$, the overage cost is $C_o = 6 - 2.5 = 3.5$, and the critical ratio is $6/(3.5 + 6) = 0.6316$. Looking in the demand forecast table, we see that $F(25{,}000) = 0.6289$ and $F(30{,}000) = 0.7852$, so use the round-up rule and order 30,000 Elvis wigs.

Part c

We want to find a Q such that $F(Q) = 0.90$. From the demand forecast table, we see that $F(35{,}000) = 0.8894$ and $F(40{,}000) = 0.9489$, so use the round-up rule and order 40,000 Elvis wigs. The actual in-stock probability is then 94.89 percent.

Part d

If $Q = 50{,}000$, then expected lost sales from the table are only 61 units. Expected leftover inventory $= Q - \mu +$ Expected lost sales $= 50{,}000 - 25{,}000 + 61 = 25{,}061$.

Part e

A 100 percent in-stock probability requires an order quantity of 75,000 units. With $Q = 75{,}000$, then expected lost sales from the table are only two units. Use Exhibit 12.5 to evaluate expected sales, expected leftover inventory, and expected profit. Expected sales are expected demand minus expected lost sales $= 25{,}000 - 2 = 24{,}998$. Expected leftover inventory is $75{,}000 - 24{,}998 = 50{,}002$.

$$\begin{aligned}\text{Expected profit} &= (\text{Price} - \text{Cost}) \times \text{Expected sales} \\ &\quad - (\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory} \\ &= (12 - 6) \times 24{,}998 - (6 - 2.5) \times 50{,}002 \\ &= -25{,}019\end{aligned}$$

So a 100 percent in-stock probability is a money-losing proposition.

Q12.4 (Flextrola)

Part a

It is within 25 percent of the forecast if it is greater than 750 and less than 1,250. Use Exhibit 12.2. The z-statistic for 750 is $z = (750 - 1{,}000)/600 = -0.42$ and the z-statistic for 1,250 is $z = (1{,}250 - 1{,}000)/600 = 0.42$. From the Standard Normal Distribution Function Table, we see that $\Phi(-0.42) = 0.3372$ and $\Phi(0.42) = 0.6628$. So there is a 33.72 percent chance demand is less than 750 and a 66.28 percent chance it is less than 1,250. The chance it is between 750 and 1,250 is the difference in those probabilities: $0.6628 - 0.3372 = 0.3256$.

Part b

The forecast is for 1,000 units. Demand is greater than 40 percent of the forecast if demand exceeds 1,400 units. Use Exhibit 12.2. Find the z-statistic that corresponds to 1,400 units:

$$z = \frac{Q - \mu}{\sigma} = \frac{1,400 - 1,000}{600} = 0.67$$

From the Standard Normal Distribution Function Table, $\Phi(0.67) = 0.7486$. Therefore, there is almost a 75 percent probability that demand is less than 1,400 units. The probability that demand is greater than 1,400 units is $1 - \Phi(0.67) = 0.2514$, or about 25 percent.

Part c

To find the expected profit-maximizing order quantity, first identify the underage and overage costs. The underage cost is $C_u = 121 - 72 = 49$ because each lost sale costs Flextrola its gross margin. The overage cost is $C_o = 72 - 50 = 22$ because each unit of leftover inventory can only be sold for $50. Now evaluate the critical ratio:

$$\frac{C_u}{C_o + C_u} = \frac{49}{22 + 49} = 0.6901$$

Look up the critical ratio in the Standard Normal Distribution Function Table: $\Phi(0.49) = 0.6879$ and $\Phi(0.50) = 0.6915$, so choose $z = 0.50$. Now convert the z-statistic into an order quantity: $Q = \mu + z \times \sigma = 1,000 + 0.5 \times 600 = 1,300$.

Part d

Use Exhibit 12.4 to evaluate expected lost sales and then Exhibit 12.5 to evaluate expected sales. If $Q = 1,200$, then the corresponding z-statistic is $z = (Q - \mu)/\sigma = (1,200 - 1,000)/600 = 0.33$. From the Standard Normal Distribution Loss Table, we see that $L(0.33) = 0.2555$. Expected lost sales are then $\sigma \times L(z) = 600 \times 0.2555 = 153.3$. Finally, recall that expected sales equal expected demand minus expected lost sales: Expected sales $= 1,000 - 153.3 = 846.7$.

Part e

Flextrola sells its leftover inventory in the secondary market, which equals Q minus expected sales $1,200 - 846.7 = 353.3$.

Part f

To evaluate the expected gross margin percentage, we begin with

$$\begin{aligned}\text{Expected revenue} &= (\text{Price} \times \text{Expected sales}) \\ &\quad + (\text{Salvage value} \times \text{Expected leftover inventory}) \\ &= (121 \times 846.7) + (50 \times 353.3) \\ &= 120,116\end{aligned}$$

Then we evaluate expected cost $= Q \times c = 1,200 \times 72 = 86,400$. Finally, expected gross margin percentage $= 1 - 86,400/120,116 = 28.1$ percent.

Part g

Use Exhibit 12.5 and the results from parts d and e to evaluate expected profit:

$$\begin{aligned}\text{Expected profit} &= (\text{Price} - \text{Cost}) \times \text{Expected sales} \\ &\quad - (\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory} \\ &= (121 - 72) \times 846.7 - (72 - 50) \times 353.3 \\ &= 33,716\end{aligned}$$

Part h

Solectric's expected profit is $1{,}200 \times (72 - 52) = 24{,}000$ because units are sold to Flextrola for \$72 and each unit has a production cost of \$52.

Part i

Flextrola incurs 400 or more units of lost sales if demand exceeds the order quantity by 400 or more units; that is, if demand is 1,600 units or greater. The z-statistic that corresponds to 1,600 is $z = (Q - \mu)/\sigma = (1{,}600 - 1{,}000)/600 = 1$. In the Standard Normal Distribution Function Table, $\Phi(1) = 0.8413$. Demand exceeds 1,600 with the probability $1 - \Phi(1) = 15.9$ percent.

Part j

The critical ratio is 0.6901. From the graph of the distribution function, we see that the probability demand is less than 1,150 with the log normal distribution about 0.70. Hence, the optimal order quantity with the log normal distribution is about 1,150 units.

Q12.5 (Fashionables)

Part a

The underage cost is $C_u = 70 - 40 = 30$ and the overage cost is $C_o = 40 - 20 = 20$. The critical ratio is $C_u/(C_o + C_u) = 30/50 = 0.6$. From the Standard Normal Distribution Function Table, $\Phi(0.25) = 0.5987$ and $\Phi(0.26) = 0.6026$, so we choose $z = 0.26$. Convert that z-statistic into an order quantity $Q = \mu + z \times \sigma = 500 + 0.26 \times 200 = 552$. Note that the cost of a truckload has no impact on the profit-maximizing order quantity.

Part b

We need to find the z in the Standard Normal Distribution Function Table such that $\Phi(z) = 0.9750$ because $\Phi(z)$ is the in-stock probability. We see that $\Phi(1.96) = 0.9750$, so we choose $z = 1.96$. Convert to $Q = \mu + z \times \sigma = 500 + 1.96 \times 200 = 892$.

Part c

If 725 units are ordered, then the corresponding z-statistic is $z = (Q - \mu)/\sigma = (725 - 500)/200 = 1.13$. We need to evaluate lost sales, expected sales, and expected leftover inventory before we can evaluate the expected profit. Expected lost sales with the standard normal is obtained from the Standard Normal Loss Function Table, $L(1.13) = 0.0646$. Expected lost sales are $\sigma \times L(z) = 200 \times 0.0646 = 12.9$. Expected sales are $500 - 12.9 = 487.1$. Expected leftover inventory is $725 - 487.1 = 237.9$. Expected profit is

$$\begin{aligned}\text{Expected profit} &= (70 - 40) \times 487.1 - (40 - 20) \times 237.9 \\ &= 9{,}855\end{aligned}$$

So the expected profit per sweater type is 9,855. The total expected profit is five times that amount, minus 2,000 times the number of truckloads required.

Part d

The stockout probability is the probability demand exceeds the order quantity 725, which is $1 - \Phi(1.13) = 12.9$ percent.

Part e

If we order the expected profit-maximizing order quantity for each sweater, then that equals $5 \times 552 = 2{,}760$ sweaters. With an order quantity of 552 sweaters, expected lost sales are $56.5 = 200 \times L(0.26) = 200 \times 0.2824$, expected sales are $500 - 56.5 = 443.5$, and expected leftover inventory is $552 - 443.5 = 108.5$. Expected profit per sweater type is

$$\begin{aligned}\text{Expected profit} &= (70 - 40) \times 443.5 - (40 - 20) \times 108.5 \\ &= 11{,}135\end{aligned}$$

Because two truckloads are required, the total profit is then $5 \times 11{,}136 - 2 \times 2{,}000 = 51{,}675$. If we order only 500 units per sweater type, then we can evaluate the expected profit per sweater to be 11,010. Total profit is then $5 \times 11{,}010 - 2{,}000 = 53{,}050$. Therefore, we are better off just ordering one truckload with 500 sweaters of each type.

Chapter 13

Q13.1 (Teddy Bower)

Part a

Teddy will order from the American supplier if demand exceeds 1,500 units. With $Q = 1{,}500$, the z-statistic is $z = (1{,}500 - 2{,}100)/1{,}200 = -0.5$. From the Standard Normal Distribution Function Table, we see that $\Phi(-0.50) = 0.3085$, which is the probability that demand is 1,500 or fewer. The probability that demand exceeds 1,500 is $1 - \Phi(-0.50) = 0.6915$, or about 69 percent.

Part b

The supplier's expected demand equals Teddy's expected lost sales with an order quantity of 1,500 parkas. From the Standard Normal Loss Function Table, $L(-0.50) = 0.6978$. Expected lost sales are $\sigma \times L(z) = 1{,}200 \times 0.6978 = 837.4$.

Part c

The overage cost is $C_o = 10 - 0 = 10$ because leftover parkas must have been purchased in the first order at a cost of \$10 and they have no value at the end of the season. The underage cost is $C_u = 15 - 10 = 5$ because there is a \$5 premium on units ordered from the American vendor. The critical ratio is $5/(10 + 5) = 0.3333$. From the Standard Normal Distribution Function Table, we see that $\Phi(-0.44) = 0.3300$ and $\Phi(-0.43) = 0.3336$, so choose $z = -0.43$. Convert to Q: $Q = 2{,}100 - 0.43 \times 1{,}200 = 1{,}584$.

Part d

First evaluate some performance measures. We already know that with $Q = 1{,}584$ the corresponding z is -0.43. From the Standard Normal Loss Function Table, $L(-0.43) = 0.6503$. Expected lost sales are then $1{,}200 \times 0.6503 = 780.4$; that is the expected order quantity to the American vendor. If the American vendor were not available, then expected sales would be $2{,}100 - 780.4 = 1{,}319.6$. Expected leftover inventory is then $1{,}584 - 1{,}319.6 = 264.4$. Now evaluate expected profit with the American vendor option available. Expected revenue is $2{,}100 \times 22 = \$46{,}200$. The cost of the first order

is 1,584 × 10 = $15,840. Salvage revenue from leftover inventory is 264.4 × 0 = 0. Finally, the cost of the second order is 780.4 × 15 = $11,706. Thus, profit is 46,200 − 15,840 − 11,706 = $18,654.

Part e

If Teddy only sources from the American supplier, then expected profit would be ($22 − $15) × 2,100 = $14,700 because expected sales would be 2,100 units and the gross margin on each unit is $22 − $15 = $7.

Q13.2 (Flextrola)

Part a

Expected sales = 1,000 and the gross margin per sale is 121 − 83.5 = $37.5. Expected profit is then 1,000 × $37.5 = $37,500.

Part b

$C_o = 72 - 50 = 22$; $C_u = 83.5 - 72 = 11.5$; therefore, the premium on orders from XE is $11.5. The critical ratio is 11.5/(22 + 11.5) = 0.3433. From the Standard Normal Distribution Function Table, $\Phi(-0.41) = 0.3409$ and $\Phi(-0.40) = 0.3446$, so $z = -0.40$. Convert to *Q:* $Q = 1{,}000 - 0.4 \times 600 = 760$.

Part c

The underage cost on an option is the change in profit if one additional option had been purchased that could be exercised. For example, if 700 options are purchased, but demand is 701, then 1 additional option could have been purchased. The cost of the option plus exercising it is $25 + $50 = $75. The cost of obtaining the unit without the option is $83.5, so purchasing the option would have saved C_u = $83.5 − $75 = $8.5. The overage cost on an option is the extra profit that could have been earned if the option were not purchased assuming it isn't needed. For example, if demand were 699, then the last option would not be necessary. The cost of that unnecessary option is C_o = $25. The critical ratio is 8.5/(25 + 8.5) = 0.2537. From the Standard Normal Distribution Function Table, $\Phi(-0.67) = 0.2514$ and $\Phi(-0.66) = 0.2546$, so $z = -0.66$. Convert to *Q:* $Q = 1{,}000 - 0.66 \times 600 = 604$.

Part d

Evaluate some performance measures. Expected number of units ordered beyond the purchased options (expected lost sales) is $\sigma \times L(-0.66) = 600 \times 0.8128 = 487.7$. Expected number of options exercised (expected sales) is 1,000 − 487.7 = 512.3. Expected revenue is 1,000 × $121 = $121,000. So profit is revenue minus the cost of purchasing options (604 × $25 = $15,100), minus the cost of exercising options (512.3 × $50 = $25,615), minus the cost of units purchased without options (487.7 × $83.5 = $40,723): Profit = 121,000 − 15,100 − 25,615 − 40,723 = $39,562.

Q13.3 (Wildcat Cellular)

Part a

The underage cost is C_u=0.4−0.05 = $0.35: if her usage exceeds the minutes she purchases then she could have lowered her cost by $0.35 per minute if she had purchased more minutes. The overage cost is 0.05oC because each minute purchased but not used provides no value. The critical ratio is 0.35/(0.05 + 0.35) = 0.8749. From the Standard Normal

Distribution Funtion Table Func Table $\Phi(1.15) = 0.8749$ and $\Phi(1.16) = 0.8770$, so $z = 1.16$. Convert to *Q:* $Q = 250 + 1.16 \times 24 = 278$.

Part b

We need to evaluate the number of minutes used beyond the quantity purchased (Expected lost sales). $z = (240 - 250)/24 = -0.42$, $L(-0.42) = 0.6436$, and expected lost sales $= 24 \times 0.6436 = 15.4$ minutes. Each minute costs \$0.4, so the total surcharge is $15.4 \times \$0.4 = \$6.16$.

Part c

Find the corresponding *z*-statistic: $z = (280 - 250)/24 = 1.25$. Now evaluate performance measures. $L(1.25) = 0.0506$, and Expected lost sales $= 24 \times 0.0506 = 1.2$ minutes, that is, only 1.2 minutes are needed on average beyond the 280 purchased. The minutes used out of the 280 (Expected sales) is $250 - 1.2 = 248.8$. The unused minutes (Expected left over inventory) is $280 - 248.8 = 31.2$.

Part d

Find the corresponding *z*-statistic: $z = (260 - 250)/24 = 0.42$. The number of minutes needed beyond the 260 is Expected lost sales: $L(0.42) = 0.2236$, and Expected lost sales $= 24 \times 0.2236 = 5.4$ minutes. Total bill is $260 \times 0.05 + 5.4 \times 0.4 = \$15.16$

Part e

From the Standard Normal Distribution Function Table $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$, so with $z = 1.65$ there is a 95.05 percent chance the outcome of a Standard Normal is less than *z.* Convert to *Q:* $Q = 250 + 1.65 \times 24 = 290$.

Part f

With "Pick Your Minutes," the optimal number of minutes is 278. The expected bill is then \$14.46: $z = (278 - 250)/24 = 1.17$; $L(1.17) = 0.0596$; Expected surcharge minutes $= 24 \times 0.0596 = 1.4$; Expected surcharge $= \$0.4 \times 1.4 = \$0.56$; Purchase cost is $278 \times 0.05 = \$13.9$; so the total is \$13.9 + 0.56. With "No Minimum," the total bill is \$22.5: minutes cost $\$0.07 \times 250 = \$17.5$; plus the fixed fee, \$5. So she should stick with the original plan.

Q13.9 (Steve Smith)

For every car Smith sells, he gets \$350 and an additional \$50 for every car sold over five cars. Look in the Poisson Loss Function Table for mean 5.5: the expected amount by which the outcome exceeds zero is $L(0) = 5.5$ (same as the mean) and the expected amount by which the outcome exceeds five is $L(5) = 1.178$. Therefore, the expected commission is $(350 \times 5.5) + (50 \times 1.178) = 1{,}984$.

Chapter 14

Q14.1 (Furniture Store)

Part a

Inventory position = Inventory level + On-order $= 100 + 85 = 185$. Order enough to raise the inventory position to the order-up-to level, in this case, $220 - 185 = 35$ desks.

Part b

As in part a, Inventory position $= 160 + 65 = 225$. Because the inventory position is above the order-up-to level, 220, you do not order additional inventory.

Part c

Use Exhibit 14.5. From the Standard Normal Distribution Function Table: $\Phi(2.05) = 0.9798$ and $\Phi(2.06) = 0.9803$, so choose $z = 2.06$. The lead time l is 2, so $\mu = (2 + 1) \times 40 = 120$ and $\sigma = \sqrt{2 + 1} \times 20 = 34.64$.

$$S = \mu + z \times \sigma = 120 + 2.06 \times 34.64 = 191.36$$

Part d

Use Exhibit 14.4. The z-statistic that corresponds to $S = 120$ is $S = (120 - 120)/34.64 = 0$. Expected back order is $\sigma \times L(0) = 34.64 \times 0.3989 = 13.82$. Expected on-hand inventory is $S - \mu +$ Expected back order $= 120 - 120 + 13.82 = 13.82$.

Part e

From part d, on-hand inventory is 13.82 units, which equals $13.82 \times \$200 = \$2{,}764$. Cost of capital is 15 percent, so the cost of holding inventory is $0.15 \times \$2{,}764 = \$414.60$.

Q14.2 (Campus Bookstore)

Part a

Use Exhibit 14.5. Mean demand over $l + 1$ periods is $0.5 \times (4 + 1) = 2.5$ units. From the Poisson Distribution Function Table, with mean 2.5 we have $F(6) = 0.9858$ and $F(7) = 0.9958$, so choose $S = 7$ to achieve a 99 percent in-stock.

Part b

Use Exhibit 14.4. Pipeline inventory is $l \times$ Expected demand in one period $= 4 \times 0.5 = 2$ units. The order-up-to level has no influence on the pipeline inventory.

Part c

Use Exhibit 14.4. From the Poisson Loss Function Table with mean 2.5, Expected back order $= L(5) = 0.06195$. Expected on-hand inventory $= 5 - 2.5 + 0.06195 = 2.56$ units.

Part d

A stockout occurs if demand is seven or more units over $l + 1$ periods, which is one minus the probability demand is six or fewer in that interval. From the Poisson Distribution Function Table with mean 2.5, we see that $F(6) = 0.9858$ and $1 - F(6) = 0.0142$; that is, there is about a 1.4 percent chance of a stockout occurring.

Part e

The store is out of stock if demand is six or more units over $l + 1$ periods, which is one minus the probability demand is five or fewer in that interval. From the Poisson Distribution Function Table with mean 2.5, we see that $F(5) = 0.9580$ and $1 - F(5) = 0.0420$; that is, there is about a 4.2 percent chance of being out of inventory at the end of any given week.

Part f

The store has one or more units of inventory if demand is five or fewer over $l + 1$ periods. From part e, $F(5) = 0.9580$; that is, there is about a 96 percent chance of having one or more units at the end of any given week.

Part g

Use Exhibit 14.5. Now the lead time is two periods (each period is two weeks and the total lead time is four weeks, or two periods). Demand over one period is 1.0 unit. Demand over $l + 1$ periods is $(2 + 1) \times 1 = 3.0$ units. From the Poisson Distribution Function Table with mean 3.0, we have $F(7) = 0.9881$ and $F(8) = 0.9962$, so choose $S = 8$ to achieve a 99 percent in-stock.

Part h

Use Exhibit 14.4. Pipeline inventory is average demand over l periods $= 2 \times 1 = 2.0$ units.

Q14.3 (Quick Print)

Part a

If $S = 700$ and the inventory position is $523 + 180 = 703$, then 0 units should be ordered because the inventory position exceeds the order-up-to level.

Part b

Use Exhibit 14.5. From the Standard Normal Distribution Function Table, $\Phi(2.32) = 0.9898$ and $\Phi(2.33) = 0.9901$, so choose $z = 2.33$. Convert to $S = \mu + z \times \sigma = 600 + 2.33 \times 159.22 = 971$.

Q14.4 (Main Line Auto Distributor)

Part a

Use equation (14.2). The critical ratio is \$25/(\$0.5 + \$25) = 0.98039. The lead time is $l = 0$, so demand over $(l + 1)$ periods is Poisson with mean 1.5. From the Poisson Distribution Function Table with mean 1.5, we see $F(3) = 0.9344$ and $F(4) = 0.9814$, so choose $S = 4$. There is currently no unit on order or on hand, so order to raise the inventory position to four: order four units.

Part b

The in-stock probability is the probability demand is satisfied during the week. With $S = 3$ the in-stock is $F(3) = 0.9344$, that is, a 93 percent probability.

Part c

Demand is not satisfied if demand is five or more units, which is $1 - [F(4) = 0.9814] = 1 - 0.9814 = 0.0186$, or about 1.9 percent.

Part d

Use Exhibit 14.5. From the Poisson Distribution Function Table with mean 1.5, $F(4) = 0.9814$ and $F(5) = 0.9955$, so choose $S = 5$ to achieve a 99.5 percent in-stock probability.

Part e

Use Exhibit 14.4. If $S = 5$, then from the Poisson Loss Function Table with mean 1.5, we see expected back order $= L(5) = 0.0056$. Expected on-hand inventory is $S -$ Demand over $(l + 1)$ periods + Expected back order $= 5 - 1.5 + 0.0056 = 3.51$ units. The holding cost is $3.51 \times \$0.5 = \$1.76$.

Q14.5 (Hotspices.com)

Part a

From the Standard Normal Distribution Function Table, $\Phi(2.43) = 0.9925$; so choose $z = 2.43$. Convert to *S*: $S = \mu + z \times \sigma = 159.62 + 2.43 \times 95.51 = 392$.

Part b

Use equation (14.3). The holding cost is $h = 0.75$ and the back-order penalty cost is 50. The critical ratio is $50/(0.75 + 50) = 0.9852$. From the Standard Normal Distribution Function Table, $\Phi(2.17) = 0.9850$ and $\Phi(2.18) = 0.9854$, so choose $z = 2.18$. Convert to $S = \mu + z \times \sigma = 159.62 + 2.18 \times 95.51 = 368$.

Part c

Use equation (14.3). The holding cost is $h = 0.05$ and the back-order penalty cost is 5. The critical ratio is $5/(0.05 + 5) = 0.9901$. Lead time plus one demand is Poisson with mean $1 \times 3 = 3$. From the Poisson Distribution Function Table, with $\mu = 3$, $F(7) = 0.9881$ and $F(8) = 0.9962$, so $S = 8$ is optimal.

Chapter 15

Q15.1 (Egghead)

Part a

New standard deviation is $30 \times \sqrt{50} = 212$.

Part b

Pipeline inventory = Expected demand per week × Lead time = $200 \times 50 \times 10 = 100{,}000$.

Q15.2 (Two Products)

The coefficient of total demand (pooled demand) is the coefficient of the product's demand times the square root of (1 + Correlation)/2. Therefore, $\sqrt{(1 - 0.7)/2} \times 0.6 = 0.23$.

Q15.3 (Fancy Paints)

Part a

Assume Fancy Paints implements the order-up-to inventory model. Find the appropriate order-up-to level. With a lead time of 4 weeks, the relevant demand is demand over $4 + 1 = 5$ weeks, which is $5 \times 1.25 = 6.25$. From the Poisson Distribution Function Table, $F(10) = 0.946$ and $F(11) = 0.974$, a base stock level $S = 11$ is needed to achieve at least a 95 percent in-stock probability. On-hand inventory at the end of the week is $S - 6.25 -$ Expected back order. From the Poisson Distribution Function Loss Function Table, the Expected back order is $L(11) = 0.04673$. Thus, on-hand inventory for one SKU is $11 - 6.25 + 0.04673 = 4.8$ units. There are 200 SKUs, so total inventory is $200 \times 4.8 = 960$.

Part b

The standard deviation over (4 + 1) weeks is $\sigma = \sqrt{5} \times 8 = 17.89$ and $\mu = 5 \times 50 = 250$. From the Standard Normal Distribution Function Table, we see that $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$, so we choose $z = 1.65$ to achieve the 95 percent in-stock probability. The base stock level is then $S = \mu + z \times \sigma = 250 + 1.65 \times 17.89 = 279.5$. From the Standard Normal Loss Function Table, $L(1.65) = 0.0206$. So, on-hand inventory

for one product is $S - 250 +$ Expected back order $= 279.5 - 250 + 17.89 \times 0.0206 = 29.9$. There are five basic SKUs, so total inventory in the store is $29.9 \times 5 = 149.5$.

Part c

The original inventory investment is $960 \times \$14 = \$13{,}440$, which incurs holding costs of $\$13{,}440 \times 0.20 = \$2{,}688$. Repeat part b, but now the target in-stock probabi lity is 98 percent. From the Standard Normal Distribution Function Table, we see that $F(2.05) = 0.9798$ and $F(2.06) = 0.9803$, so we choose $z = 2.06$ to achieve the 98 percent in-stock probability. The base stock level is then $S = \mu + z \times \sigma = 250 + 2.06 \times 17.89 = 286.9$. From the Standard Normal Loss Function Table, $L(2.06) = 0.0072$. So, on-hand inventory for one product is $S - 250 +$ Expected back order $= 286.9 - 250 + 17.89 \times 0.0072 = 37.0$. There are five basic SKUs, so total inventory in the store is $37.0 \times 5 = 185$. With the mixing machine, the total inventory investment is $185 \times \$14 = \$2{,}590$. Holding cost is $\$2{,}590 \times 0.2 = \$518$, which is only 19 percent (518/2688) of the original inventory holding cost.

Q15.4 (Burger King)

Part a

Use the newsvendor model to determine an order quantity. Use Exhibit 12.7. From the table we see that $F(3{,}500) = 0.8480$ and $F(4{,}000) = 0.8911$, so order 4,000 for each store.

Part b

Use Exhibit 12.4 to evaluate expected lost sales and Exhibit 12.5 to evaluate the expected leftover inventory. Expected lost sales come from the table, $L(4{,}000) = 185.3$. Expected sales are $\mu - 185.3 = 2{,}251 - 185.3 = 2{,}065.7$. Expected leftover inventory is Q minus expected sales, $4{,}000 - 2{,}065.7 = 1{,}934.3$. Across 200 stores there will be $200 \times 1{,}934.3 = 386{,}860$ units left over.

Part c

The mean is 450,200. The coefficient of variation of individual stores is 1,600/ 2,251 = 0.7108. The coefficient of variation of total demand, we are told, is one-half of that, $0.7108/2 = 0.3554$. Hence, the standard deviation of total demand is $450{,}200 \times 0.3554 = 160{,}001$. To find the optimal order quantity to hit an 85 percent in-stock probability, use Exhibit 12.7. From the Standard Normal Distribution Function Table, we see $\Phi(1.03) = 0.8485$ and $\Phi(1.04) = 0.8508$, so choose $z = 1.04$. Convert to $Q = 450{,}200 + 1.04 \times 160{,}001 = 616{,}601$.

Part d

Expected lost sales $= 160{,}001 \times L(z) = 160{,}001 \times 0.0772 = 12{,}352$. Expected sales $= 450{,}200 - 12{,}352 = 437{,}848$. Expected leftover inventory $= 616{,}601 - 437{,}848 = 178{,}753$, which is only 46 percent of what would be left over if individual stores held their own inventory.

Part e

The total order quantity is $4{,}000 \times 200 = 800{,}000$. With a mean of 450,200 and standard deviation of 160,001 (from part c), the corresponding z is $(800{,}000 - 450{,}200)/160{,}001 = 2.19$. From the Standard Normal Distribution Function Table, we see $\Phi(2.19) = 0.9857$, so the in-stock probability would be 98.57 percent instead of 89.11 percent if the inventory were held at each store.

Q15.5 (Livingstion Tools)

Part a

With a lead time of 3 weeks, μ $(3 + 1) \times 5{,}200 = 20{,}800$ and $\sigma = \sqrt{3 + 1} \times 3{,}800 = 7{,}600$. The target expected back orders is $(5{,}200/7{,}600) \times (1 - 0.999) = 0.0007$. From the Standard Normal Distribution Function Table, we see that $\Phi(3.10) = 0.9990$, so we choose $z = 3.10$ to achieve the 99.9 percent in-stock probability. Convert to $S = 20{,}800 + 3.10 \times 7{,}600 = 44{,}360$. Expected back order is $7{,}600 \times 0.0003 = 2.28$. Expected on-hand inventory for each product is $44{,}360 - 20{,}800 + 2.28 = 23{,}562$. The total inventory for the two is $2 \times 23{,}562 = 47{,}124$.

Part b

Weekly demand for the two products is $5{,}200 \times 2 = 10{,}400$. The standard deviation of the two products is $\sqrt{2 \times (1 - \text{Correlation})} \times \text{Standard deviation of one product} = \sqrt{2 \times (1 - 0.20)} \times 3{,}800 = 4{,}806.66$. Lead time plus one expected demand is $10{,}400 \times 4 = 41{,}600$. Standard deviation over $(I + 1)$ periods is $\sqrt{(3+1)} \times 4{,}806.66 = 9{,}613$. Now repeat the process in part a with the new demand parameters. Convert to $S = 41{,}600 + 3.10 \times 9{,}613 = 71{,}401$. Expected back order is $9{,}613 \times 0.0003 = 2.88$. Expected on-hand inventory is $71{,}401 - 41{,}600 + 2.88 = 29{,}804$. The inventory investment is reduced by $(47{,}124 - 29{,}804)/47{,}124 = 37$ percent.

Q15.9 (Consulting Services)

Option a provides the longest chain, covering all four areas. This gives the maximum flexibility value to the firm, so that should be the chosen configuration. To see that it forms a long chain, Alice can do Regulations, as well as Bob. Bob can do Taxes, as well as Doug. Doug can do Strategy, as well as Cathy. Cathy can do Quota, as well as Alice. Hence, there is a single chain among all four consultants. The other options do not form a single chain.

Chapter 16

Q16.1 (The Inn at Penn)

Part a

The booking limit is capacity minus the protection level, which is $150 - 50 = 100$; that is, allow up to 100 bookings at the low fare.

Part b

Use Exhibit 16.1. The underage cost is $C_u = 200 - 120 = 80$ and the overage cost is $C_o = 120$. The critical ratio is $80/(120 + 80) = 0.4$. From the Standard Normal Distribution Function Table, we see $\Phi(-0.26) = 0.3974$ and $\Phi(-0.25) = 0.4013$, so choose $z = -0.25$. Evaluate Q: $Q = 70 - 0.25 \times 29 = 63$.

Part c

Decreases. The lower price for business travelers leads to a lower critical ratio and hence to a lower protection level; that is, it is less valuable to protect rooms for the full fare.

Part d

The number of unfilled rooms with a protection level of 61 is the same as expected leftover inventory. Evaluate the critical ratio, $z = (61 - 70)/29 = -0.31$. From the Standard Normal Loss Function Table, $L(z) = 0.5730$. Expected lost sales are $29 \times 0.5730 = 16.62$ and expected leftover inventory is $61 - 70 + 16.62 = 7.62$. So we can expect 7.62 rooms to remain empty.

Part e

$70 \times \$200 + (150 - 70) \times \$120 = \$23{,}600$ because, on average, 70 rooms are sold at the high fare and $150 - 70 = 80$ are sold at the low fare.

Part f

$150 \times \$120 = \$18{,}000$.

Part g

If 50 are protected, we need to determine the number of rooms that are sold at the high fare. The z statistic is $(50 - 70)/29 = -0.69$. Expected lost sales are $29 \times L(-0.69) = 24.22$. Expected sales are $70 - 24.22 = 45.78$. Revenue is then $(150 - 50) \times \$120 + 45.78 \times \$200 = \$21{,}155$.

Q16.2 (Overbooking The Inn at Penn)

Part a

Use Exhibit 16.2. The underage cost is \$120, the discount fare. The overage cost is \$325. The critical ratio is $120/(325 + 120) = 0.2697$. From the table, $F(12) = 0.2283$ and $F(13) = 0.3171$, so the optimal overbook quantity is 13.

Part b

A reservation cannot be honored if there are nine or fewer no-shows. $F(9) = 0.0552$, so there is a 5.5 percent chance the hotel will be overbooked.

Part c

It is fully occupied if there are 15 or fewer no-shows, which has probability $F(15) = 0.5170$.

Part d

Bumped customers equal 20 minus the number of no-shows, so it is equivalent to left-over inventory. Lost sales are $L(20) = 0.28$, expected sales are $15.5 - 0.28 = 15.22$, and expected leftover inventory/bumped customers $= 20 - 15.22 = 4.78$. Each one costs \$325, so the total cost is $\$325 \times 4.78 = \$1{,}554$.

Q16.3 (WAMB)

Part a

First evaluate the distribution function from the density function provided in the table: $F(8) = 0$, $F(9) = F(8) + 0.05 = 0.05$, $F(10) = F(9) + 0.10 = 0.15$, and so on. Let Q denote the number of slots to be protected for sale later and let D be the demand for slots at \$10,000 each. If $D > Q$, we reserved too few slots and the underage penalty is $C_u = \$10{,}000 - \$4{,}000 = \$6{,}000$. If $D < Q$, we reserved too many slots and the overage penalty is $C_o = \$4{,}000$. The critical ratio is $6{,}000/(4{,}000 + 6{,}000) = 0.6$. From the table, we find $F(13) = 0.6$, so the optimal protection quantity is 13. Therefore, WAMB should sell $25 - 13 = 12$ slots in advance.

Part b

The underage penalty remains the same. The overage penalty is now $C_o = \$4{,}000 - \$2{,}500 = \$1{,}500$. Setting the protection level too high before meant lost revenue on the slot, but now at least \$2,500 can be gained from the slot, so the loss is only \$1,500. The critical ratio is $6{,}000/(1{,}500 + 6{,}000) = 0.8$. From the table, $F(15) = 0.8$, so protect 15 slots and sell $25 - 15 = 10$ in advance.

Part c

If the booking limit is 10, there are 15 slots for last-minute sales. There will be standby messages if there are 14 or fewer last-minute sales, which has probability $F(14) = 0.70$.

Part d

Over-overbooking means the company is hit with a \$10,000 penalty, so $C_o = 10{,}000$. Under-overbooking means slots that could have sold for \$4,000 are actually sold at the standby price of \$2,500, so $C_u = 4{,}000 - 2{,}500 = 1{,}500$. The critical ratio is $1{,}500/(10{,}000 + 1{,}500) = 0.1304$. From the Poisson Distribution Function Table with mean 9.0, $F(5) = 0.1157$ and $F(6) = 0.2068$, so the optimal overbooking quantity is six, that is, sell up to 31 slots.

Part e

The overage cost remains the same: we incur a penalty of \$10,000 for each bumped customer (and we refund the \$1,000 deposit of that customer, too). The underage cost also remains the same. To explain, suppose they overbooked by two slots but there are three withdrawals. Because they have one empty slot, they sell it for \$2,500. Had they overbooked by one more (three slots), then they would have collected \$4,000 on that last slot instead of the \$2,500, so the difference is C_u = \$4,000-\$2,500 = \$1,500. Note, the nonrefundable amount of \$1,000 is collected from the three withdrawals in either scenario, so it doesn't figure into the change in profit by overbooking one more unit. The critical ratio is $1{,}500/(10{,}000 + 1{,}500) = 0.1304$. From the Poisson Distribution Function Table with mean 4.5, $F(1) = 0.0611$ and $F(2) = 0.17358$, so the optimal overbooking quantity is two, that is, sell up to 27 slots.

Q16.4 (Designer Dress)

Part a

The z-statistic is $(100 - 70)/40 = 0.75$. Expected lost sales are $40 \times L(z) = 40 \times 0.1312 = 5.248$. Expected sales are $70 - 5.248 = 64.752$. Expected leftover inventory is $100 - 64.752 = 35.248$.

Part b

Expected revenue is \$10,000 × 64.752 = \$647,520.

Part c

Use Exhibit 16.1. The underage cost is \$10,000 − \$6,000 = \$4,000 because under-protecting boutique sales means a loss of \$4,000 in revenue. Overprotecting means a loss of \$6,000 in revenue. The critical ratio is $4{,}000/(6{,}000 + 4{,}000) = 0.4$. From the Standard Normal Distribution Function Table, we see $\Phi(-0.26) = 0.3974$ and $\Phi(-0.25) = 0.4013$, so choose $z = -0.25$. Evaluate Q: $Q = 40 - 0.25 \times 25 = 33.75$. So protect 34 dresses for sales at the boutique, which means sell $100 - 34 = 66$ dresses at the show.

Part d

If 34 dresses are sent to the boutique, then expected lost sales are $\sigma \times L(z) = 25 \times L(-0.25) = 25 \times 0.5363 = 13.41$. Expected sales are $40 - 13.41 = 26.59$. So revenue is $26.59 \times \$10{,}000 + (100 - 34) \times 6{,}000 = \$661{,}900$.

Part e

From part d, expected sales are 26.59, so expected leftover inventory is $34 - 26.59 = 7.41$ dresses.

Q16.5 (Overbooking, PHL-LAX)

Part a

Use Exhibit 16.2. The overage cost is \$800 (over-overbooking means a bumped passenger, which costs \$800). The underage cost is \$475 (an empty seat). The critical ratio is $475/(800 + 475) = 0.3725$. From the Standard Normal Distribution Function Table, we see $\Phi(-0.33) = 0.3707$ and $\Phi(-0.32) = 0.3745$, so choose $z = -0.32$. Evaluate Y: $Y = 30 - 0.32 \times 15 = 25.2$. So the maximum number of reservations to accept is $200 + 25 = 225$.

Part b

$220 - 200 = 20$ seats are overbooked. The number of bumped passengers equals 20 minus the number of no-shows, which is equivalent to leftover inventory with an order quantity of 20. The z-statistic is $(20-30)/15 = -0.67$. $L(-0.67) = 0.8203$, so lost sales are $15 \times 0.8203 = 12.3$. Sales are $30 - 12.3 = 17.7$ and expected leftover inventory is $20 - 17.7 = 2.3$. If 2.3 customers are bumped, then the payout is $\$800 \times 2.3 = \$1{,}840$.

Part c

You will have bumped passengers if there are 19 or fewer no-shows. The z-statistic is $(19 - 30)/15 = -0.73$. $\Phi(-0.73) = 0.2317$, so there is about a 23 percent chance there will be bumped passengers.

Chapter 17

Q17.1 (Buying Tissues)

Part a

If orders are made every week, then the average order quantity equals one week's worth of demand, which is 25 cases. If at the end of the week there is one week's worth of inventory, then the average inventory is $25/2 + 25 = 37.5$. (In this case, inventory "sawtooths" from a high of two weeks' worth of inventory down to one week, with an average of 1.5 weeks.) On average the inventory value is $37.5 \times 9.25 = \$346.9$. The holding cost per year is $52 \times 0.4\% = 20.8$ percent. Hence, the inventory holding cost with the first plan is $20.8\% \times \$346.9 = \$72$. Purchase cost is $52 \times 25 \times \$9.25 = \$12{,}025$. Total cost is $\$12{,}025 + \$72 = \$12{,}097$.

Part b

Four orders are made each year; each order on average is for $(52/4) \times 25 = 325$ units. Average inventory is then $325/2 + 25 = 187.5$. The price paid per unit is $\$9.40 \times 0.95 = \$8.93$. The value of that inventory is $187.5 \times \$8.93 = \$1{,}674$. Annual holding costs are $\$1{,}674 \times 20.8\% = \$348$. Purchase cost is $52 \times 25 \times \$8.93 = \$11{,}609$. Total cost is $\$348 + \$11{,}609 = \$11{,}957$.

Part c

P&G prefers our third plan as long as the price is higher than in the second plan, \$8.93. But the retailer needs a low enough price so that its total cost with the third plan is not greater than in the second plan, \$11,957 (from part b). In part a, we determined that the annual holding cost with a weekly ordering plan is approximately \$72. If we lower the price, the annual holding cost will be a bit lower, but \$72 is a conservative approximation of the

holding cost. So the retailer's purchase cost should not exceed \$11,957 − \$72 = \$11,885. Total purchase quantity is 25 × 52 = 1,300 units. So if the price is \$11,885/1,300 = \$9.14, then the retailer will be slightly better off (relative to the second plan) and P&G is much better off (revenue of \$12,012 instead of \$11,885).

Q17.2 (Returning Books)

Part a

Use the newsvendor model. The overage cost is C_o = Cost − Salvage value = \$20 − \$28/4 = \$13. The underage cost is C_u = Price − Cost = \$28 − \$20 = \$8. The critical ratio is 8/(13 + 8) = 0.3810. Look up the critical ratio in the Standard Normal Distribution Function Table to find the appropriate z statistic = −0.30. The optimal order quantity is $Q = \mu + z \times \sigma = 100 - 0.30 \times 42 = 87$.

Part b

Expected lost sales = $L(z) \times \sigma = 0.5668 \times 42 = 23.81$, where we find $L(z)$ from the Standard Normal Loss Function Table and $z = -0.30$ (from part a). Expected sales = μ − Expected lost sales = 100 − 23.81 = 76.2. Expected leftover inventory = Q − Expected sales = 87 − 76.2 = 10.8. Profit = Price × Expected sales + Salvage value × Expected leftover inventory − Q × Cost = \$28 × 76.2 + \$7 × 10.8 − 87 × \$20 = \$469.

Part c

The publisher's profit = Q × (Wholesale price − Cost) = 87 × (\$20 − \$7.5) = \$1,087.5.

Part d

The underage cost remains the same because a lost sale still costs Dan the gross margin, C_u = \$8. However, the overage cost has changed because Dan can now return books to the publisher. He buys each book for \$20 and then returns leftover books for a net salvage value of \$15 − \$1 (due to the shipping cost) = \$14. So his overage cost is now C_o = Cost − Salvage value = \$20 − \$14 = \$6. The critical ratio is 8/(6 + 8) = 0.5714. Look up the critical ratio in the Standard Normal Distribution Function Table to find the appropriate z statistic = 0.18. The optimal order quantity is $Q = \mu + z \times \sigma = 100 + 0.18 \times 42 = 108$.

Part e

Expected lost sales = $L(z) \times \sigma = 0.3154 \times 42 = 13.2$, where we find $L(z)$ from the Standard Normal Loss Function Table and $z = 0.18$ (from part d). Expected sales = μ − Expected lost sales = 100 − 13.2 = 86.8. Expected leftover inventory = Q − Expected sales = 108 − 86.8 = 21.2. Profit = Price × Expected sales + Salvage value × Expected leftover inventory − Q × Cost = \$28 × 86.8 + \$14 × 21.2 − 108 × \$20 = \$567.

Part f

The publisher's sales revenue is \$20 × 108 = \$2,160. Production cost is \$7.5 × 108 = \$810. The publisher pays Dan \$15 × 21.2 = \$318. The publisher's total salvage revenue on returned books is \$6 × 21.2 = \$127.2. Profit is then \$2,160 − \$810 − \$318 + \$127.2 = \$1,159. Note that both the publisher and Dan are better off with this buy-back arrangement.

Part g

Equation (17.1) in the text gives the buy-back price that coordinates the supply chain (that is, maximizes the supply chain's profit). That buy-back price is \$1 + \$28 − (\$28 − \$20) × (\$28 − \$6)/(\$28 − \$7.5) = \$20.41. Note, the publisher's buy-back price is actually higher than the wholesale price because the publisher needs to subsidize Dan's shipping cost to return books: Dan's net loss on each book returned is \$20 − (20.41 − 1) = \$0.59.

Chapter 18

Q18.1 (Bauxite to New Zealand)

Total emissions on the journey is 1,400,000 × 38.2 = 53,480,000 kgs CO_2. The journey transports 300,000 × 3,000 = 900,000,000 tonne kms. So emissions are 53,480,000 kgs CO_2/900,000,000 tonne kms = 0.059 kgs CO_2 per tonne km.

Glossary

A

abandoning Refers to flow units leaving the process because of lengthy waiting times.

activity Value-adding steps in a process where resources process flow units.

activity on node (AON) representation A way to graphically illustrate the project dependencies in which activities correspond to nodes in a graph.

activity time The duration that a flow unit has to spend at a resource, not including any waiting time; also referred to as service time or processing time.

A/F ratio The ratio of actual demand (A) to forecasted demand (F). Used to measure forecast accuracy.

Andon cord A cord running adjacent to assembly lines that enables workers to stop production if they detect a defect. Just like the jidoka automatic shut-down of machines, this procedure dramatizes manufacturing problems and acts as a pressure for process improvements.

appointment Predefined times at which the flow unit supposedly enters the process; used to reduce variability (and seasonality) in the arrival process.

assemble-to-order Also known as make-to-order. A manufacturing system in which final assembly of a product only begins once a firm order has been received. Dell Inc. uses assemble-to-order with personal computers.

asset turns See capital turns.

assignable causes of variation Those effects that result in changes of the parameters of the underlying statistical distribution of the process. Thus, for assignable causes, a change in process performance is not driven simply by common-cause variation.

attribute-based control charts A special form of control chart that only distinguishes between defective and nondefective items. Such control charts should be used if it is not possible to capture the quality conformance of a process outcome in one variable.

authorization level For a fare class, the percentage of capacity that is available to that fare class or lower. An authorization level is equivalent to a booking limit expressed as a percentage of capacity.

availability The proportion of a time a process (single resource or buffer) is able to either process (in the case of a resource) or admit (in the case of a buffer) incoming flow units.

average labor utilization Measures the percentage of paid labor time that is spent on actual production as opposed to idle time; measures the efficiency of the process as well as the balance of work across workers.

B

back order If demand occurs and inventory is not available, then the demand can be back-ordered until inventory becomes available.

back-order penalty cost The cost incurred by a firm per back order. This cost can be explicit or implicit (e.g., lost goodwill and future business).

balancing resources Attempting to achieve an even utilization across the resources in a process. This is equivalent to minimizing idle time by reallocating work from one resource to another.

base stock level Also known as the order-up-to level. In the implementation of an order-up-to policy, inventory is ordered so that inventory position equals the base stock level.

batch A collection of flow units.

batch flow operation Those processes where flow units are batched to benefit from scale economies of production and/or transportation. Batch flow operations are known to have long flow times.

batch ordering A firm batch orders when it orders in integer multiples of a fixed batch size. For example, if a firm's batch size is 20 cases, then the firm orders either 0, 20, 40, 60, . . . cases.

bid price With bid price control, a bid price is assigned to each segment of capacity and a reservation is accepted only if its fare exceeds the bid prices of the segments of capacity that it uses.

bid price control A method for controlling whether or not to accept a reservation. This method explicitly recognizes that not all customers paying the same fare for a segment of capacity are equally valuable to the firm.

blocking The situation in which a resource has completed its work on a flow unit, yet cannot move the flow unit to the next step (resource or inventory) downstream as there is not space available.

booking limit The maximum number of reservations that are allowed for a fare class or lower.

bottleneck The resource with the lowest capacity in the process.

buckets A booking limit is defined for a bucket that contains multiple fare class–itinerary combinations.

buffer Another word for inventory, which is used especially if the role of the buffer is to maintain a certain throughput level despite the presence of variability.

buffer inventory Allows resources to operate independent from each other, thereby avoiding blocking and starving (in which case we speak of a decoupling buffer).

buffer-or-suffer principle The inherent tension between inventory and flow rate. In a process that suffers from setup times or variability, adding inventory can increase the flow rate.

bullwhip effect The propagation of demand variability up the supply chain.

business model Articulates how a firm aims to create a positive net utility to the customer while making a profit. Typically involves choices with respect to the process timing, the process location, and the process standardization.

business model innovation A substantial shift in a firm's business model either relative to others in the industry or to its previous practice.

buy-back contract A contract in which a supplier agrees to purchase leftover inventory from a retailer at the end of the selling season.

C

capability index The ratio between the tolerance level and the actual variation of the process.

capacity Measures the maximum flow rate that can be supported by a resource.

capacity-constrained A process for which demand exceeds the process capacity.

capacity pooling The practice of combining multiple capacities to deliver one or more products or services.

capital turns The ratio between revenues and invested capital; measures how much capital is needed to support a certain size of a company.

carbon dioxide equivalent (CO_2e) For a given gas, the weight of CO_2 that has the equivalent warming potential as 1 kilogram of that gas.

carbon footprint The total creation of carbon dioxide equivalents an organization is responsible for. Can be analyzed in the form of scope 1, 2, or 3.

channel assembly The practice in the PC industry of having final assembly completed by a distributor (e.g., Ingram Micron) rather than the manufacturer (e.g., IBM).

channel stuffing The practice of inducing retailers to carry more inventory than needed to cover short-term needs.

coefficient of variation A measure of variability. Coefficient of variation = Standard deviation divided by the mean; that is, the ratio of the standard deviation of a random variable to the mean of the random variable. This is a relative measure of the uncertainty in a random variable.

collaborative planning, forecasting, and replenishment A set of practices designed to improve the exchange of information within a supply chain.

common causes of variation Constant variation reflecting pure randomness in the process. Such causes are hence a result of pure randomness as opposed to being the result of an assignable cause.

conservation-of-matter law A law that states that, on average, the flow into a system must equal the flow out of the system, otherwise the quantity within the system will not be stable.

consignment inventory Inventory that is kept at a customer's location but is owned by the supplier.

consolidated distribution The practice of delivering from a supplier to multiple locations (e.g., retail stores) via a distribution center.

continuous process A process in which the flow unit continuously flows from one resource to the next; different from discrete process, in which the flow units are separate entities.

contract manufacturer A firm that manufactures or assembles a product for another firm. Contract manufacturers typically manufacture products from multiple competitors, they are generally responsible for procurement, but they do not design or distribute the products they assemble.

control charts Graphical tools to statistically distinguish between assignable and common causes of variation. Control charts visualize variation, thereby enabling the user to judge whether the observed variation is due to common causes or assignable causes.

control limits Part of control charts that indicate to what extent a process outcome falls in line with the common cause variation of the process versus being a result of an assignable cause. Outcomes above the upper control limit (UCL) or below the lower control limit (LCL) indicate the presence of an assignable cause.

cost of direct labor Measures the per-unit cost of labor, which includes both the labor content (the actual labor going into completing a flow unit) and the idle time that occurs across all workers per completed flow unit.

critical path A project management term that refers to all those activities that—if delayed—would delay the overall completion of the project.

critical ratio The ratio of the underage cost to the sum of the overage and underage costs. It is used in the newsvendor model to choose the expected profit-maximizing order quantity.

cross docking The practice of moving inventory in a distribution facility from the inbound dock directly to the outbound loading dock without placing the inventory in storage within the distribution facility.

cycle inventory The inventory that results from receiving (producing) several flow units in one order (batch) that are then used over a time period of no further inflow of flow units.

cycle time The time that passes between two consecutive flow units leaving the process. Cycle time = 1/Flow rate.

D

decision tree A scenario-based approach to map out the discrete outcomes of a particular uncertainty.

decoupling inventory See buffer inventory.

demand pooling combining demands from multiple sources to reduce overall variability.

discovery-driven planning A process that emphasizes learning about unknown variables related to a project with the goal of deciding whether or not to invest further resources in the project.

demand aggregation The idea to supply multiple demand streams with the same resource and thereby generate economies of scale.

demand-constrained A process for which the flow rate is limited by demand.

demand-pull An inventory policy in which demand triggers the ordering of replenishments.

density function The function that returns the probability the outcome of a random variable will exactly equal the inputted value.

dependency matrix Describes the dependencies among the activities in a project.

design specifications Establish how much a process outcome is allowed to vary before it is labeled a defect. Design specifications are driven by customer requirements, not by control limits.

distribution function The function that returns the probability the outcome of a random variable will equal the inputted value or lower.

diversion The practice by retailers of purchasing product from a supplier only to resell the product to another retailer.

diverters Firms that practice diversion.

double marginalization The phenomenon in a supply chain in which one firm takes an action that does not optimize supply chain performance because the firm's margin is less than the supply chain's total margin.

DuPont model A financial framework built around the idea that the overall ROIC of a business can be decomposed into two financial ratios, the margins and the asset turns.

E

earliest completion time (ECT) The earliest time a project activity can be completed, which can be computed as the sum of the earliest start time and the duration of the activity.

earliest start time (EST) The earliest time a project activity can start, which requires that all information providing activities are completed.

economies of scale Obtaining lower cost per unit based on a higher flow rate. Can happen, among other reasons, because of a spread of fixed cost, learning, statistical reasons (pooling), or the usage of dedicated resources.

Efficient Consumer Response The collective name given to several initiatives in the grocery industry to improve the efficiency of the grocery supply chain.

efficient frontier All locations in a space of performance measures (e.g., time and cost) that are efficient, that is, improvement along one dimension can occur only if the level of another dimension is reduced.

electronic data interchange (EDI) A technology standard for the communication between firms in the supply chain.

elimination of flow units Discarding defective flow units instead of reworking them.

e-lot system A decoupling buffer that is tracked to detect any systemic variation that could suggest a defect in the process; it is thereby a way to direct managerial attention toward defects in the process that would otherwise be hidden by inventory without immediately losing flow rate.

empirical distribution function A distribution function constructed with historical data.

EOQ (economic order quantity) The quantity that minimizes the sum of inventory costs and fixed ordering cost.

erlang loss formula Computes the proportion of time a resource has to deny access to incoming flow units in a system of multiple parallel servers and no space for inventory.

expected leftover inventory The expected amount of inventory left over at the end of the season when a fixed quantity is chosen at the start of a single selling season with random demand.

expected lost sales The expected amount of demand that is not satisfied when a fixed quantity is chosen at the start of a single selling season and demand is random.

Expected Marginal Seat Analysis A technique developed by Peter Belobaba of MIT to assign booking limits to multiple fare classes.

expected sales The expected amount of demand that is satisfied when a fixed quantity is chosen at the start of a single selling season and demand is random.

exponential distribution Captures a random variable with distribution $\text{Prob}\{x < t\} = 1 - \exp(-t/a)$, where a is the mean as well as the standard deviation of the distribution. If interarrival times are exponentially distributed, we speak of a Poisson arrival process. The exponential distribution is known for the memoryless property; that is, if an exponentially distributed service time with mean five minutes has been going on for five minutes, the expected remaining duration is still five minutes.

external setups Those elements of setup times that can be conducted while the machine is processing; an important element of setup time reduction/SMED.

F

FCFS (first-come, first-served) Rule that states that flow units are processed in the order of their arrivals.

fences Restrictions imposed on a low-fare class to prevent high-fare customers from purchasing the low fare. Examples include advanced purchase requirements and Saturday night stay over.

FIFO (first-in, first out) See FCFS.

fill rate The fraction of demand that is satisfied; that is, that is able to purchase a unit of inventory.

flexibility The ability of a process to meet changes in demand and/or a high amount of product variety.

flow rate (*R*) Also referred to as throughput. Flow rate measures the number of flow units that move through the process in a given unit of time. *Example:* The plant produces at a flow rate of 20 scooters per hour. Flow rate = Min{Demand, Capacity}.

flow time (*T*) Measures the time a flow unit spends in the process, which includes the time it is worked on at various resources as well as any time it spends in inventory. *Example:* A customer spends a flow time of 30 minutes on the phone in a call center.

flow unit The unit of analysis that we consider in process analysis; for example, patients in a hospital, scooters in a kick-scooter plant, and callers in a call center.

forecast The estimate of demand and potentially also of the demand distribution.

forward buying If a retailer purchases a large quantity during a trade promotion, then the retailer is said to forward buy.

G

gamma distribution A continuous distribution. The sum of exponential distributions is a gamma distribution. This is a useful distribution to model demands with high coefficients of variation (say about 0.5).

Gantt chart A graphical way to illustrate the durations of activities as well as potential dependencies between the activities.

H

heijunka A principle of the Toyota Production System, proposing that models are mixed in the production process according to their mix in customer demand.

hockey stick phenomenon A description of the demand pattern that a supplier can receive when there is a substantial amount of order synchronization among its customers.

holding cost rate The cost incurred to hold one unit of inventory for one period of time.

horizontal pooling Combining a sequence of resources in a queuing system that the flow unit would otherwise visit sequentially; increases the span of control; also related to the concept of a work cell.

I

idle time The time a resource is not processing a flow unit. Idle time should be reduced as it is a non-value-adding element of labor cost.

ikko-nagashi An element of the Toyota Production System. It advocates the piece-by-piece transfer of flow units (transfer batches of one).

implied utilization The workload imposed by demand of a resource relative to its available capacity. Implied utilization = Demand rate/Capacity.

incentive conflicts In a supply chain, firms may have conflicting incentives with respect to which actions should be taken.

independent arrivals A requirement for both the waiting time and the loss formulas. Independent arrivals mean that the probability of having an arrival occur in the next x minutes is independent of how many arrivals have occurred in the last y minutes.

information turnaround time (ITAT) The delay between the occurrence of a defect and its detection.

innovation A novel match between a solution and a need that creates value.

in-stock probability The probability all demand is satisfied over an interval of time.

integrated supply chain The supply chain considered as a single integrated unit, that is, as if the individual firms were owned by a single entity.

interarrival time The time that passes between two consecutive arrivals.

internal setups Those elements of setup times that can only be conducted while the machine is not producing. Internal setups should be reduced as much as possible and/or converted to external setups wherever possible (SMED).

inventory (*I*) The number of flow units that are in the process (or in a particular resource). Inventory can be expressed in (a) flow units (e.g., scooters), (b) days of supply (e.g., three days of inventory), or (c) monetary units ($1 million of inventory).

inventory cost The cost a process incurs as a result of inventory. Inventory costs can be computed on a per-unit basis (see Exhibit 2.1) or on a per-unit-of-time basis.

inventory level The on-hand inventory minus the number of units back-ordered.

inventory policy The rule or method by which the timing and quantity of inventory replenishment are decided.

inventory position The inventory level plus the number of units on order.

inventory turns How often a company is able to turn over its inventory. Inventory turns = 1/Flow time, which—based on Little's Law—is COGS/Inventory.

Ishikawa diagram Also known as fishbone diagram or cause–effect diagram; graphically represents variables that are causally related to a specific outcome.

J

jidoka In the narrow sense, a specific type of machine that can automatically detect defects and automatically shut down itself. The basic idea is that shutting down the machine forces human intervention in the process, which in turn triggers process improvement.

just-in-time The idea of producing units as close as possible to demand, as opposed to producing the units earlier and then leaving them in inventory or producing them later and thereby leaving the unit of demand waiting. Just-in-time is a fundamental part of the Toyota Production System as well as of the matching supply with demand framework postulated by this book.

K

kaizen The continuous improvement of processes, typically driven by the persons directly involved with the process on a daily basis.

kanban A production and inventory control system in which the production and delivery of parts are triggered by the consumption of parts downstream (pull system).

key performance indicator An operational variable with a strong marginal impact on ROIC (value driver) that is used as an indicator of current operational performance.

L

labor content The amount of labor that is spent on a flow unit from the beginning to the end of the process. In a purely manual process, we find labor content as the sum of all the activity times.

labor productivity The ratio between revenue and labor cost.

labor utilization How well a process uses the labor involved in the process. Labor utilization can be found based on activity times and idle times.

latest completion time (LCT) The latest time a project activity has to be completed by to avoid delaying the overall completion time of the project.

latest start time (LST) The latest time a project activity can start without delaying the overall completion time of the project.

lead time The time between when an order is placed and when it is received. Process lead time is frequently used as an alternative word for flow time.

lean operations An operations paradigm built around the idea of waste reduction; inspired by the Toyota Production System.

line balancing The process of evenly distributing work across the resources of a process. Line balancing reduces idle time and can (a) reduce cycle time or (b) reduce the number of workers that are needed to support a given flow rate.

Little's Law The average inventory is equal to the average flow rate times the average flow time ($I = R \times T$).

location pooling The combination of inventory from multiple locations into a single location.

loss function A function that returns the expected number of units by which a random variable exceeds the inputted value.

M

machine-paced line A process design in which flow units are moved from one resource to another by a constant speed dictated by the conveyor belt. There is typically no inventory between the resources connected by a conveyor belt.

make-to-order A production system, also known as assemble-to-order, in which flow units are produced only once the customer order for that flow unit has been received. Make-to-order production typically requires wait times for the customer, which is why it shares many similarities with service operations. Dell Inc. uses make-to-order with personal computers.

make-to-stock A production system in which flow units are produced in anticipation of demand (forecast) and then held in finished goods inventory.

margin arithmetic Equations that evaluate the percentage increase in profit as a function of the gross margin and the percentage increase in revenue.

marginal cost pricing The practice of setting the wholesale price to the marginal cost of production.

materials requirement planning A system to control the timing and quantity of component inventory replenishment based on forecasts of future demand and production schedules.

maximum profit In the context of the newsvendor model, the expected profit earned if quantity can be chosen after observing demand. As a result, there are no lost sales and no leftover inventory.

mean The expected value of a random variable.

mismatch cost The sum of the underage cost and the overage cost. In the context of the newsvendor model, the mismatch cost is the sum of the lost profit due to lost sales and the total loss on leftover inventory.

mixed model production See heijunka.

MRP jitters The phenomenon in which multiple firms operate their MRP systems on the same cycle, thereby creating order synchronization.

muda One specific form of waste, namely waste in the form of non-value-adding activities. Muda also refers to unnecessary inventory (which is considered the worst form of muda), as unnecessary inventory costs money without adding value and can cover up defects and other problems in the process.

multiple flow units Used in a process that has a mix of products or customers flowing through it. Most computations, including the location of the bottleneck, depend on the mix of products.

N

negative binomial distribution A discrete distribution function with two parameters that can independently change the mean of the distribution as well as the standard deviation. In contrast, the Poisson distribution has only one parameter and can only regulate its mean.

nested booking limits Booking limits for multiple fare classes are nested if each booking limit is defined for a fare class or lower. With nested booking limits, it is always the case that an open fare class implies all higher fare classes are open and a closed fare class implies all lower fare classes are closed.

newsvendor model A model used to choose a single order quantity before a single selling season with stochastic demand.

nonlinear The relationship between variables if the graph of the two variables is not a straight line.

normal distribution A continuous distribution function with the well-known bell-shaped density function.

no-show A customer that makes a reservation but cancels or fails to arrive for service.

O

on allocation A product whose total amount demanded exceeds available capacity.

on-hand inventory The number of units currently in inventory.

on-order inventory Also known as pipeline inventory. The number of units of inventory that have been ordered but have not been received.

one-for-one ordering policy Another name for an order-up-to policy. (With this policy, one unit is ordered for every unit of demand.)

options contract With this contract, a buyer pays a price per option purchased from a supplier and then pays an additional price later on to exercise options. The supplier is responsible for building enough capacity to satisfy all of the options purchased in case they are all exercised.

order batching A cause of the bullwhip effect. A firm order batches when it orders only in integer multiples of some batch quantity.

order inflation The practice of ordering more than desired in anticipation of receiving only a fraction of the order due to capacity constraints upstream.

order synchronization A cause of the bullwhip effect. This describes the situation in which two or more firms submit orders at the same moments in time.

order-up-to level Also known as the base stock level. In the implementation of an order-up-to policy, inventory is ordered so that inventory position equals the order-up-to level.

order-up-to model A model used to manage inventory with stochastic demand, positive lead times, and multiple replenishments.

origin-destination control A revenue management system in the airline industry that recognizes passengers that request the same fare on a particular segment may not be equally valuable to the firm because they differ in their itinerary and hence total revenue.

out-of-stock When a firm has no inventory.

overage cost In the newsvendor model, the cost of purchasing one too many units. In other words, it is the increase in profit if the firm had purchased one fewer unit without causing a lost sale (i.e., thereby preventing one additional unit of leftover inventory).

overbooking The practice of accepting more reservations than can be accommodated with available capacity.

P

pallet Literally the platform used (often wood) by a forklift to move large quantities of material.

par level Another name for the order-up-to level in the order-up-to model.

Pareto principle Principle that postulates that 20 percent of the causes account for 80 percent of all problems (also known as the 80-20 rule).

period In the order-up-to model, time is divided into periods of time of equal length. Typical period lengths include one day, one week, and one month.

phantom orders An order that is canceled before delivery is taken.

pipeline inventory The minimum amount of inventory that is required to operate the process. Since there is a minimum flow time that can be achieved (i.e., sum of the activity times), because of Little's Law, there is also a minimum required inventory in the process. Also known as on-order inventory, it is the number of units of inventory that have been ordered but have not been received.

point-of-sale (POS) Data on consumer transactions.

Poisson distribution A discrete distribution function that often provides an accurate representation of the number of events in an interval of time when the occurrences of the events are independent of each other. In other words, it is a good distribution to model demand for slow-moving items.

Poisson process An arrival process with exponentially distributed interarrival times.

poka-yoke A Toyota technique of "fool-proofing" many assembly operations, that is, by making mistakes in assembly operations physically impossible.

pooling The concept of combining several resources (including their buffers and their arrival processes) into one joint resource. In the context of waiting time problems, pooling reduces the expected wait time.

preference fit The firm's ability to provide consumers with the product or service they want or need.

price protection The practice in the PC industry of compensating distributors due to reductions in a supplier's wholesale price. As a result of price protection, the price a distributor pays to purchase inventory is effectively always the current price; that is, the supplier rebates the distributor

whenever a price reduction occurs for each unit the distributor is holding in inventory.

precedence relationship The connection between two activities in a project in which one activity must be completed before the other can begin.

priority rules Used to determine the sequence with which flow units waiting in front of the same resource are served. There are two types of priority rules: service time independent (e.g., FCFS rule) and service time dependent (e.g., SPT rule).

process Resources, inventory locations, and a flow that describe the path of a flow unit from its transformation as input to output.

process analysis Concerned with understanding and improving business processes. This includes determining the location of the bottleneck and computing the basic performance measures inventory, flow rate, and flow time.

process capability The tolerance level of the process relative to its current variation in outcomes. This is frequently measured in the form of the capability index.

process capacity Capacity of an entire process, which is the maximum flow rate that can be achieved in the process. It is based on the capacity of the bottleneck.

process flow diagram Maps resources and inventory and shows graphically how the flow unit travels through the process in its transformation from input to output.

process utilization To what extent an entire process uses its capacity when supporting a given flow rate. Process utilization = Flow rate/Process capacity.

processing time The duration that a flow unit has to spend at a resource, not including any waiting time; also referred to as activity time or service time.

product pooling The practice of using a single product to serve two demand segments that were previously served by their own product version.

production batch The collection of flow units that are produced within a production cycle.

production cycle The processing and setups of all flow units before the resource starts to repeat itself.

production smoothing The practice of smoothing production relative to demand. Used to manage seasonality. In anticipation of a peak demand period production is maintained above the current flow rate, building up inventory. During the peak demand, production is not increased substantially, but rather, the firm satisfies the gap by drawing down previously accumulated inventory.

productivity ratio The ratio between revenue as a measure of output and some cost (for example labor cost) as a measure of input.

project A temporary (and thus nonrepetitive) operation consisting of a set of activities; different from a process, which is a repetitive operation.

protection level The number of reservations that must always be available for a fare class or higher. For example, if a flight has 120 seats and the protection level is 40 for the high-fare class, then it must always be possible to have 40 high-fare reservations.

pull system A manufacturing system in which production is initiated by the occurrence of demand.

push system A manufacturing system in which production is initiated in anticipation of demand.

Q

quality at the source The idea of fixing defects right when and where they occur. This is a fundamental idea of the Toyota Production System. Fixing defects later on in the process is difficult and costly.

quantity discount Reduced procurement costs as a result of large order quantities. Quantity discounts have to be traded off against the increased inventory costs.

quantity flexibility contracts With this contract, a buyer provides an initial forecast to a supplier. Later on the buyer is required to purchase at least a certain percentage of the initial forecast (e.g., 75 percent), but the buyer also is allowed to purchase a certain percentage above the forecast (e.g., 125 percent of the forecast). The supplier must build enough capacity to be able to cover the upper bound.

quartile analysis A technique to empirically analyze worker productivity (e.g., in the form of their processing times) by comparing the performance of the top quartile with the performance of the bottom quartile.

queue The accumulation of flow units waiting to be processed or served.

queuing system A sequence of individual queues in which the outflow of one buffer/server is the inflow to the next buffer/server.

Quick Response A series of practices in the apparel industry used to improve the efficiency of the apparel supply chain.

R

random variable A variable that represents a random event. For example, the random variable X could represent the number of times the value 7 is thrown on two dice over 100 tosses.

range of a sample The difference between the highest and the lowest value in the sample.

R-bar charts Track the variation in the outcome of a process. R-bar charts require that the outcome of a process be evaluated based on a single variable.

reactive capacity Capacity that can be used after useful information regarding demand is learned; that is, the capacity can be used to react to the learned demand information.

resource The entity of a process that the flow unit has to visit as part of its transformation from input to output.

returns policy See buy-back contract.

revenue management Also known as yield management. The set of tools used to maximize revenue given a fixed supply.

revenue-sharing contracts With this contract, a retailer pays a supplier a wholesale price per unit purchased plus a fraction of the revenue the retailer realizes from the unit.

rework An approach of handling defective flow units that attempts to invest further resource time into the flow unit in the attempt to transform it into a conforming (nondefective) flow unit.

rework loops An iteration/repetition of project or process activities done typically because of quality problems.

ROIC Return on invested capital, defined as the ratio between financial returns (profits) and the invested capital.

round-up rule When looking for a value inside a table, it often occurs that the desired value falls between two entries in the table. The round-up rule chooses the entry that leads to the larger quantity.

S

safety inventory The inventory that a firm holds to protect itself from random fluctuations in demand.

salvage value The value of leftover inventory at the end of the selling season in the newsvendor model.

scale economies Cost savings that can be achieved in large operations. Examples are pooling benefits in waiting time problems and lower per-unit setup costs in batch flow operations.

scope 1 emissions All direct emissions of an organization, such as fuel burned in its own trucks, or oil burned in its own machines.

scope 2 emissions All emissions of an organization associated with purchased electricity, heat, steam, or other forms of energy.

scope 3 emissions All emissions of an organization, including emissions created upstream in the value chain (suppliers) as well as downstream in the value chain (customers using the product or service).

seasonal arrivals Systemic changes in the interarrival times (e.g., peak times during the day, the week, or the year).

seasonal inventory Arises if the flow rate exceeds the demand rate in anticipation of a time period when the demand rate exceeds the flow rate.

second buy An opportunity to request a second replenishment, presumably after some demand information is learned.

service level The probability with which a unit of incoming demand will receive service as planned. In the context of waiting time problems, this means having a waiting time less than a specified target wait time; in other contexts, this also can refer to the availability of a product.

service time The duration that a flow unit has to spend at a resource, not including any waiting time; also referred to as activity time or processing time.

setup cost Costs that are incurred in production whenever a resource conducts a setup and in transportation whenever a shipment is done. Setup costs drive batching. It is important to include only out-of-pocket costs in the setup costs, not opportunity costs.

setup time The duration of time a resource cannot produce as it is either switched from one setting to the other (e.g., from producing part A to producing part B, in which case we speak of a changeover time) or not available for production for other reasons (e.g., maintenance step). Setup times reduce capacity and therefore create an incentive to produce in batches.

setup time reduction See SMED.

shortage gaming A cause of the bullwhip effect. In situations with a capacity constraint, retailers may inflate their orders in anticipation of receiving only a portion of their order.

single segment control A revenue management system in the airline industry in which all passengers on the same segment paying the same fare class are treated equally.

six sigma In its narrow sense, refers to a process capability of two. This means that a process outcome can fall six standard deviations above or below the mean and still be within tolerance (i.e., still not be a defect). In its broader meaning, refers to quality improvement projects that are using statistical process control.

slack time The difference between the earliest completion time and the latest completion time; measures by how much an activity can be delayed without delaying the overall project.

SMED (single minute exchange of dies) The philosophy of reducing setup times instead of just finding optimal batch sizes for given setup times.

span of control The scope of activities a worker or a resource performs. If the resource is labor, having a high span of control requires extensive training. Span of control is largest in a work cell.

specification levels The cut-off points above (in the case of upper specification level) and below (in the case of lower specification level) which a process outcome is labeled a defect.

SPT (shortest processing time) rule A priority rule that serves flow units with the shortest processing time first. The SPT rule is known to minimize the overall waiting time.

standard deviation A measure of the absolute variability around a mean. The square of the standard deviation equals the variance.

standard normal A normal distribution with mean 0 and standard deviation 1.

starving The situation in which a resource has to be idle as there is no flow unit completed in the step (inventory, resource) upstream from it.

stationary arrivals When the arrival process does not vary systemically over time; opposite of seasonal arrivals.

statistical process control (SPC) A set of statistical tools that is used to measure the capability of a process and to help monitor the process, revealing potential assignable causes of variation.

stochastic An event that is random, that is, its outcome cannot be predicted with certainty.

stockout Occurs if a customer demands a unit but a unit of inventory is not available. This is different from "being out of stock," which merely requires that there is no inventory available.

stockout probability The probability a stockout occurs over a predefined interval of time.

supply chain The series of firms that deliver a good or service from raw materials to customer fulfillment.

supply chain efficiency The ratio of the supply chain's actual profit to the supply chain's optimal profit.

supply-constrained A process for which the flow rate is limited by either capacity or the availability of input.

sustainable operations An operations paradigm built around the idea of not depleting or destroying scarce resources, including the atmosphere, water, materials, land, and people.

T

takotei-mochi A Toyota technique to reduce worker idle time. The basic idea is that a worker can load one machine and while this machine operates, the worker—instead of being idle—operates another machine along the process flow.

tandem queue A set of queues aligned in a series so that the output of one server flows to only one other server.

target wait time (TWT) The wait time that is used to define a service level concerning the responsiveness of a process.

tasks The atomic pieces of work that together constitute activities. Tasks can be moved from one activity/resource to another in the attempt to improve line balance.

throughput See flow rate.

tolerance levels The range of acceptable outcomes of a process. See also design specifications.

Toyota Production System A collection of practices related to production, product development, and supply chain management as developed by the Toyota Motor Corporation. Important elements discussed in this book are the idea of permanent improvement (kaizen), the reduction of waste (muda), inventory reduction (just-in-time, kanban), mixed model production (heijunka), and reduction of setup times (SMED).

trade promotion A temporary price discount off the wholesale price that a supplier offers to its retailer customers.

transactional efficiency Measures how easy it is to do business with a firm. Consists of the subdimension customer effort and the elapsed time between need articulation and need fulfillment.

transfer batch A collection of flow units that are transferred as a group from one resource to the next.

trunk inventory The inventory kept by sales representatives in the trunk of their vehicles.

tsukurikomi The Toyota idea of integrating quality inspection throughout the process. This is therefore an important enabler of the quality-at-the-source idea.

turn-and-earn An allocation scheme in which scarce capacity is allocated to downstream customers proportional to their past sales.

U

underage cost In the newsvendor model, the profit loss associated with ordering one unit too few. In other words, it is the increase in profit if one additional unit had been ordered and that unit is sold.

universal design/product A product that is designed to serve multiple functions and/or multiple customer segments.

unknown unknowns (unk-unks) Project management parlance to refer to uncertainties in a project that are not known at the outset of the project.

utilization The extent to which a resource uses its capacity when supporting a given flow rate. Utilization = Flow rate/Capacity.

V

value chain See supply chain.

value curve A graphical depiction of the key attributes of a product or service. Can be used to compare a new business model with an incumbent solution. Common attributes are the transactional efficiency, the preference fit, as well as price and quality.

value driver An operational variable that has a strong marginal effect on the ROIC.

variance A measure of the absolute variability around a mean. The square root of the variance equals the standard deviation.

vendor-managed inventory The practice of switching control of inventory management from a retailer to a supplier.

virtual nesting A revenue management system in the airline industry in which passengers on different itineraries and paying different fare classes may nevertheless be included in the same bucket for the purchase of capacity controls.

virtual pooling The practice of holding inventory in multiple physical locations that share inventory information data so that inventory can be moved from one location to another when needed.

W

waiting time The part of flow time in which the flow unit is not processed by a resource.

waiting time formula The average wait time, T_q, that a flow unit spends in a queue before receiving service.

waste An abstract word that refers to any inefficiencies that exist in the process; for example, line imbalances, inadequate batch sizes, variability in service times, and so forth. Waste can be seen as the distance between the current performance of a process and the efficient frontier. Waste is called "muda" in the Toyota Production System.

win–win A situation in which both parties in a negotiation are better off.

work cell A resource where several activities that were previously done by separate resources (workers, machines) are combined into a single resource (team of workers). Work cells have several quality advantages, as they have a short ITAT; they are also—by definition—more balanced.

work in process (WIP) The inventory that is currently in the process (as opposed to inventory that is finished goods or raw material).

worker-paced line A process layout in which a worker moves the flow unit to the next resource or buffer when he or she has completed processing it; in contrast to a machine-paced line, where the flow unit moves based on a conveyor belt.

workload The request for capacity created by demand. Workload drives the implied utilization.

X

X-bar charts Track the mean of an outcome of a process. X-bar charts require that the outcome of a process be evaluated based on a single variable.

Y

yield management Also known as revenue management. The set of tools used to maximize revenue given a fixed supply.

yield of a resource The percentage of flow units processed correctly at the resource. More generally, we also can speak of the yield of an entire process.

Z

zero-sum game A game in which the total payoff to all players equals a constant no matter what outcome occurs.

***z*-statistic** Given quantity and any normal distribution, that quantity has a unique *z*-statistic such that the probability the outcome of the normal distribution is less than or equal to the quantity equals the probability the outcome of a standard normal distribution equals the *z*-statistic.

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Index of Key "How to" Exhibits

Summary of Key Notation and Equations

Chapter 2: Process Flow

$$\text{Little's Law: Average inventory} = \text{Average flow rate} \times \text{Average time}$$

Chapter 3: Capacity/Bottleneck Analysis

$$\text{Implied utilization} = \frac{\text{Capacity requested by demand}}{\text{Available capacity}}$$

Chapter 4: Labor Content

$$\text{Flow rate} = \text{Min}\{\text{Available input}, \text{Demand}, \text{Process capacity}\}$$

$$\text{Cycle time} = \frac{1}{\text{Flow rate}}$$

$$\text{Cost of direct labor} = \frac{\text{Total wages}}{\text{Flow rate}}$$

$$\text{Idle time across all workers at resource } i = \text{Cycle time} \times (\text{Number of workers at resource } i) - \text{Processing time at resource } i$$

$$\text{Average labor utilization} = \frac{\text{Labor content}}{\text{Labor content} + \text{Total idle time}}$$

Chapter 7: Batching

$$\text{Capacity given batch size} = \frac{\text{Batch size}}{\text{Setup time} + \text{Batch size} \times \text{Time per unit}}$$

$$\text{Recommended batch size} = \frac{\text{Flow rate} \times \text{Setup time}}{1 - \text{Flow rate} \times \text{Time per unit}}$$

$$\text{Economic order quantity} = \sqrt{\frac{2 \times \text{Setup cost} \times \text{Flow rate}}{\text{Holding cost}}}$$

Chapter 8: Waiting Time Systems

m = number of servers

p = processing time

a = interarrival time

CV_a = coefficient of variation for interarrivals

CV_p = coefficient of variation of processing time

$$\text{Utilization } u = \frac{p}{a \times m}$$

$$T_q = \left(\frac{\text{Processing time}}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2(m+1)}-1}}{1 - \text{Utilization}}\right) \times \left(\frac{\text{CV}_a^2 + \text{CV}_p^2}{2}\right)$$

$$\text{Flow time } T = T_q + p$$

$$\text{Inventory in service } I_p = m \times u$$

$$\text{Inventory in the queue } I_q = T_q/a$$

$$\text{Inventory in the system } I = I_p + I_q$$

Chapter 10: Quality

$$\text{Yield of resource} = \frac{\text{Flow rate of units processed correctly at the resource}}{\text{Flow rate}}$$

Chapter 12: Newsvendor

Q = order quantity

C_u = Underage cost

C_o = Overage cost

μ = Expected demand

σ = Standard deviation of demand

$F(Q)$ = Distribution function

$\Phi(Q)$ = Distribution function of the standard normal

$L(Q)$ = Loss function

$L(z)$ = Loss function of the standard normal distribution

$$\text{Critical ratio} = \frac{C_u}{C_o + C_u}$$

$$\text{A/F ratio} = \frac{\text{Actual demand}}{\text{Forecast}}$$

$$\text{Expected profit-maximizing order quantity: } F(Q) = \frac{C_u}{C_o + C_u}$$

$$z\text{-statistic or normalized order quantity: } z = \frac{Q - \mu}{\sigma}$$

$$Q = \mu + z \times \sigma$$

$$\text{Expected lost sales with a standard normal distribution} = L(z)$$

$$\text{Expected lost sales with a normal distribution} = \sigma \times L(z)$$

$$\text{In Excel: } L(z) = \sigma\text{*Normdist}(z,0,1,0) - z\text{*}(1 - \text{Normdist}(z))$$

$$\text{Expected lost sales for nonnormal distributions} = L(Q) \text{ (from loss function table)}$$

$$\text{Expected sales} = \mu - \text{Expected lost sales}$$

$$\text{Expected leftover inventory} = Q - \text{Expected sales}$$

$$\begin{aligned}\text{Expected profit} = {} & [(\text{Price} - \text{Cost}) \times \text{Expected sales}] \\ & - [(\text{Cost} - \text{Salvage value}) \times \text{Expected leftover inventory}]\end{aligned}$$

$$\text{In-stock probability} = F(Q)$$

$$\text{Stockout probability} = 1 - \text{In-stock probability}$$

$$\text{In Excel: In-stock probability} = \text{Normdist}(z)$$

$$\text{In Excel: } z = \text{Normsinv(Target in-stock probability)}$$

Chapter 13: Reactive Capacity

$$\begin{aligned}\text{Mismatch cost} &= (C_o \times \text{Expected leftover inventory}) + (C_u \times \text{Expected lost sales}) \\ &= \text{Maximum profit} - \text{Expected profit}\end{aligned}$$

$$\text{Maximum profit} = (\text{Price} - \text{Cost}) \times \mu$$

$$\text{Coefficient of variation} = \text{Standard deviation/Expected demand}$$

Chapter 14: Order-up-to Model

l = Lead time

S = Order-up-to level

$$\begin{aligned}\text{Inventory level} &= \text{On-hand inventory} - \text{Back order} \\ \text{Inventory position} &= \text{On-order inventory} + \text{Inventory level} \\ \text{In-stock probability} &= 1 - \text{Stockout probability} \\ &= \text{Prob}\{\text{Demand over } (l+1) \text{ periods} \le S\}\end{aligned}$$

$$z\text{-statistic for normalized order quantity: } z = \frac{S - \mu}{\sigma}$$

$$\text{Expected back order with a normal distribution} = \sigma \times L(z)$$

$$\text{In Excel: Expected back order} = \sigma\text{*}(\text{Normdist}(z,0,1,0) - z\text{*}(1 - \text{Normdist}(z)))$$

$$\text{Expected back order for nonnormal distributions} = L(S) \text{ (from loss function table)}$$

$$\text{Expected inventory} = S - \text{Expected demand over } l+1 \text{ periods} + \text{Expected back order}$$

$$\text{Expected on-order inventory} = \text{Expected demand in one period} \times \text{Lead time}$$

Chapter 15: Pooling

$$\text{Expected pooled demand} = 2 \times \mu$$

$$\text{Standard deviation of pooled demand} = \sqrt{2 \times (1 + \text{Correlation})} \times \sigma$$

$$\text{Coefficient of variation of pooled demand} = \sqrt{\frac{1}{2}(1 + \text{Correlation})} \times \left(\frac{\sigma}{\mu}\right)$$

Chapter 16: Revenue Management

$$\text{Protection level: Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_h - r_l}{r_h}$$

$$\text{Low-fare booking limit} = \text{Capacity} - Q$$

$$\text{Overbooking: Critical ratio} = \frac{C_u}{C_o + C_u} = \frac{r_l}{\text{Cost per bumped customer} + r_l}$$

Index

General index for Cachon, *Matching Supply with Demand*